

# EMISSION MECHANISMS

## LESSON 4

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CHIARA FERRARI

REFERENCE TEXT:

“ASTROPHYSICAL PROCESSES” BY H. BRADT  
CAMBRIDGE UNIVERSITY PRESS (2008)



Observatoire  
de la CÔTE d'AZUR

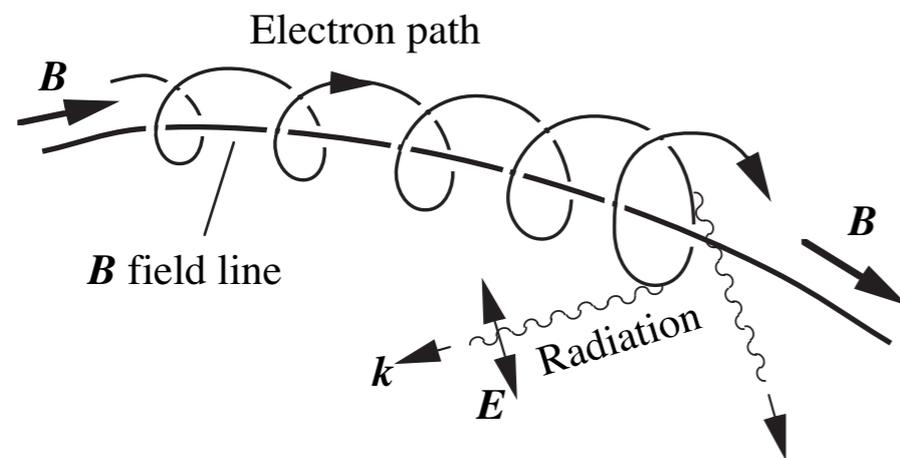


# MAGNETOBREMSSTRAHLUNG

## *Gyro, cyclotron and synchrotron radiation*

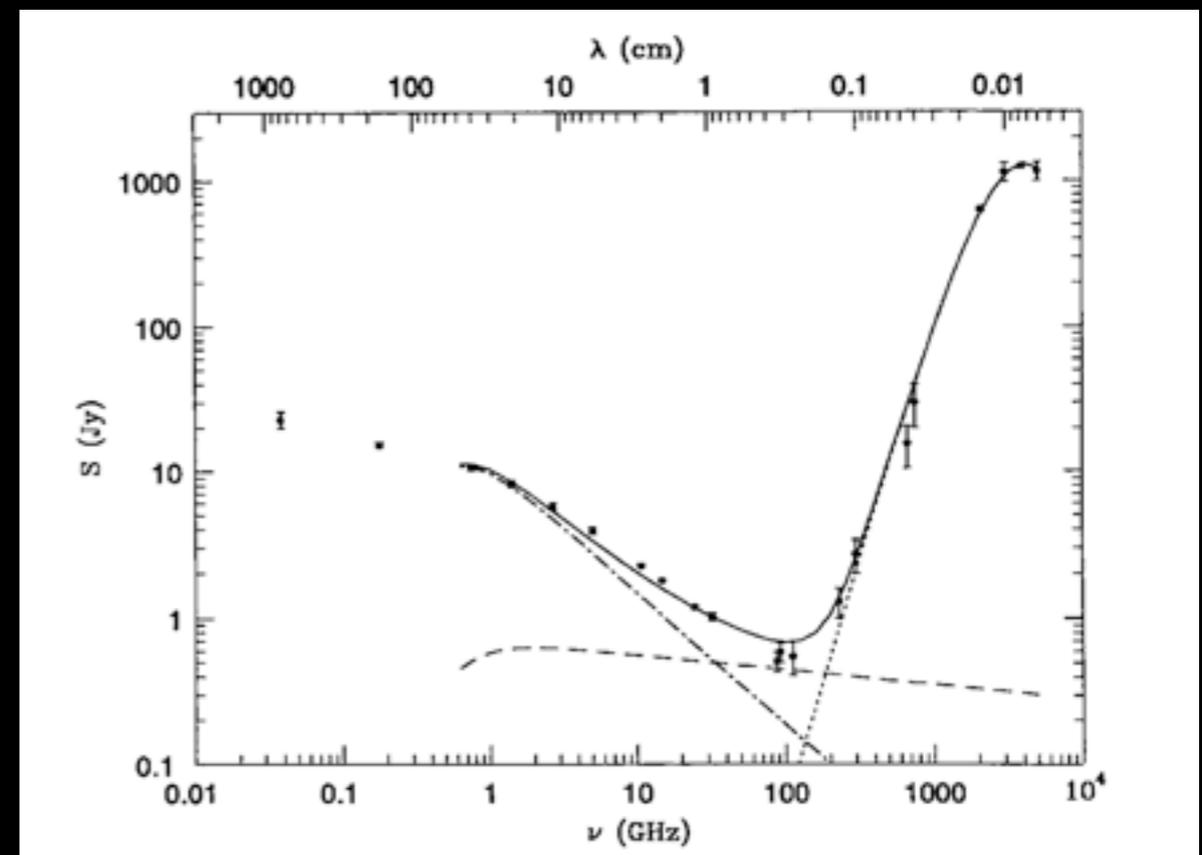
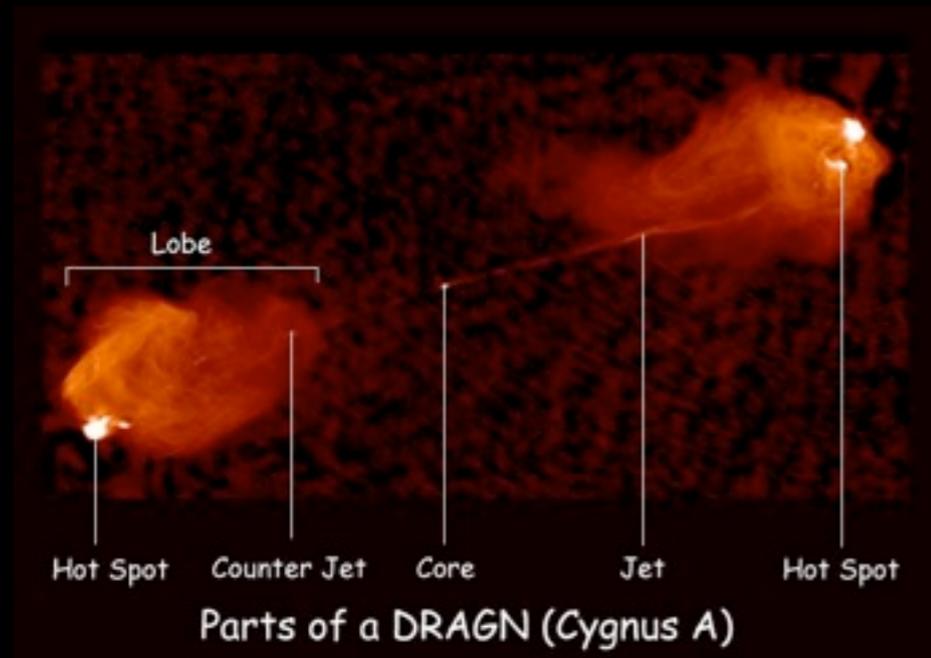
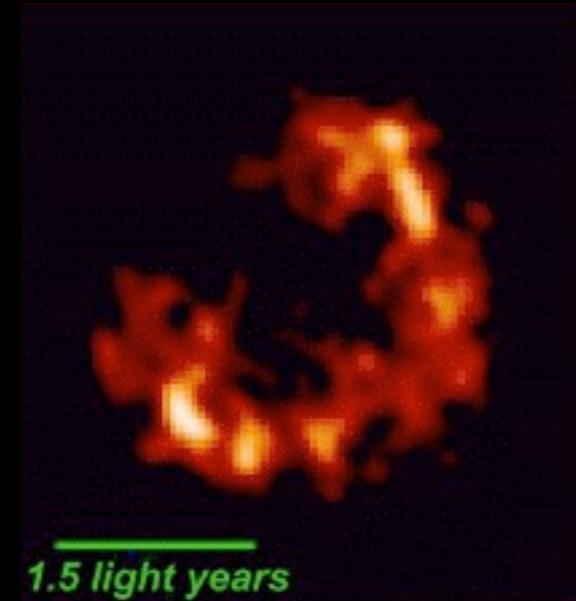
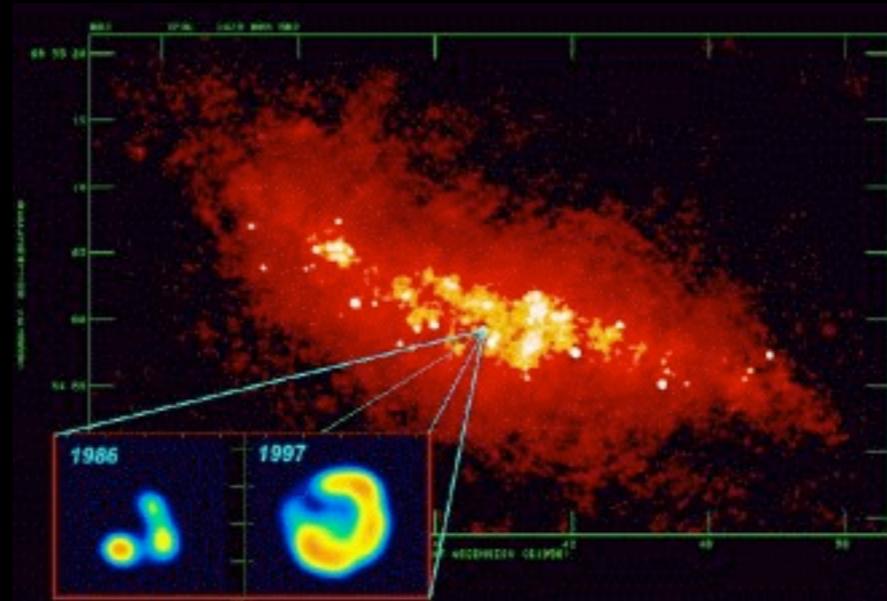
If acceleration of a charged particle by an electric field accounts for Bremsstrahlung radiation, acceleration by a magnetic field produces **Magnetobremstrahlung**

Light charged particles (e.g. electrons) are more easily accelerated than heavier particles (e.g. protons)

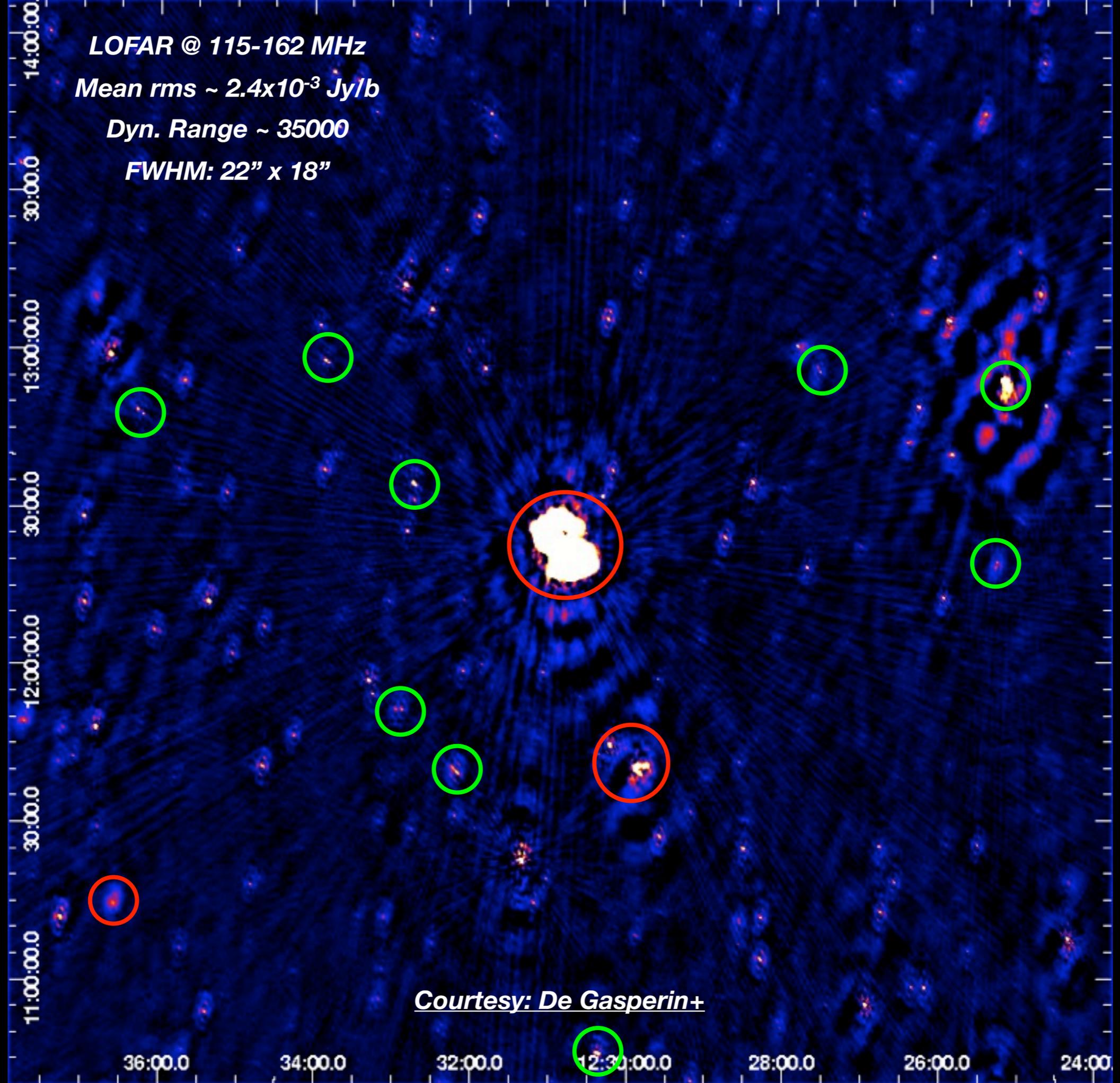


- ▶ **Gyro radiation** - Non relativistic electrons  
[ $v \ll c$ ]
- ▶ **Cyclotron radiation** - Mildly relativistic electrons  
[ $(\gamma-1)mc^2 \sim mc^2$ ]
- ▶ **Synchrotron radiation** - Ultra-relativistic electrons  
[ $(\gamma-1)mc^2 \gg mc^2$ ]

# EXTRA-GALACTIC SOURCES OF SYNCHROTRON RADIATION

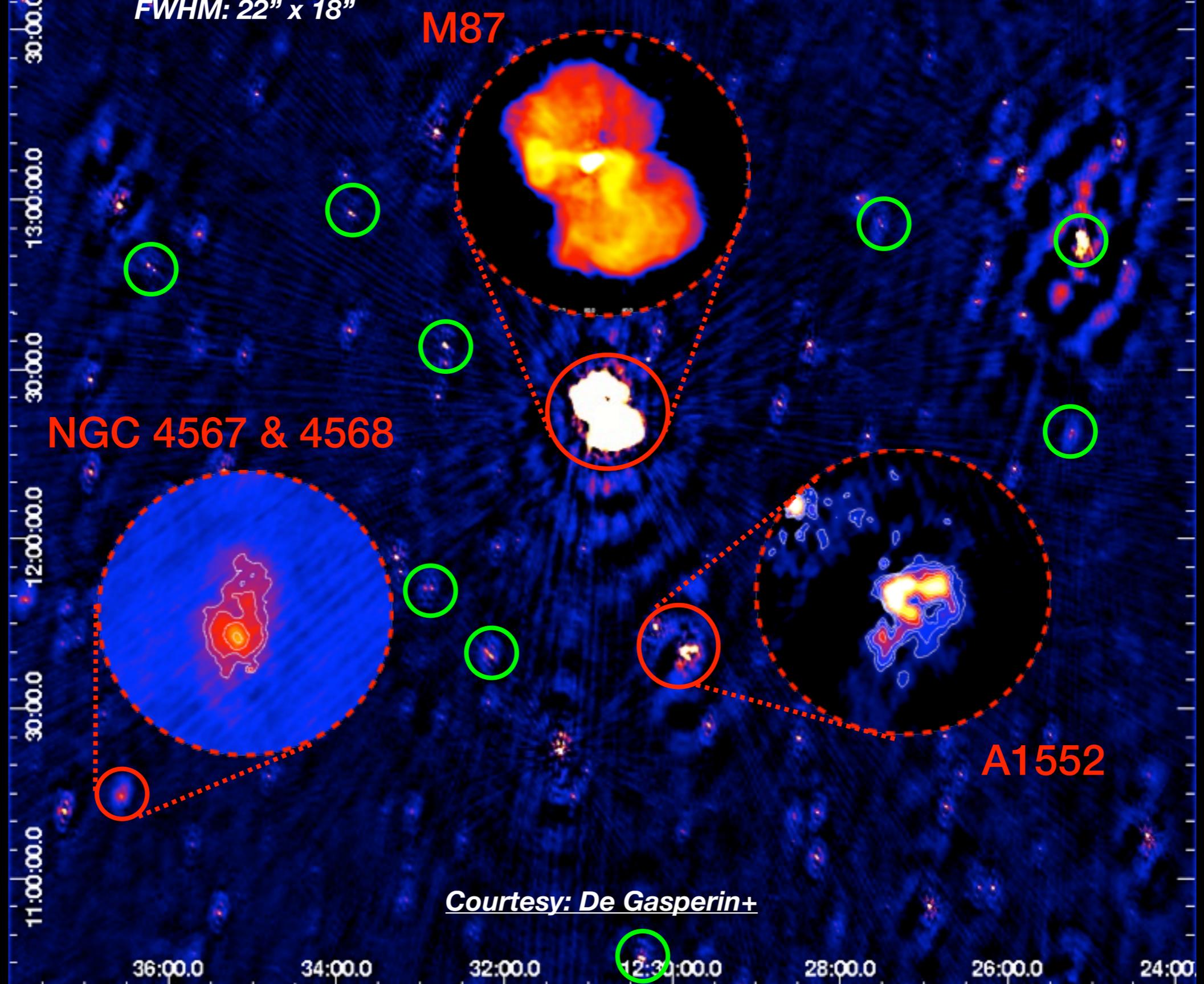


**LOFAR @ 115-162 MHz**  
**Mean rms ~  $2.4 \times 10^{-3}$  Jy/b**  
**Dyn. Range ~ 35000**  
**FWHM: 22" x 18"**



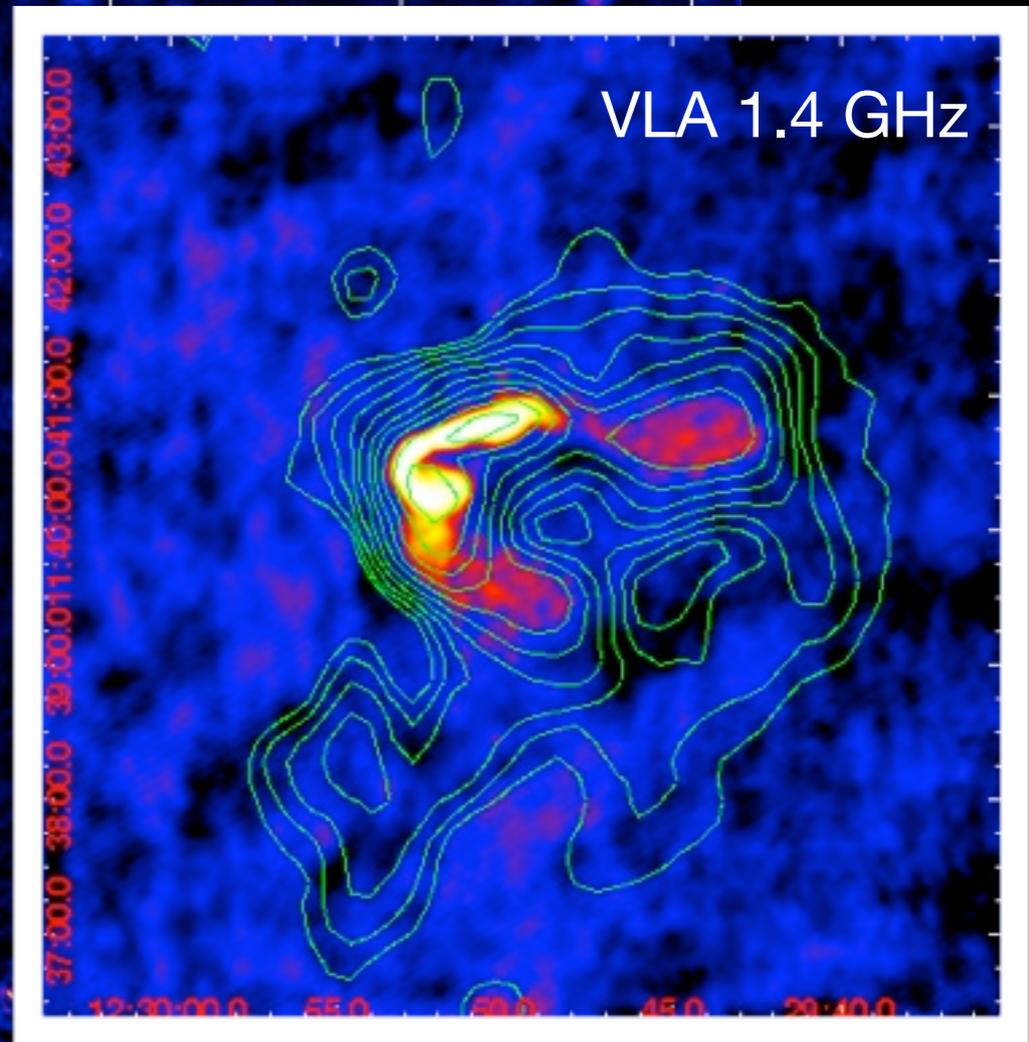
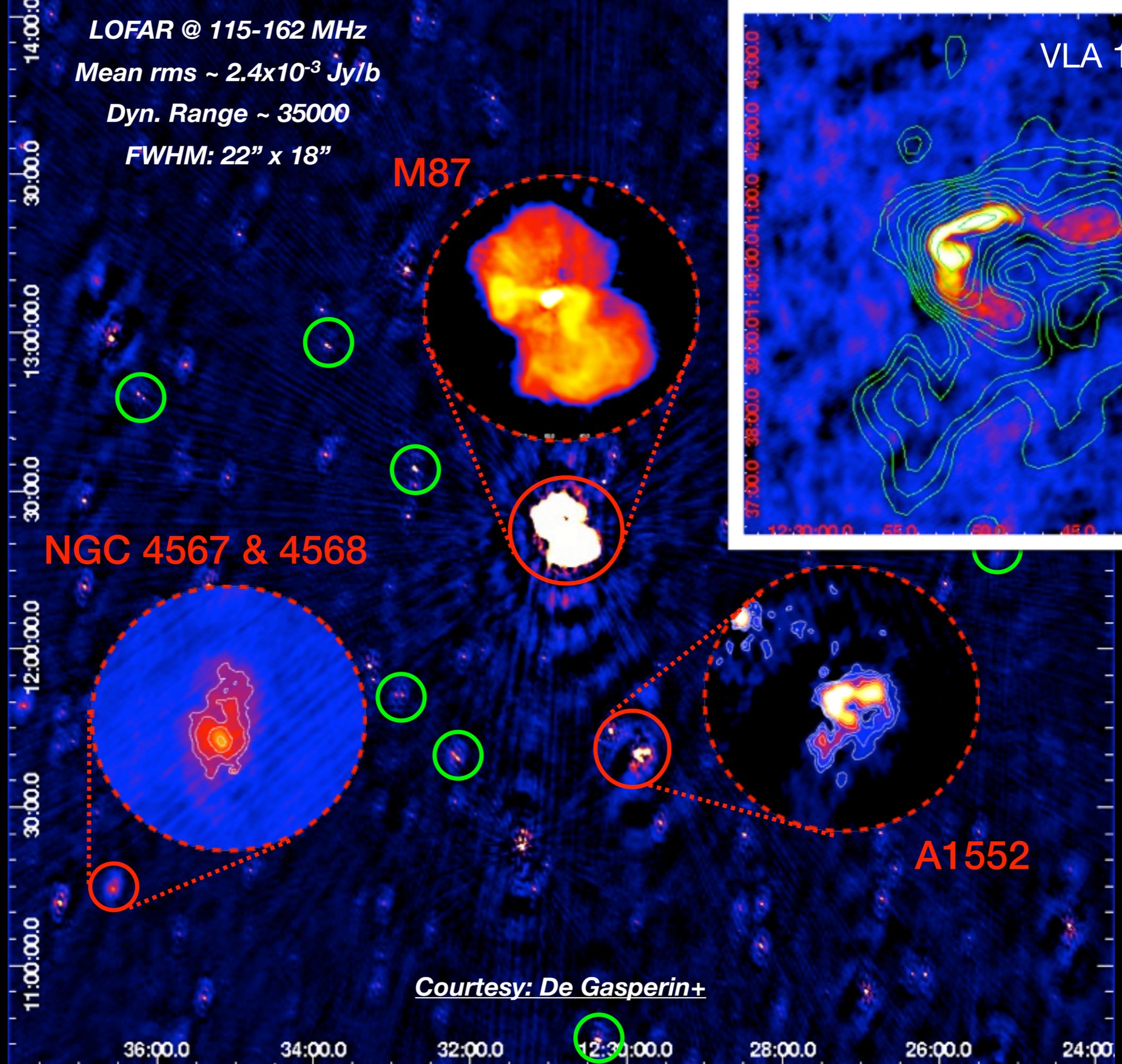
Courtesy: De Gasperin+

LOFAR @ 115-162 MHz  
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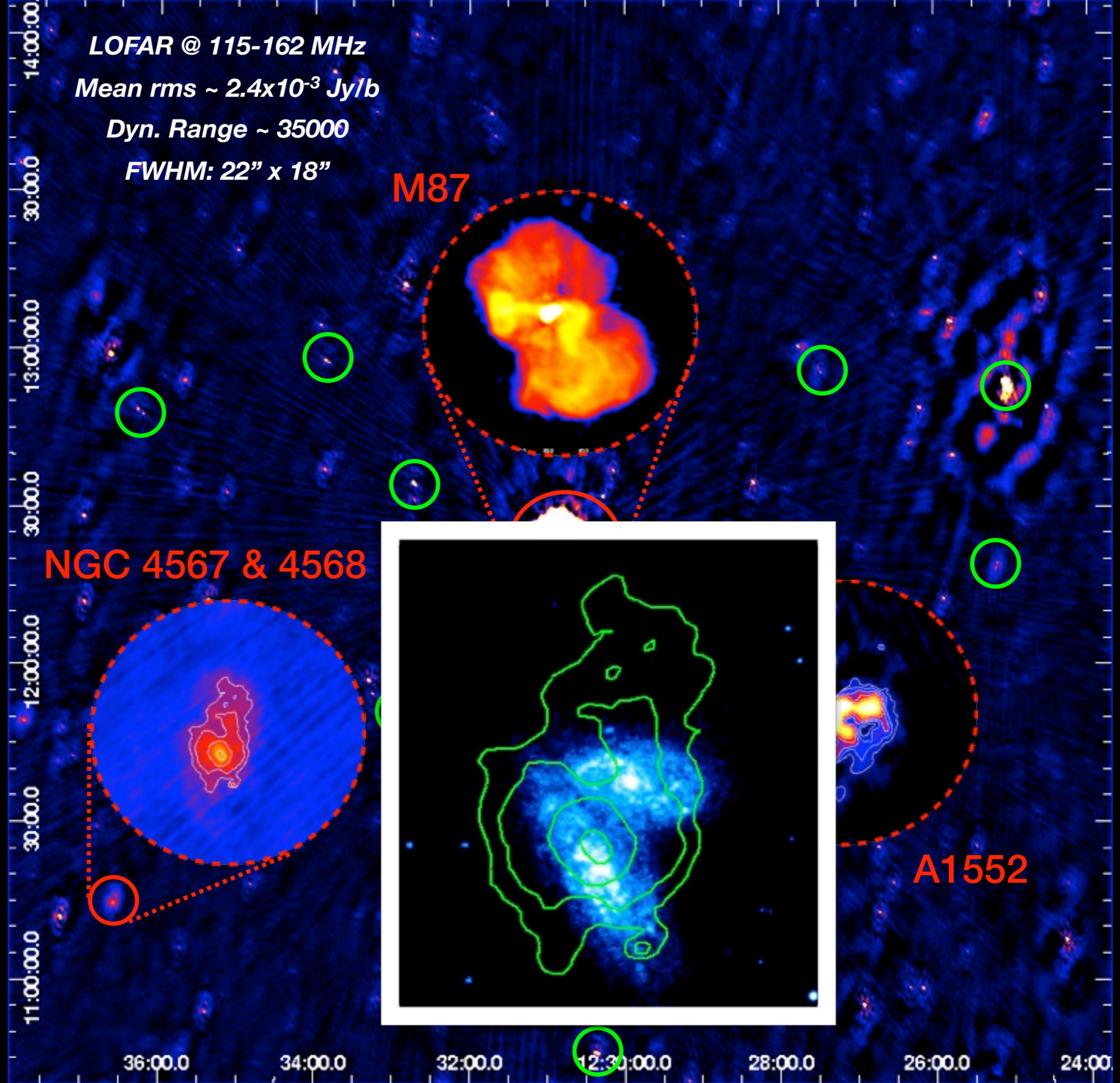
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M87

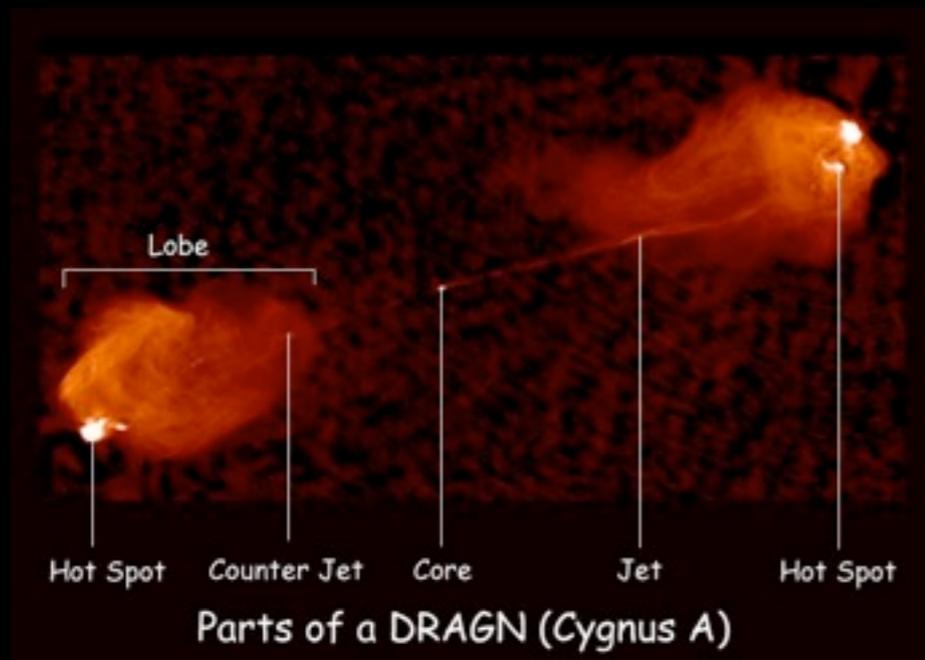
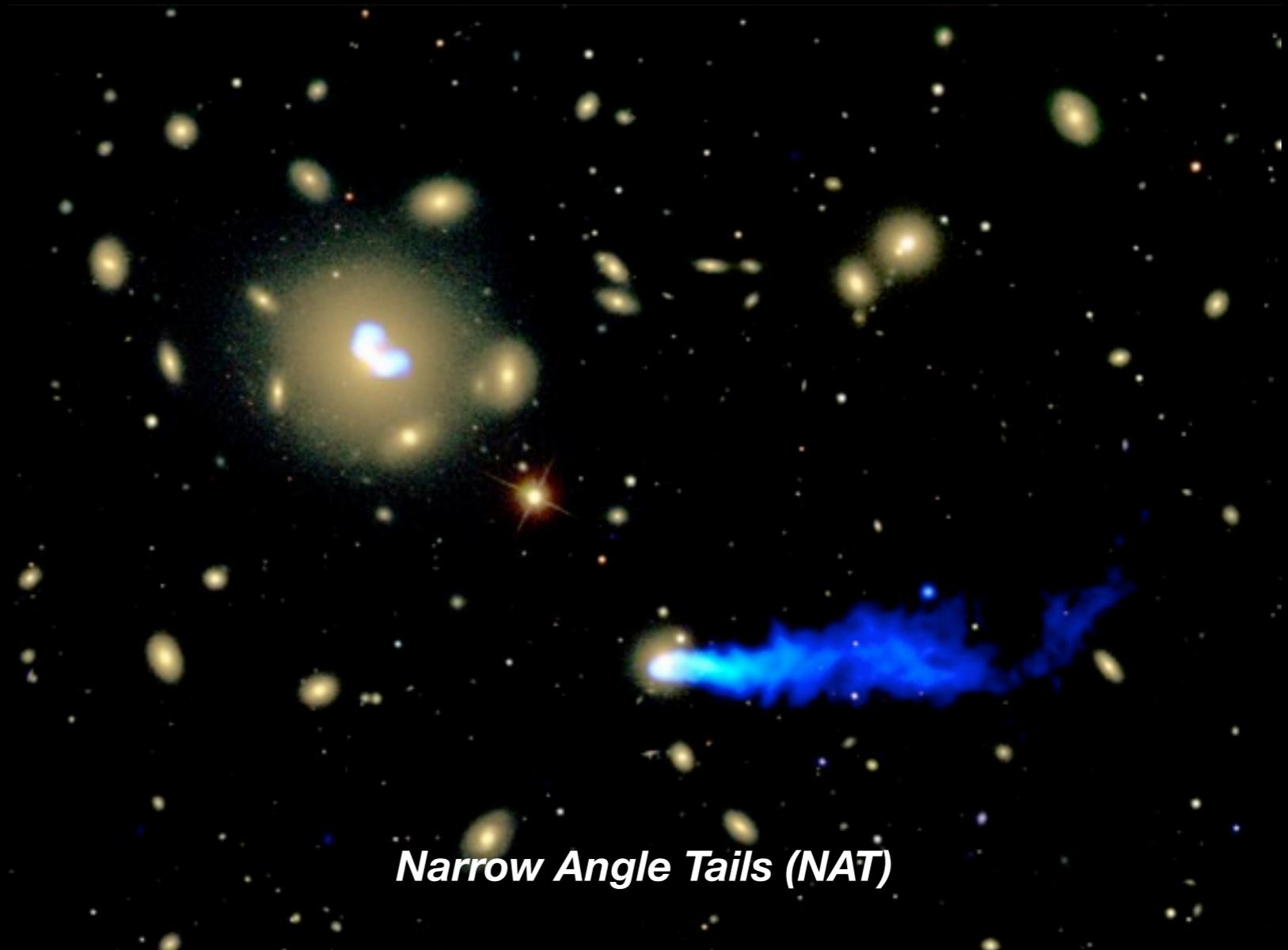
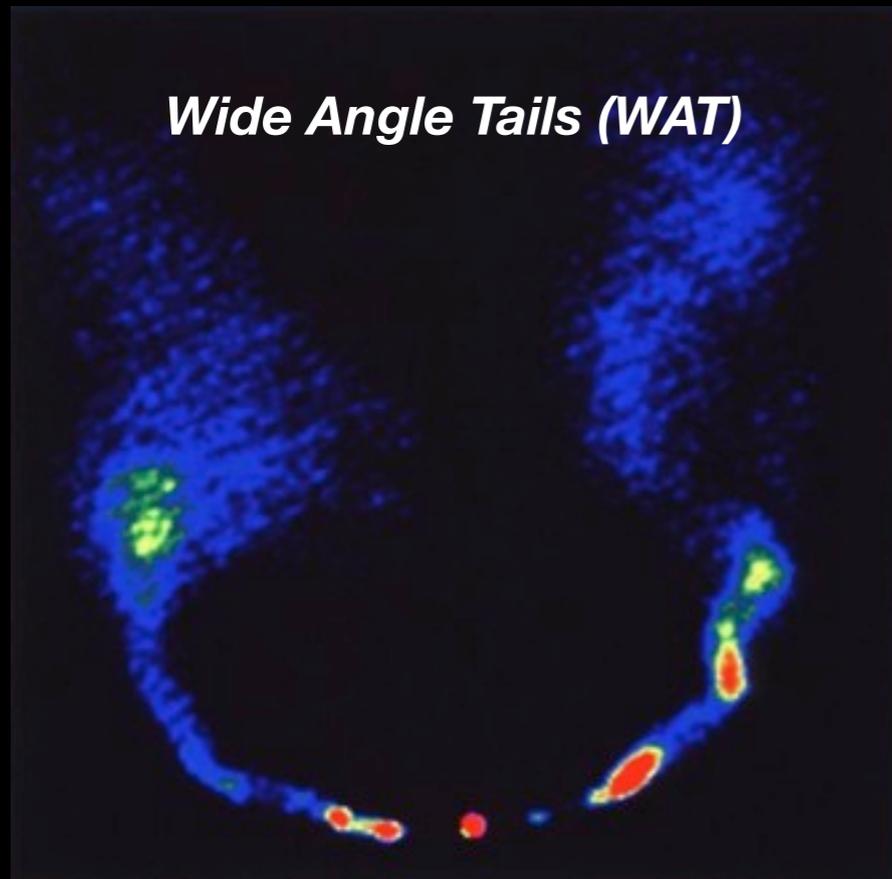
NGC 4567 & 4568

A1552



# EXTRA-GALACTIC SOURCES OF SYNCHROTRON RADIATION

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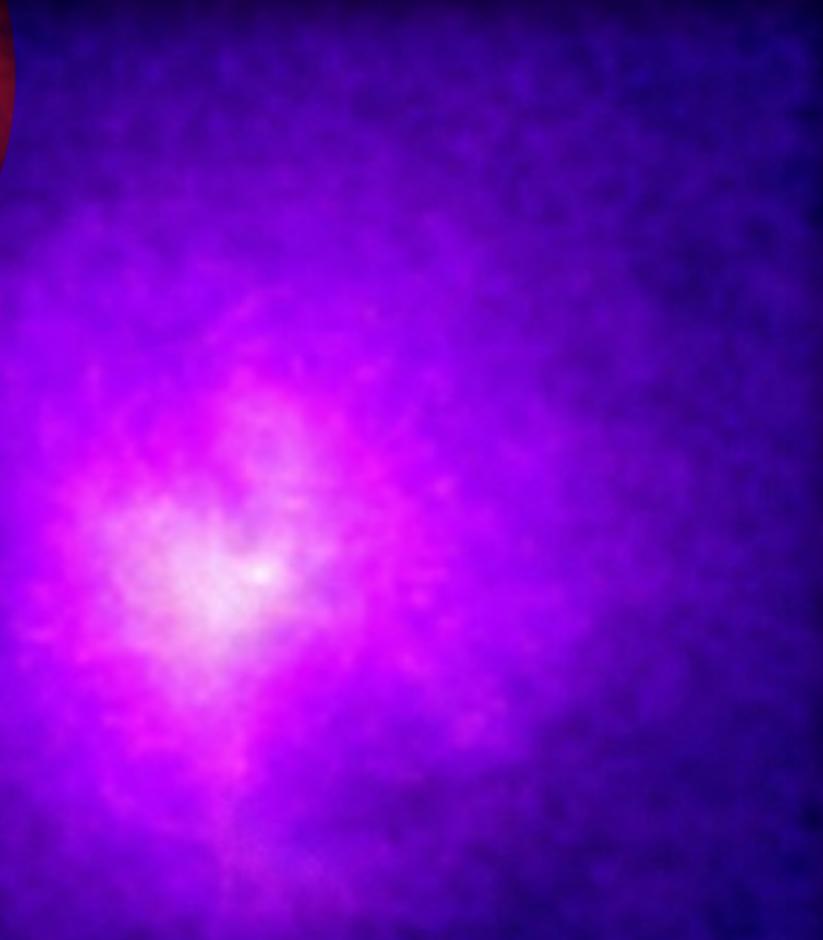
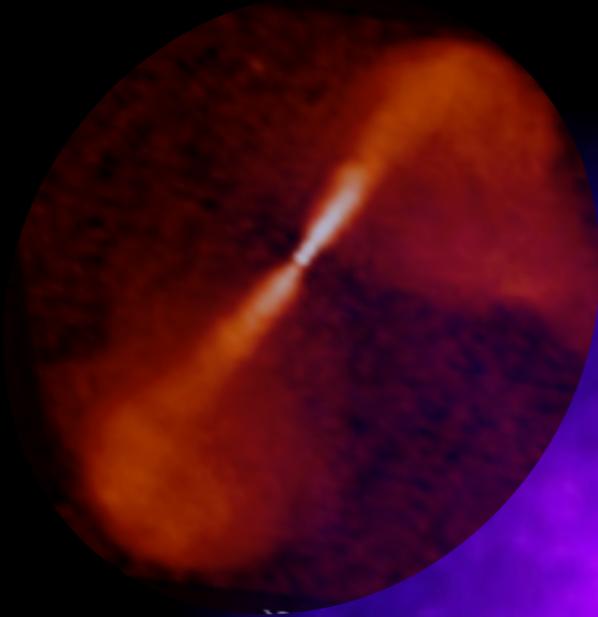


# PECULIAR RADIO GALAXIES AS TRACERS OF INTER-CLUSTER GAS

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# PECULIAR RADIO GALAXIES AS TRACERS OF INTER-CLUSTER GAS

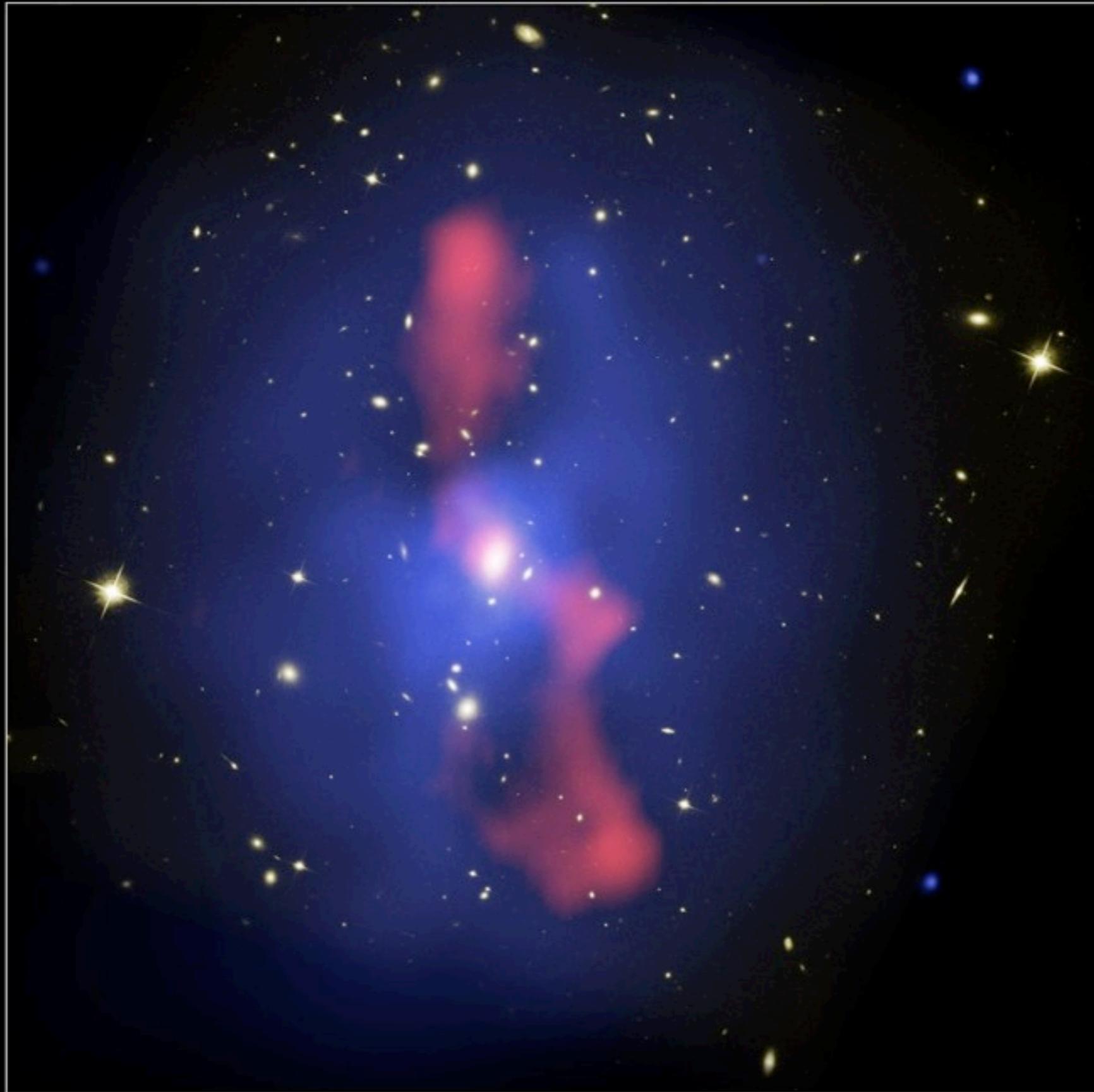
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$$P_{\text{ram}} \propto \rho v_{\text{gal}}^2$$

# Galaxy Cluster MS 0735.6+7421

CXO ■ HST ■ VLA



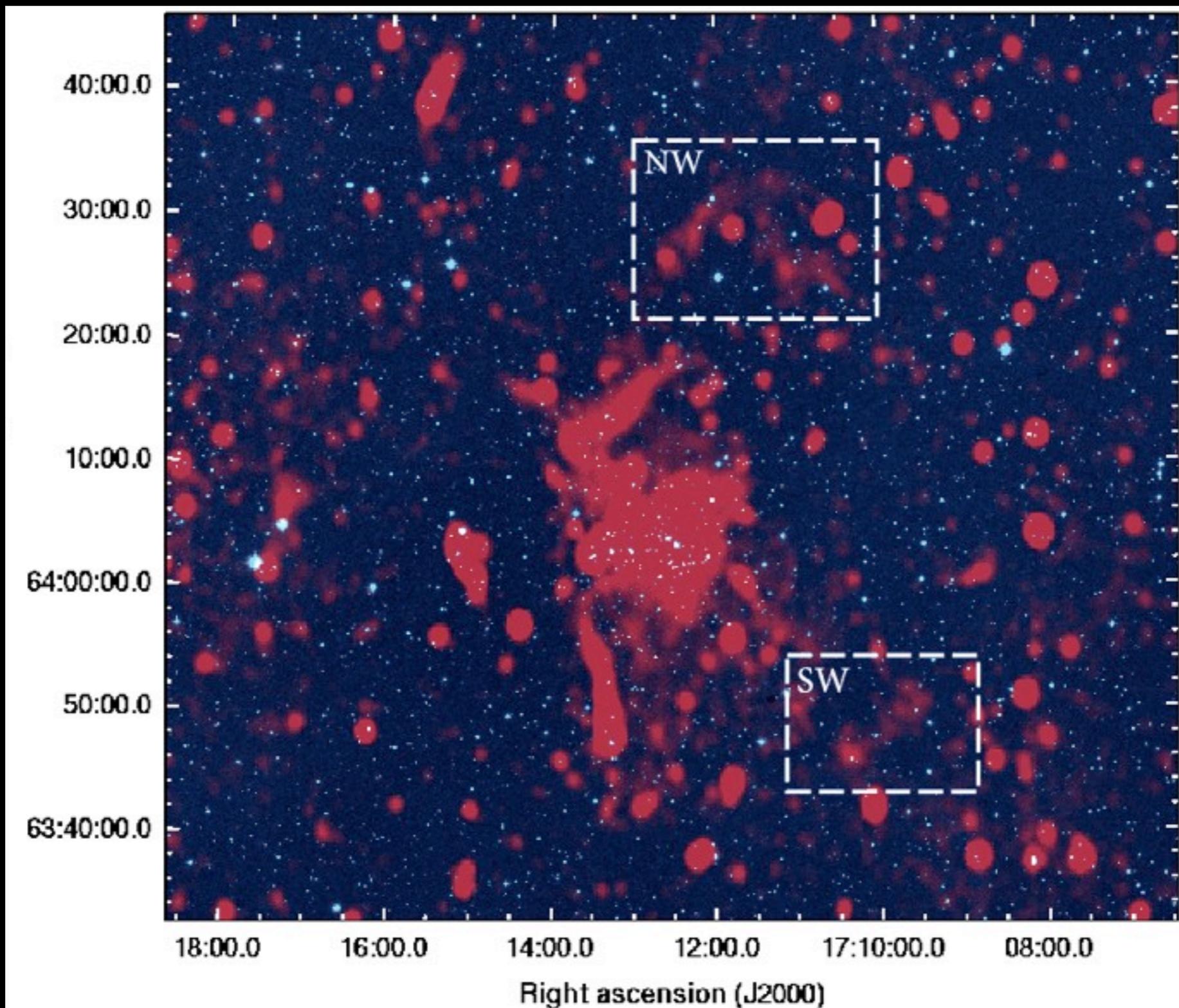
X-ray  
*Chandra X-Ray Observatory*

Visible  
*Hubble Space Telescope*

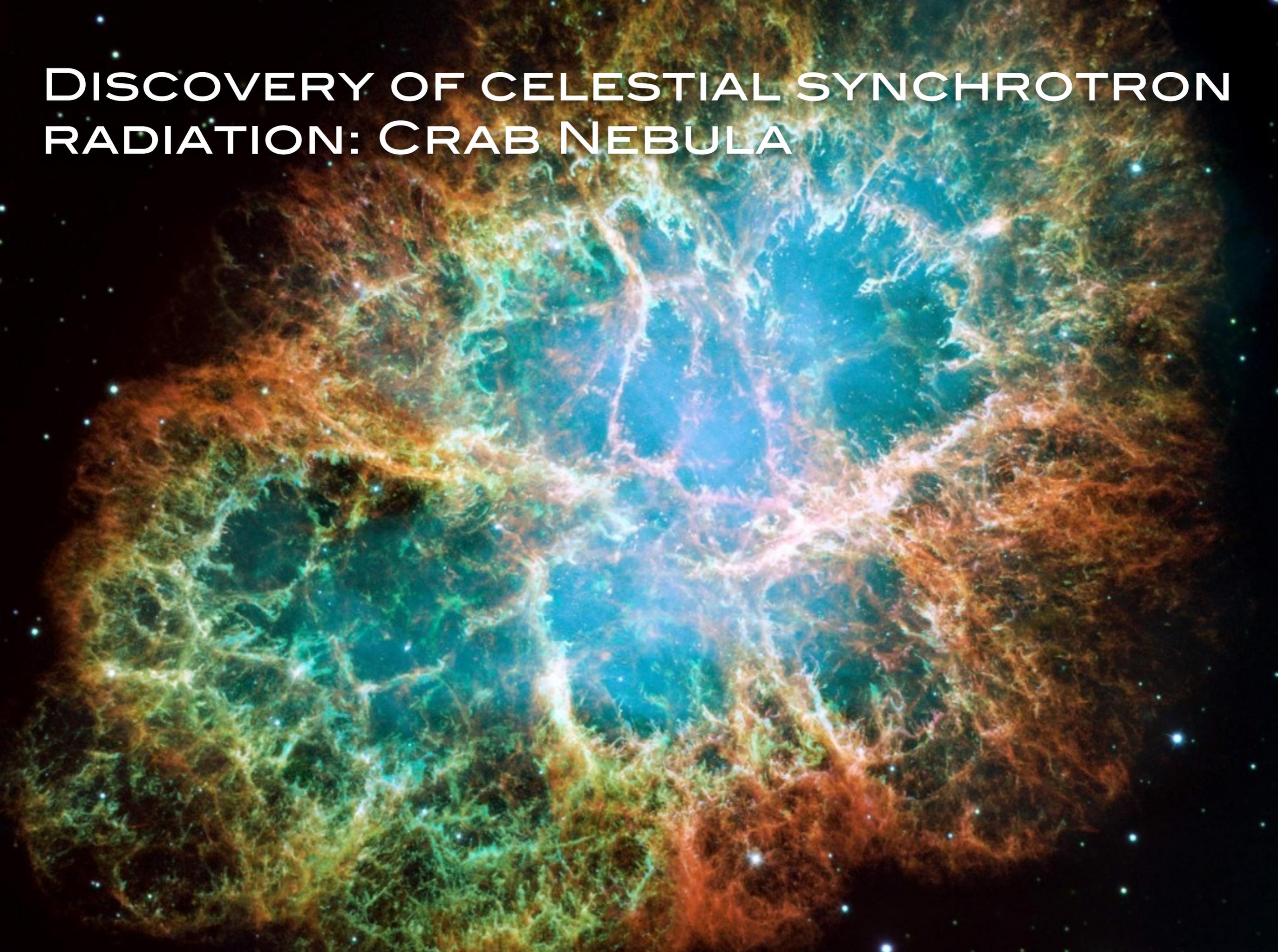
Radio  
*Very Large Array*

NASA, ESA, CXC/NRAO/STScI, B. McNamara (University of Waterloo and Ohio University)

STScI-PRC06-51



# DISCOVERY OF CELESTIAL SYNCHROTRON RADIATION: CRAB NEBULA



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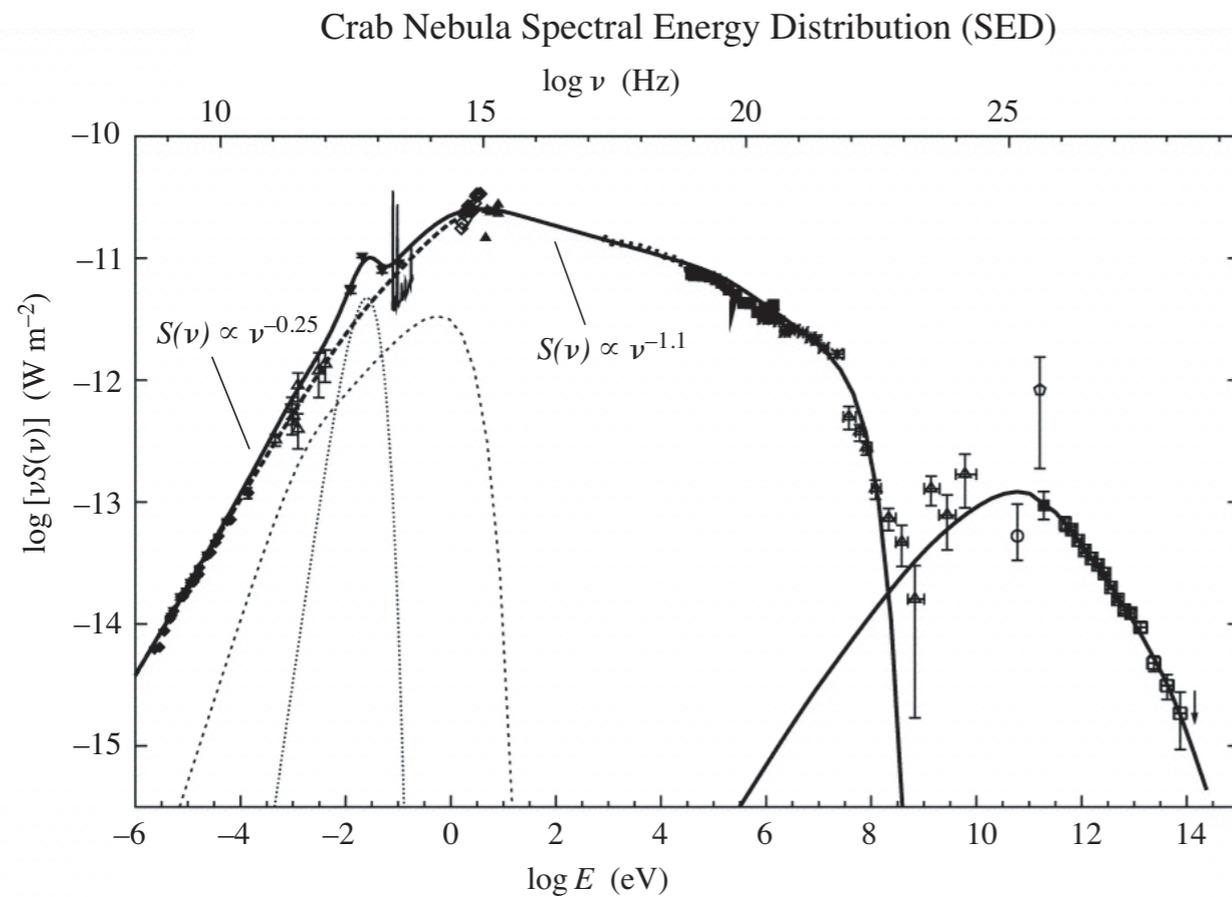


Fig. 8.2: Astrophysics Processes (CUP), H. Bradt 2008  
Compilation by HEGRA Team; F. Aharonian et al., ApJ 614, 897 (2004)

# DISCOVERY OF CELESTIAL SYNCHROTRON RADIATION: CRAB NEBULA

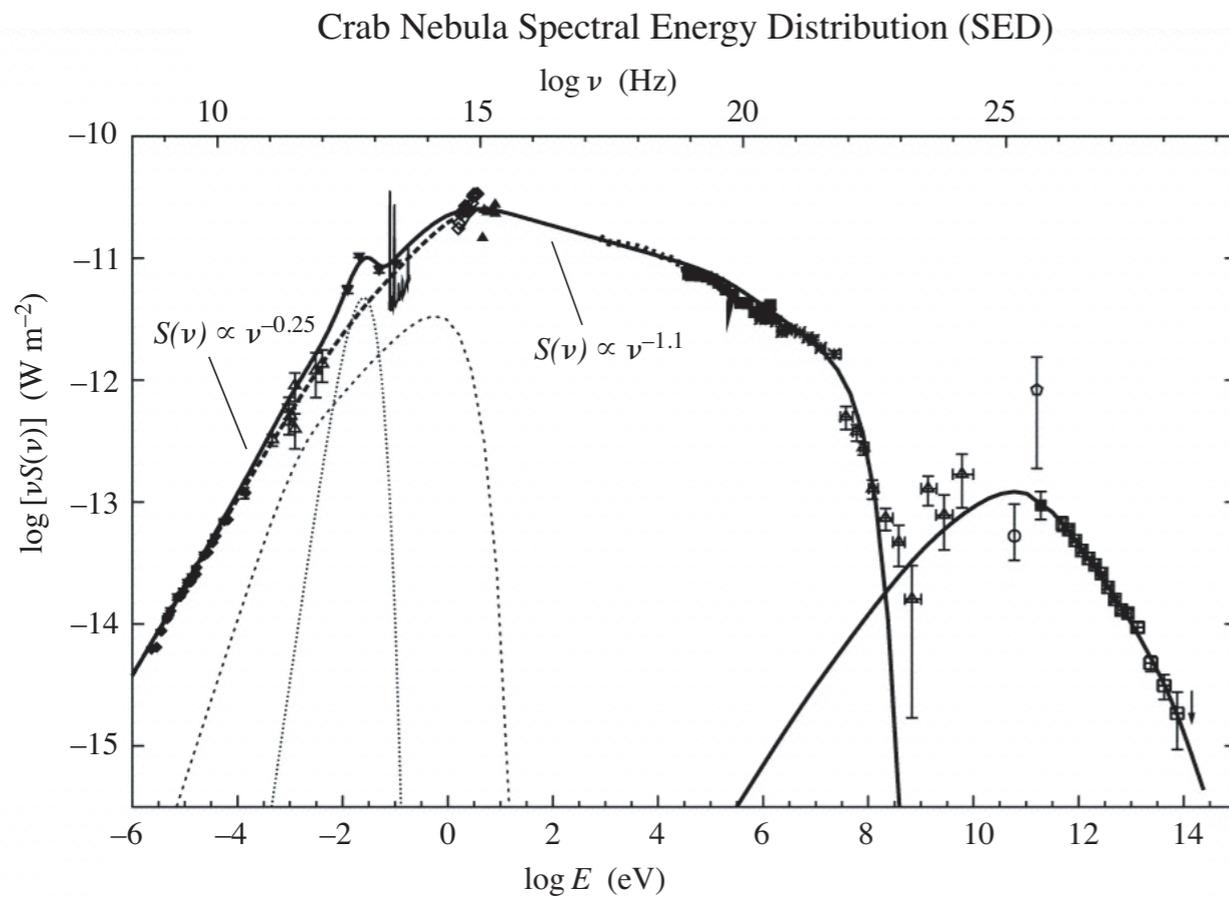


Fig. 8.2: Astrophysics Processes (CUP), H. Bradt 2008  
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Crab Optical Polarization

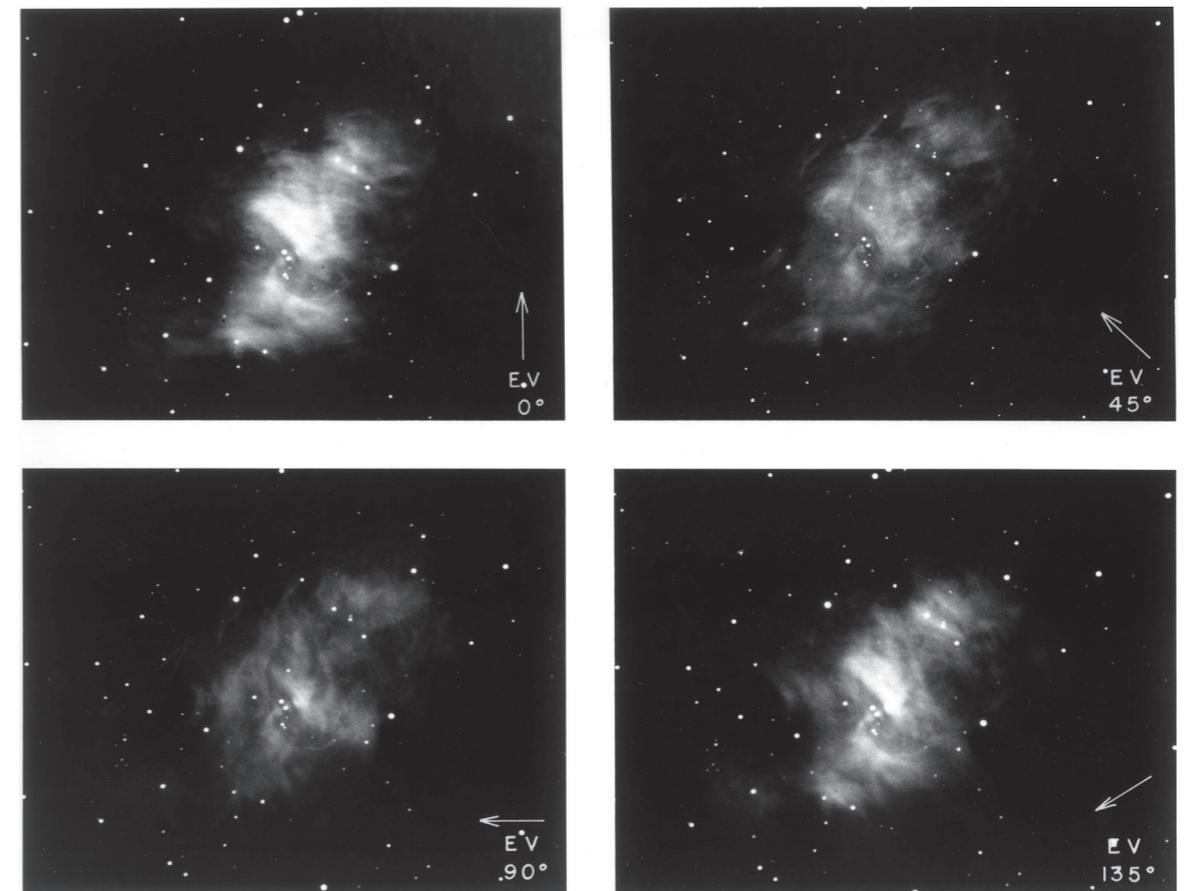
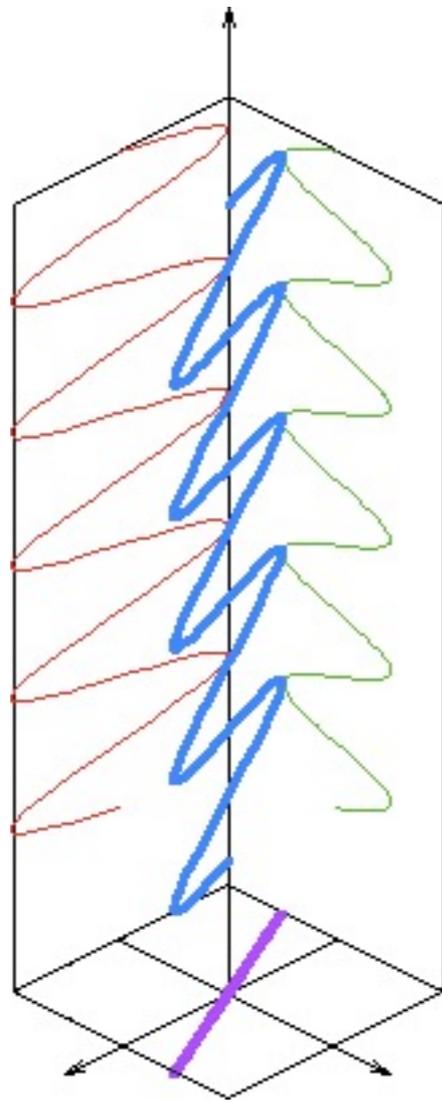


Fig. 8.3: Astrophysics Processes (CUP), H. Bradt 2008  
Palomar Observatory/CalTech

# POLARIZED EMISSION FROM THE CRAB NEBULA

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Crab Optical Polarization

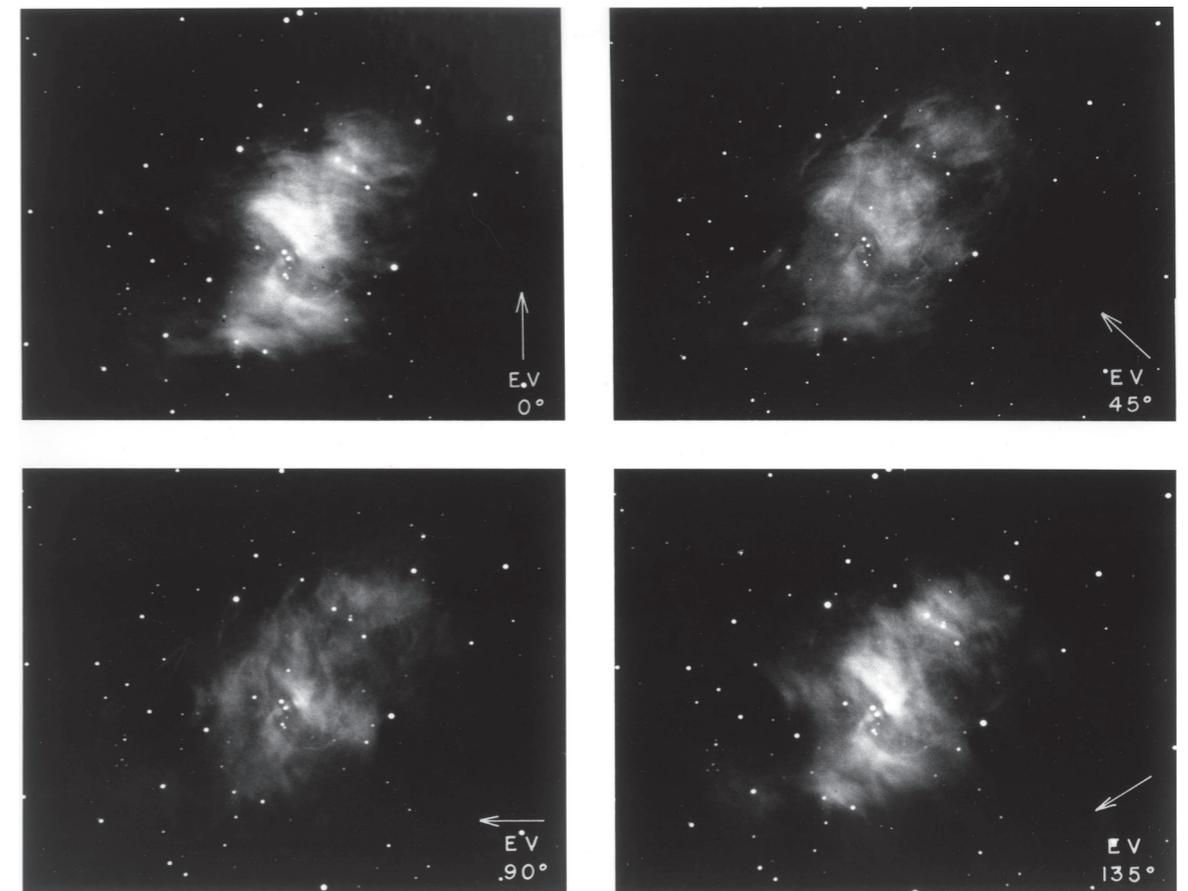
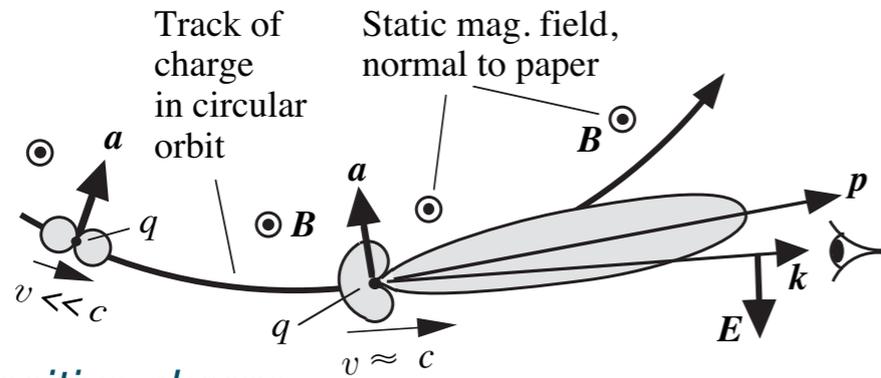


Fig. 8.3: Astrophysics Processes (CUP), H. Bradt 2008  
Palomar Observatory/CalTech

# POLARIZED EMISSION FROM THE CRAB NEBULA



Note: positive charge

Crab Optical Polarization

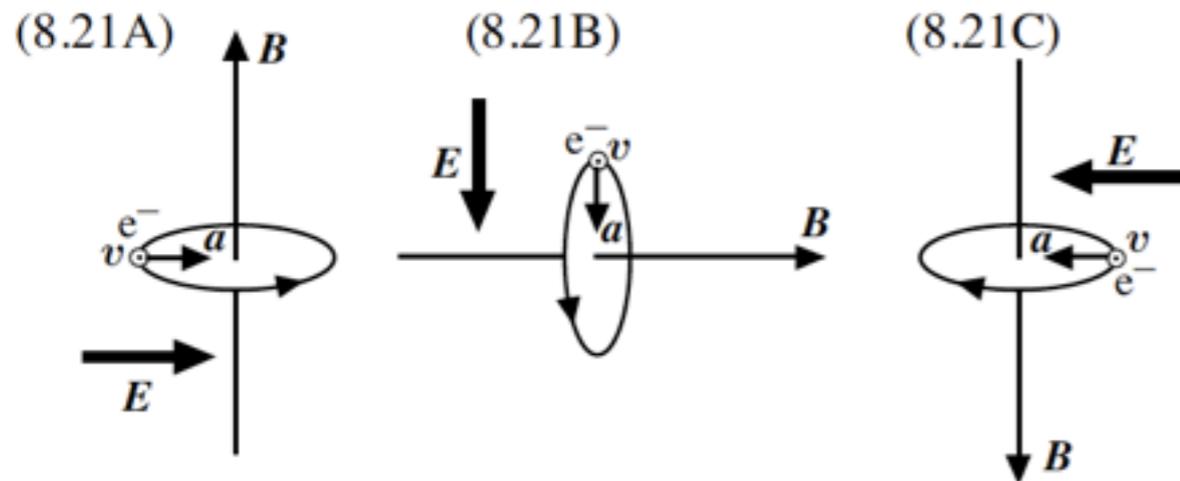
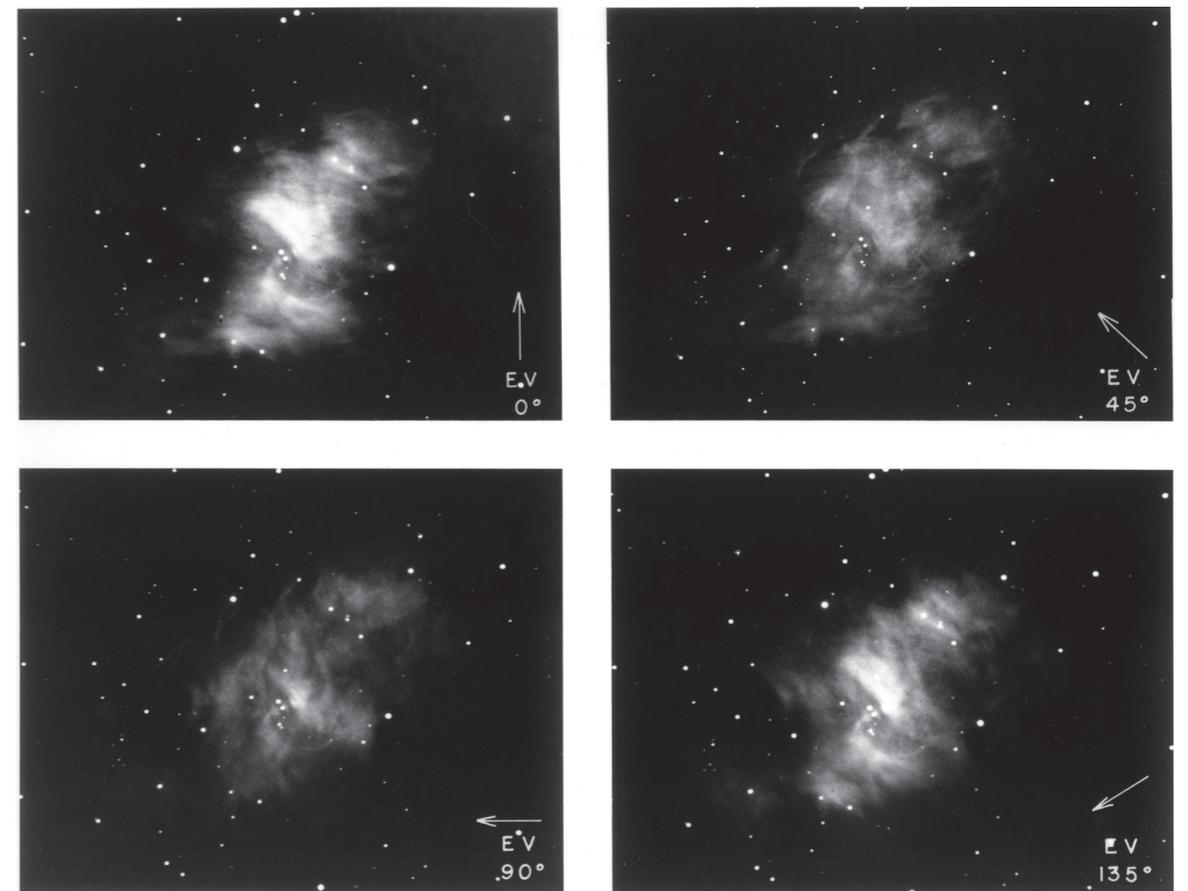


Fig. 8.3: Astrophysics Processes (CUP), H. Bradt 2008  
Palomar Observatory/CalTech

# ELECTRON ACCELERATORS: SYNCHROTRONS

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$$\mathbf{F} = q(\mathbf{v} \times \mathbf{B}) \quad (N)$$

$$\mathcal{P}(t) = \frac{1}{6\pi\epsilon_0} \frac{q^2 a(t)^2}{c^3}$$

**Power radiated BUT for  $v \ll c$**

In the relativistic case, the power radiated is much greater

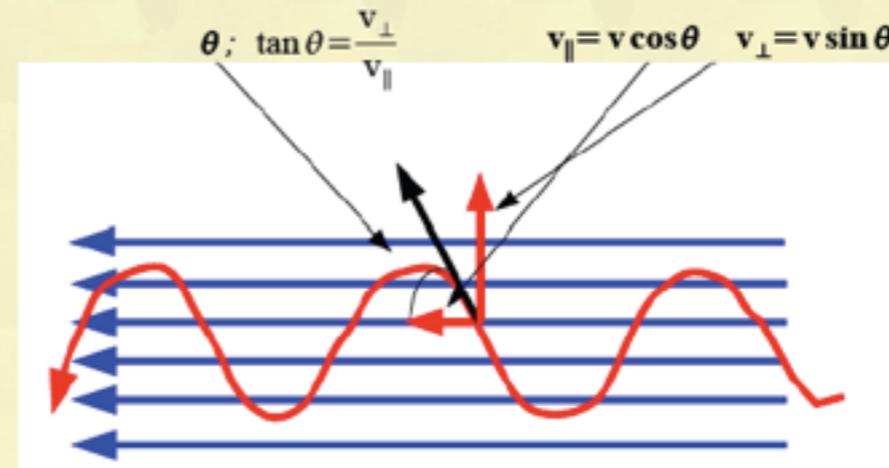
# MOTION OF A CHARGED PARTICLE IN A UNIFORM MAGNETIC FIELD

$$\mathbf{F} = q(\mathbf{v} \times \mathbf{B}) \quad (N)$$

$$\frac{d\mathbf{p}}{dt} = q(\mathbf{v} \times \mathbf{H}/c) \quad (N)$$

- ▶ Let's start with non relativistic electrons [ $v \ll c$ ]
- ▶ No power is transferred to the charged particle: its kinetic energy (velocity) remains constant

$\theta$  = pitch angle = angle between the directions of  $\vec{v}$  and  $\vec{H}$



$\frac{d(m v_{\parallel})}{dt} = 0 \rightarrow v_{\parallel}$  is constant  $\rightarrow$  uniform motion along  $\mathbf{B}$  direction (♣)

since  $v$  is constant, also  $v_{\perp}$  is constant (♦)

the pitch angle remains constant

$\frac{d(m v_{\perp})}{dt} = \frac{q}{c} \vec{v}_{\perp} \times \vec{H} + (\diamond) \rightarrow$  circular motion with a constant orbital velocity  $v_{\perp}$  and:

▶ curvature radius:

$$r_L = \frac{mc}{qH} v_{\perp}$$

▶ revolution period:

$$T_L = \frac{2\pi r_L}{v_{\perp}} = \frac{2\pi mc}{qH}$$

▶ frequency of the gyration:

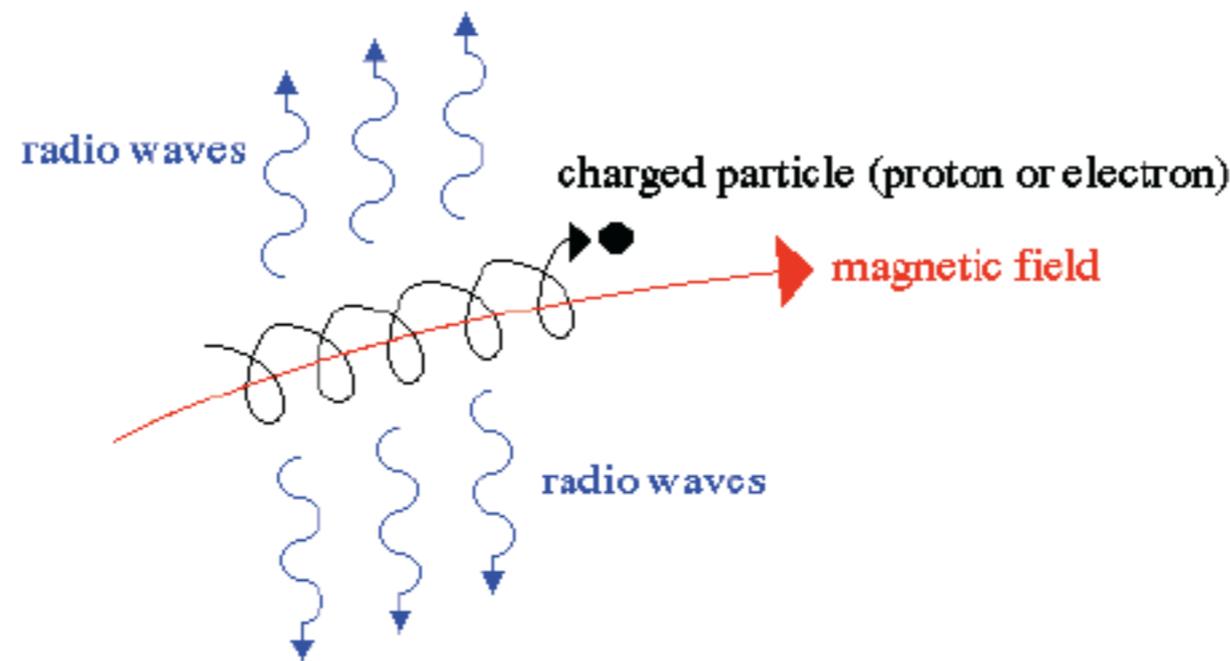
$$\omega_L = \frac{2\pi}{T_L} = \frac{qH}{mc}$$

# CORKSCREW PATTERN OF ELECTRONS

$$\frac{d(m v_{\parallel})}{dt} = 0 \rightarrow v_{\parallel} \text{ is constant} \rightarrow \text{uniform motion along } \mathbf{B} \text{ direction}$$

$$\frac{d(m v_{\perp})}{dt} = \frac{q}{c} \vec{v}_{\perp} \times \vec{H} \rightarrow \text{circular motion with a constant orbital velocity } v_{\perp}$$

Synchrotron radiation



→ The path followed by the electron is a helix winding around the magnetic field with the constant pitch angle  $\theta$

# MOTION OF A CHARGED PARTICLE IN A UNIFORM MAGNETIC FIELD

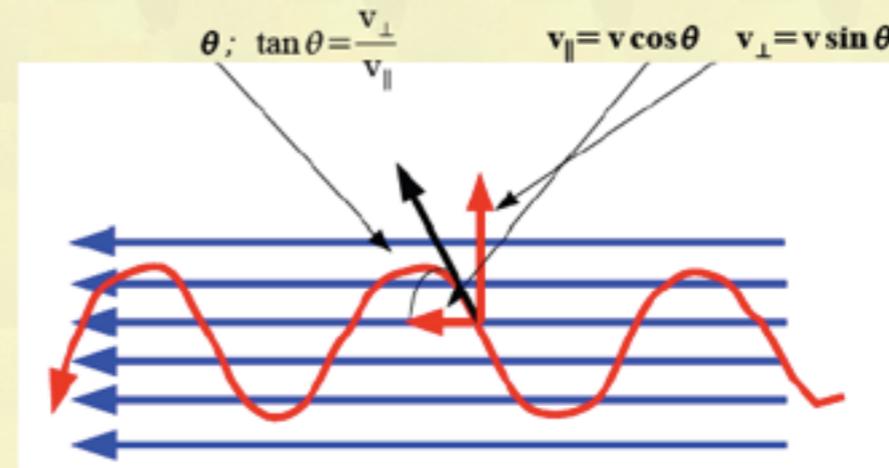
In the inertial frame moving with velocity  $v_{\parallel}$ , from Newton's second law with acceleration  $-\omega^2 r$  and:

$$\mathbf{F} = q(\mathbf{v} \times \mathbf{B}) \quad (N)$$

$$m \omega^2 R = q v B = q \omega R B \rightarrow$$

$$\omega = \frac{q B}{m}$$

$\theta$  = pitch angle = angle between the directions of  $\vec{v}$  and  $\vec{H}$



$\frac{d(m v_{\parallel})}{dt} = 0 \rightarrow v_{\parallel}$  is constant  $\rightarrow$  uniform motion along  $\mathbf{B}$  direction ( $\clubsuit$ )  
 since  $v$  is constant, also  $v_{\perp}$  is constant ( $\diamond$ )  
 the pitch angle remains constant

$\frac{d(m v_{\perp})}{dt} = \frac{q}{c} \vec{v}_{\perp} \times \vec{H} + (\diamond) \rightarrow$  circular motion with a constant orbital velocity  $v_{\perp}$  and:

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# RADIATION FROM CHARGED PARTICLE IN A UNIFORM MAGNETIC FIELD

## Circular Motion: One Charge

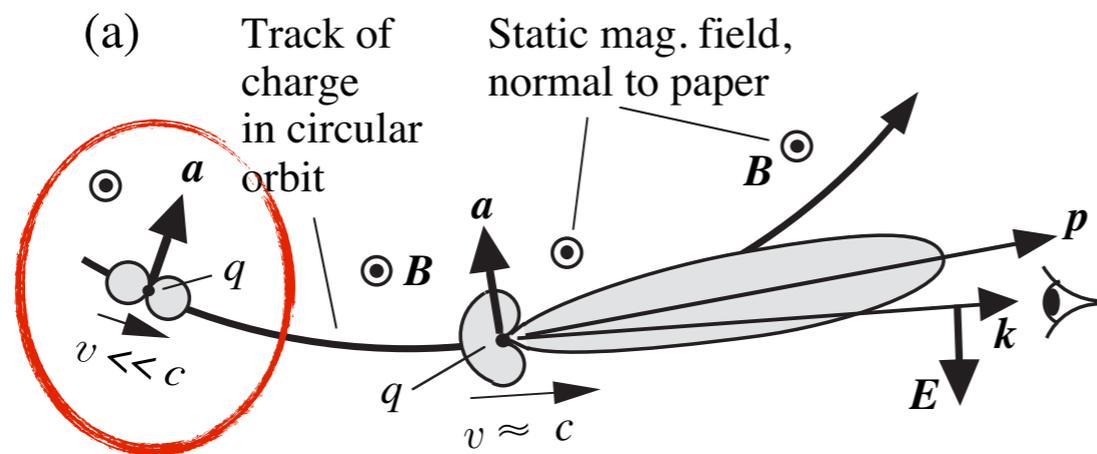
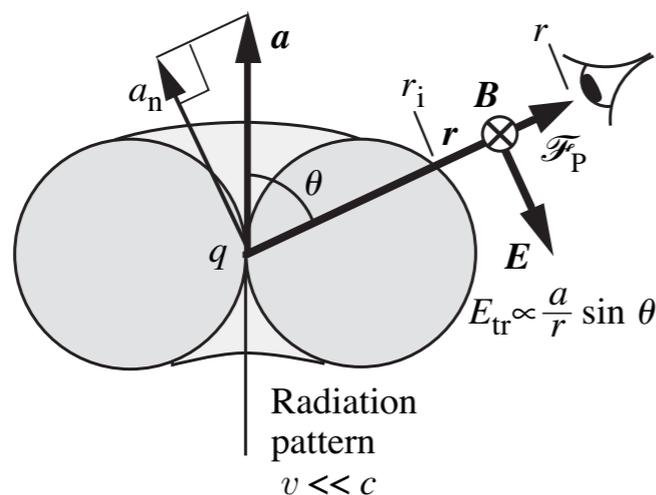


Fig. 8.4: Astrophysics Processes (CUP), © H. Bradt 2008



$$\mathbf{E}(\mathbf{r}, t) = E_{tr} \hat{\mathbf{n}} = \frac{qa(t') \sin \theta}{4\pi\epsilon_0 c^2 r} \hat{\mathbf{n}}$$

$\mathbf{E}(\mathbf{r}, t)$  = Transverse electric vector; V/m;  $v \ll c$

# RADIATION FROM CHARGED PARTICLE IN A UNIFORM MAGNETIC FIELD

## Circular Motion: One Charge

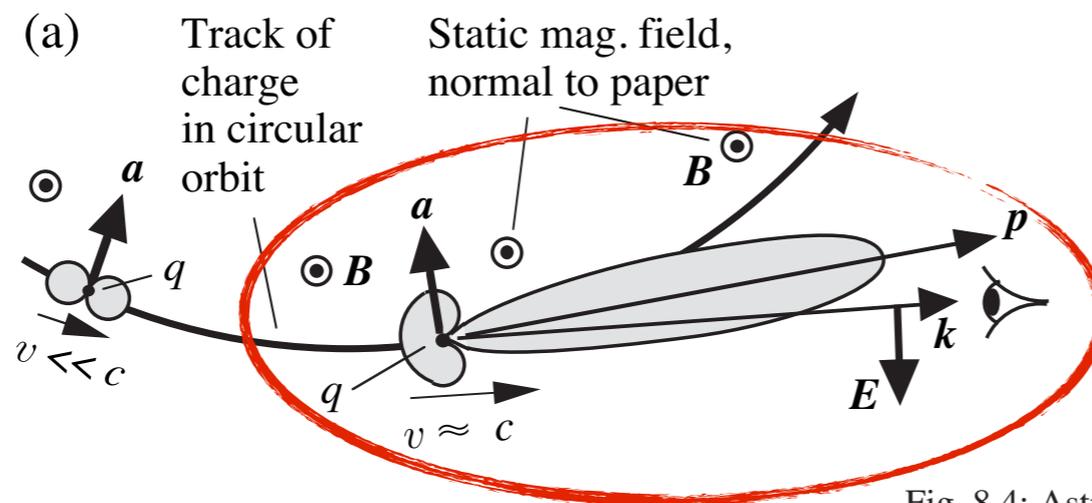


Fig. 8.4: Astrophysics Processes (CUP), © H. Bradt 2008

**Aberration:** apparent direction of radiation will differ according to observers in two frames of reference that are moving with respect to each other

$$\cos \theta = \frac{\cos \theta' + \beta}{1 + \beta \cos \theta'}$$

See Chap. 7 in "Astrophysics Processes"

# RADIATION FROM CHARGED PARTICLE IN A UNIFORM MAGNETIC FIELD

## Circular Motion: One Charge

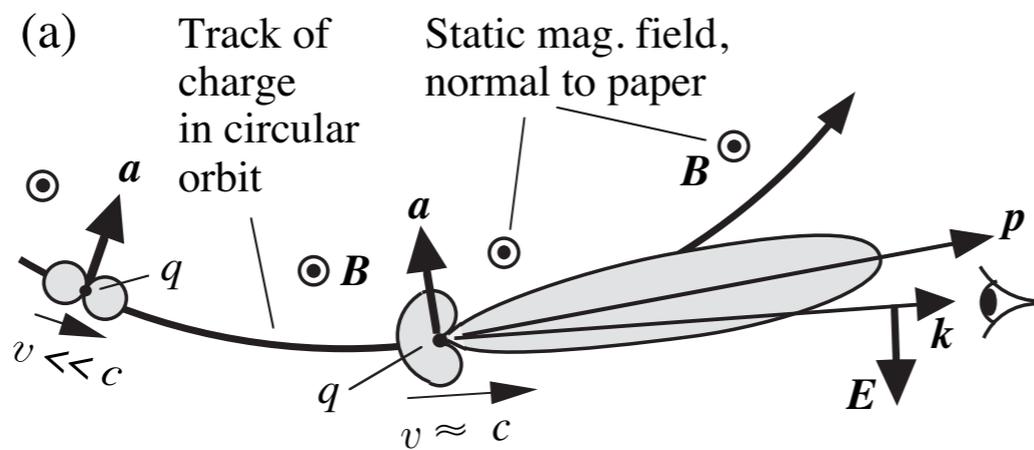


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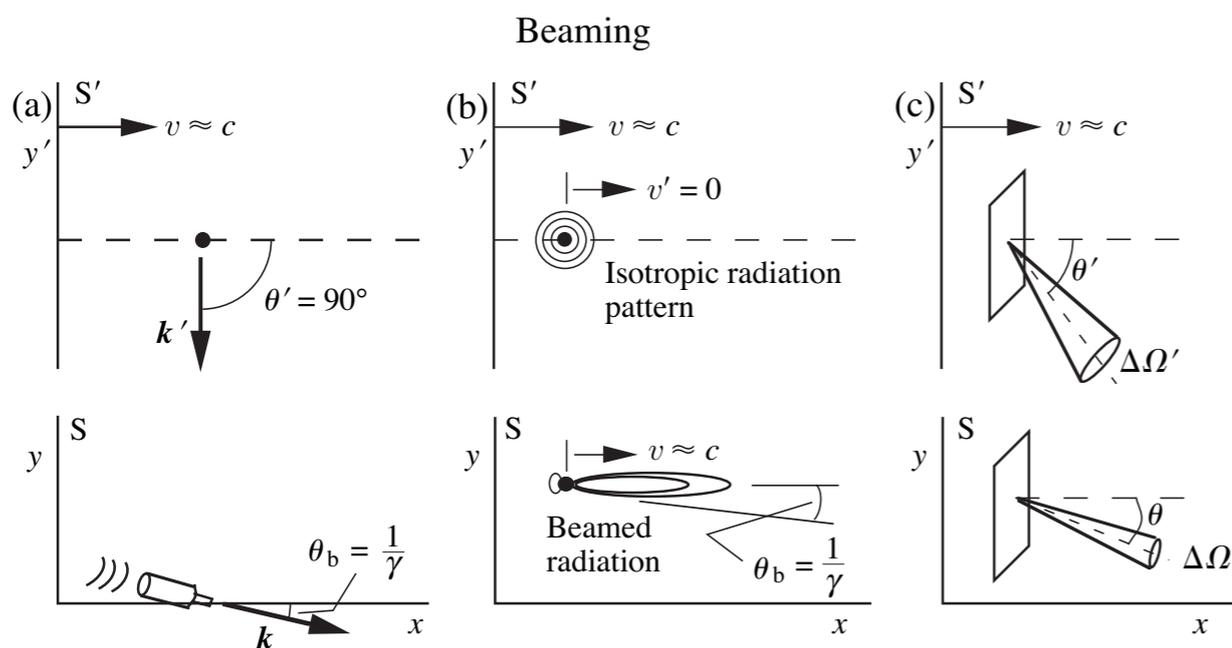


Fig. 7.9: Astrophysics Processes (CUP), © H Bradt 2008

$$\cos \theta = \frac{\cos \theta' + \beta}{1 + \beta \cos \theta'} \quad \& \quad \theta' = 90^\circ$$

# RADIATION FROM CHARGED PARTICLE IN A UNIFORM MAGNETIC FIELD

## Circular Motion: One Charge

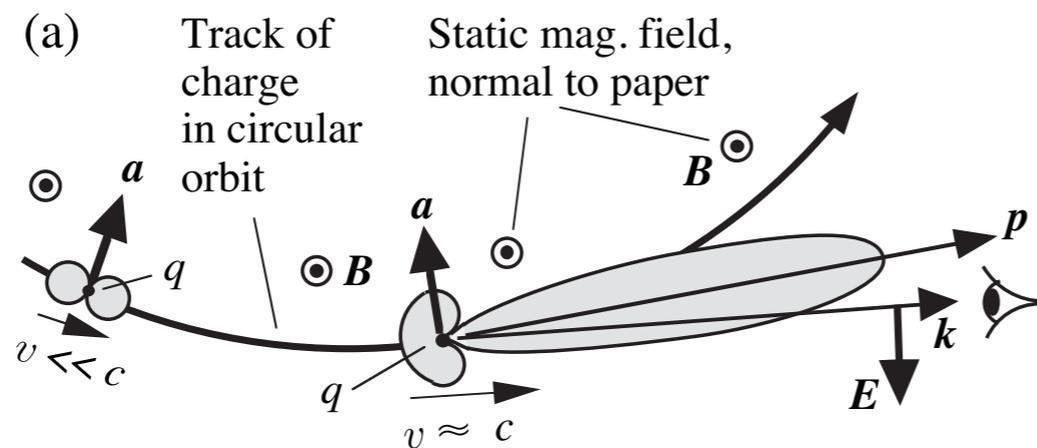


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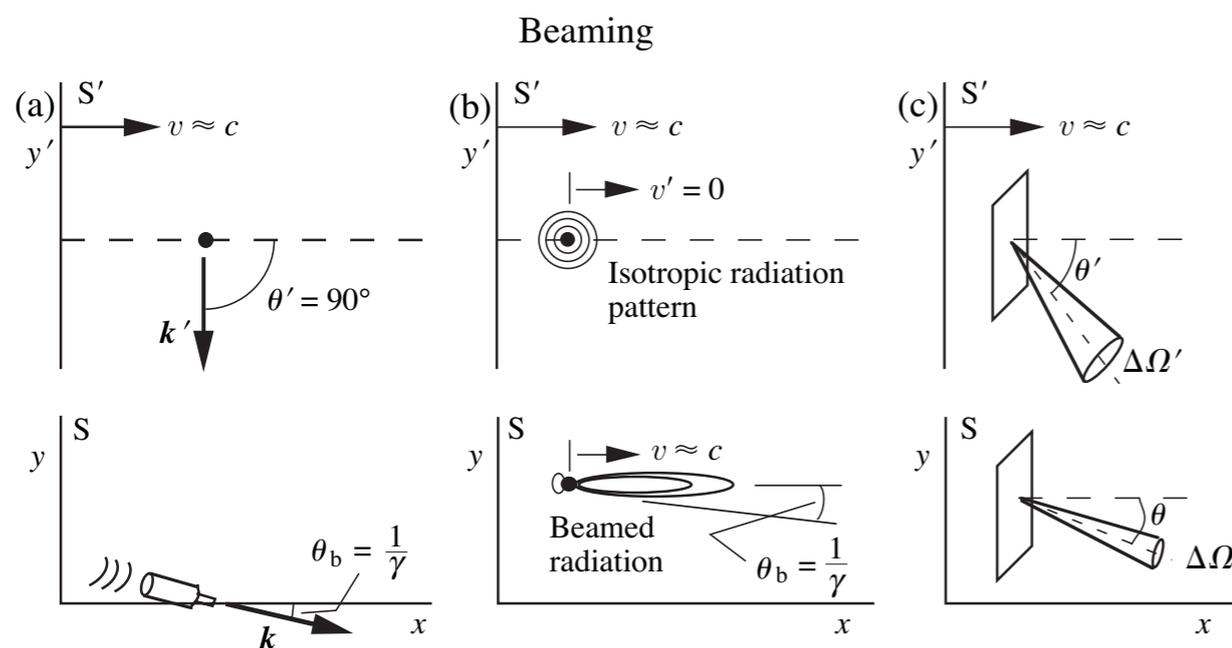


Fig. 7.9: Astrophysics Processes (CUP), © H Bradt 2008

$$\cos \theta = \frac{\cos \theta' + \beta}{1 + \beta \cos \theta'} \quad \& \quad \theta' = 90^\circ$$

$$\cos \theta_b = \beta = \left(1 - \frac{1}{\gamma^2}\right)^{1/2}$$

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## Circular Motion: One Charge

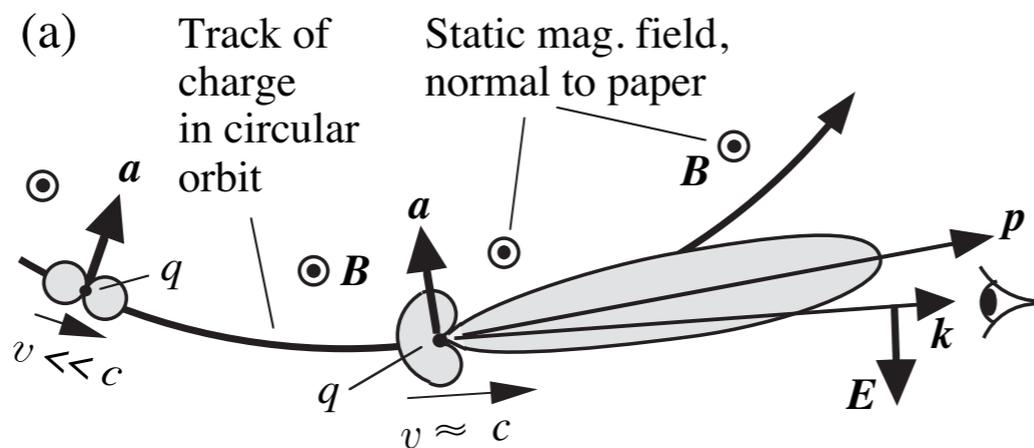


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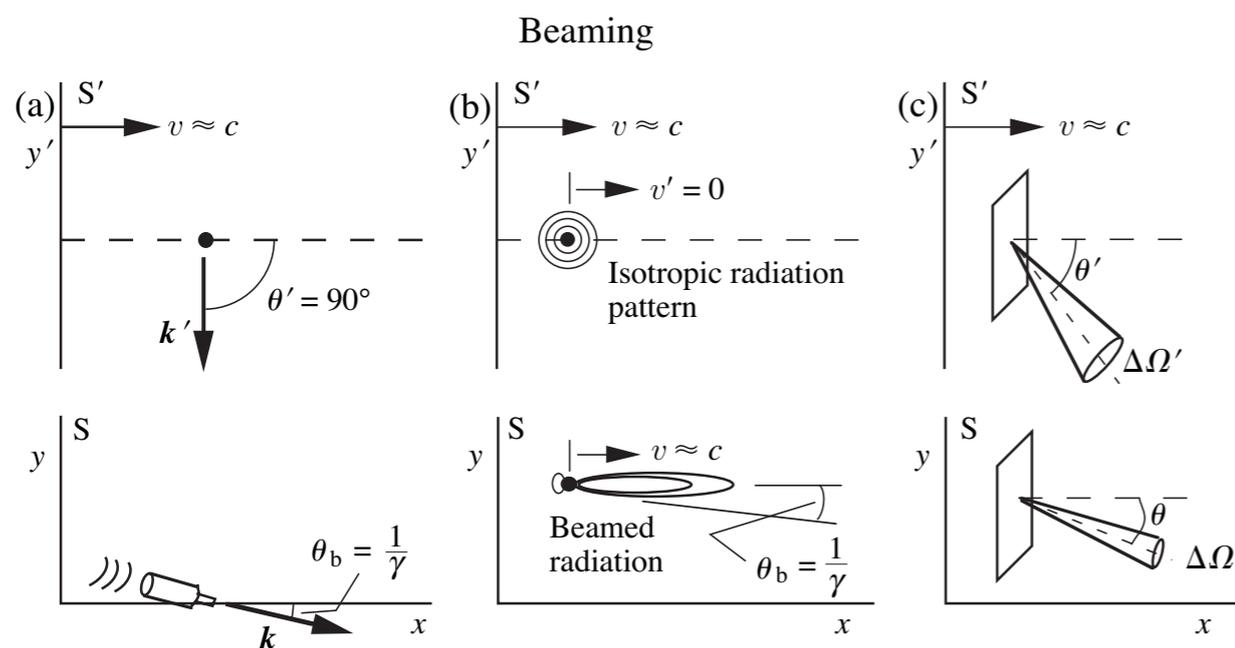


Fig. 7.9: Astrophysics Processes (CUP), © H Bradt 2008

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$$\gamma \gg 1 \rightarrow \theta_b \approx 0 \rightarrow 1 - \frac{\theta_b^2}{2} = 1 - \frac{1}{2\gamma^2}$$

# RADIATION FROM CHARGED PARTICLE IN A UNIFORM MAGNETIC FIELD

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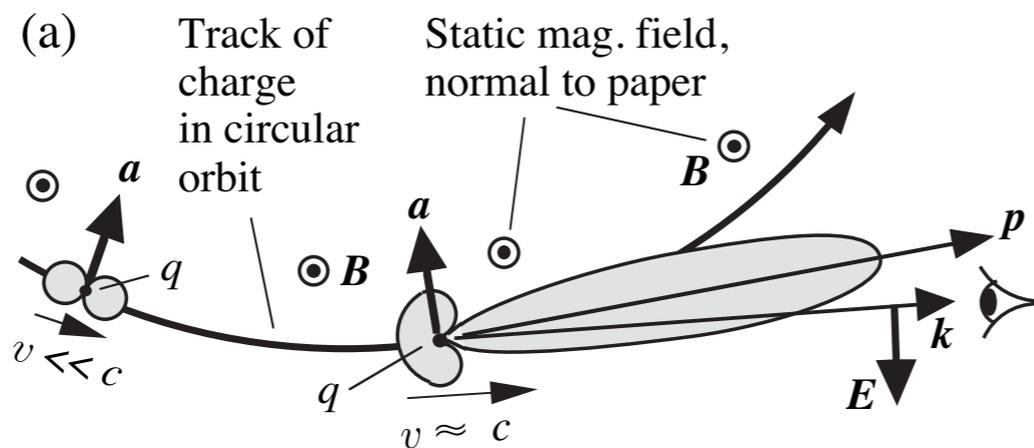


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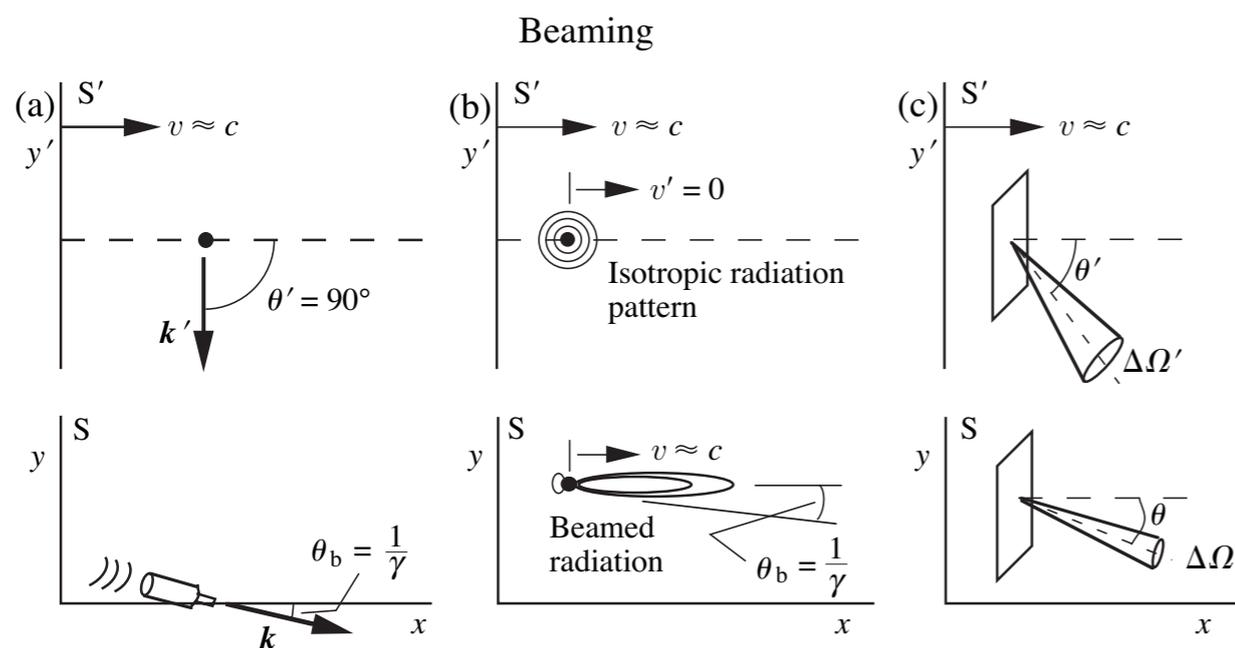


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$$\theta_b \approx \frac{1}{\gamma}$$

# RADIATION FROM CHARGED PARTICLE IN A UNIFORM MAGNETIC FIELD

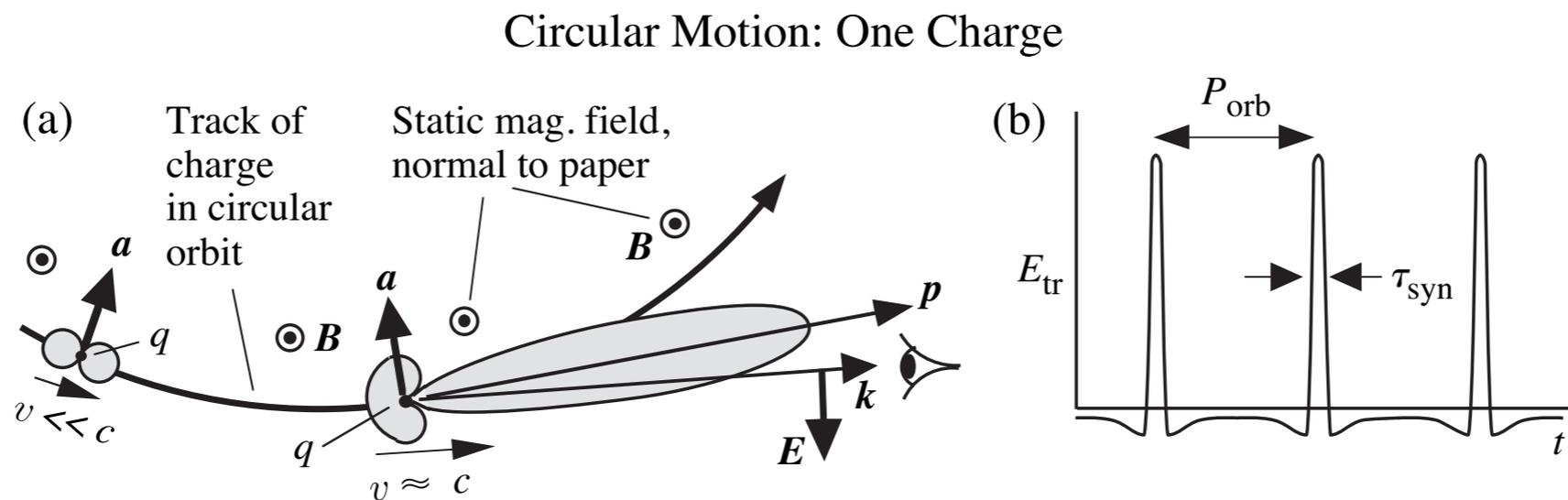


Fig. 8.4: Astrophysics Processes (CUP), © H. Bradt 2008

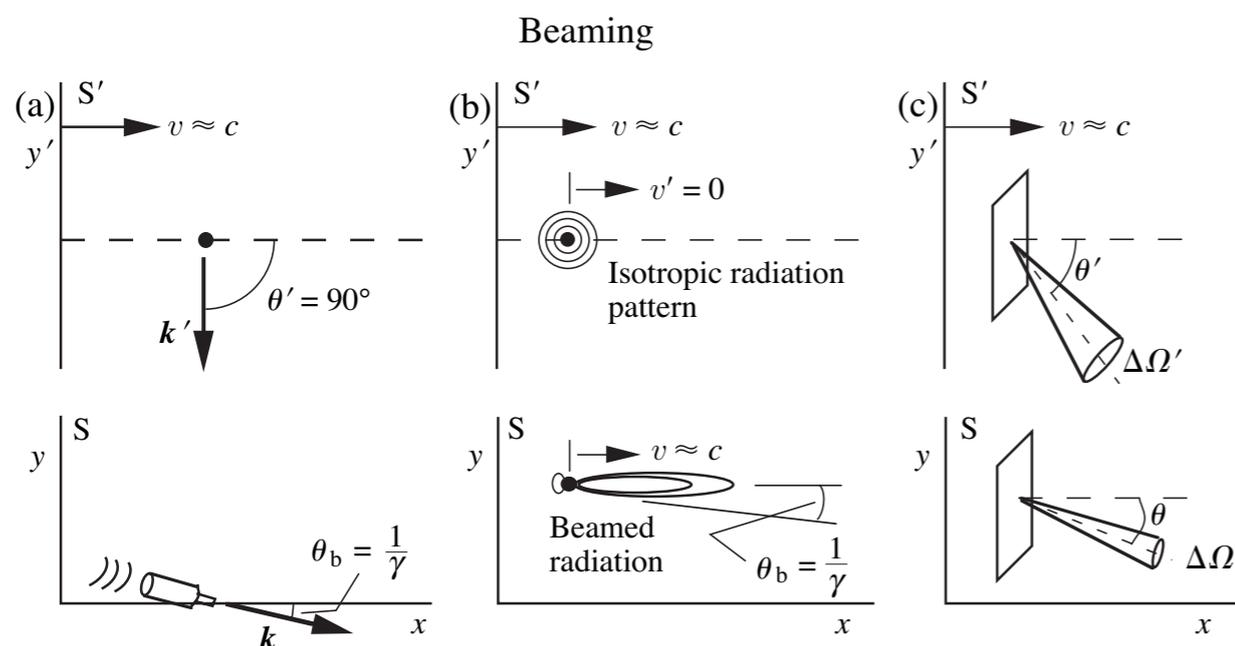


Fig. 7.9: Astrophysics Processes (CUP), © H Bradt 2008

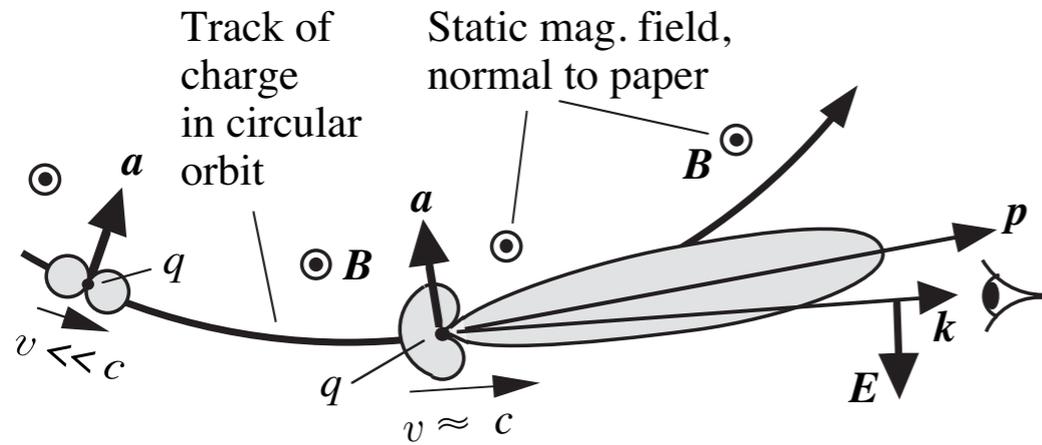
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$$\theta_b \approx \frac{1}{\gamma}$$

# DURATION OF THE PULSE

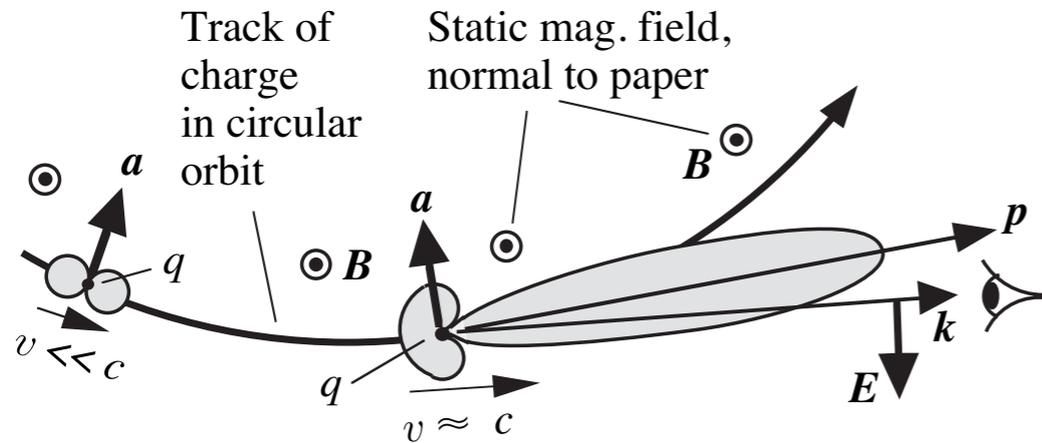


$$\theta_b \approx \frac{1}{\gamma} = \frac{m c^2}{U}$$

$$\omega_r = \frac{1}{\gamma} \omega = \frac{1}{\gamma} \frac{qB}{m}$$

$$\rightarrow \tau_0 \approx \frac{2 \theta_b}{\omega_r} = \frac{2m}{qB} = \frac{2}{\omega}$$

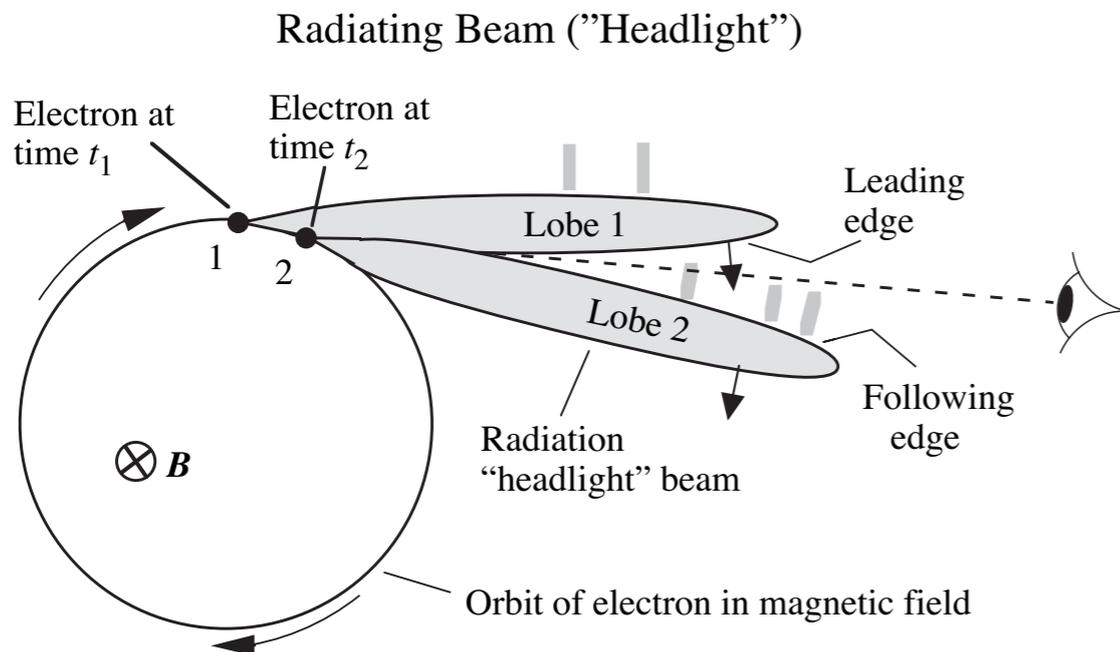
# DURATION OF THE PULSE



$$\theta_b \approx \frac{1}{\gamma} = \frac{m c^2}{U}$$

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$$\rightarrow \tau_0 \approx \frac{2 \theta_b}{\omega_r} = \frac{2m}{qB} = \frac{2}{\omega}$$



$$\rightarrow \tau_{\text{syn}} \approx \frac{\tau_0}{2 \gamma^2} = \frac{1}{\omega} \frac{1}{\gamma^2}$$

Fig. 8.6: Astrophysics Processes (CUP), © H. Bradt 2008

# DOPPLER SHIFT AND EFFECTS ON THE PULSE DURATION

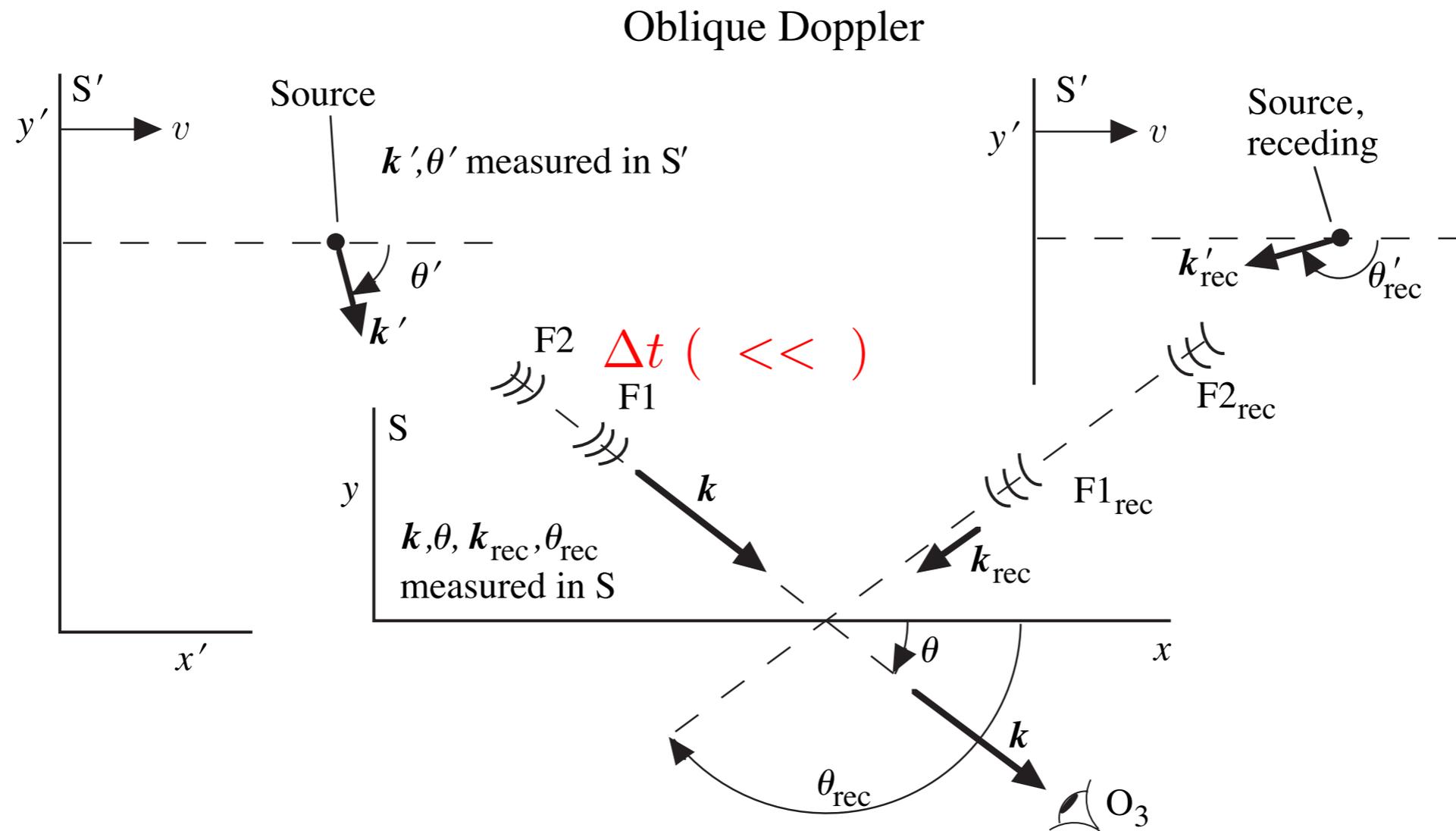


Fig. 7.4: Astrophysics Processes (CUP), © H Bradt 2008

$$\Delta t_3 = (1 - \beta \cos \theta) \Delta t$$

# DURATION OF THE PULSE

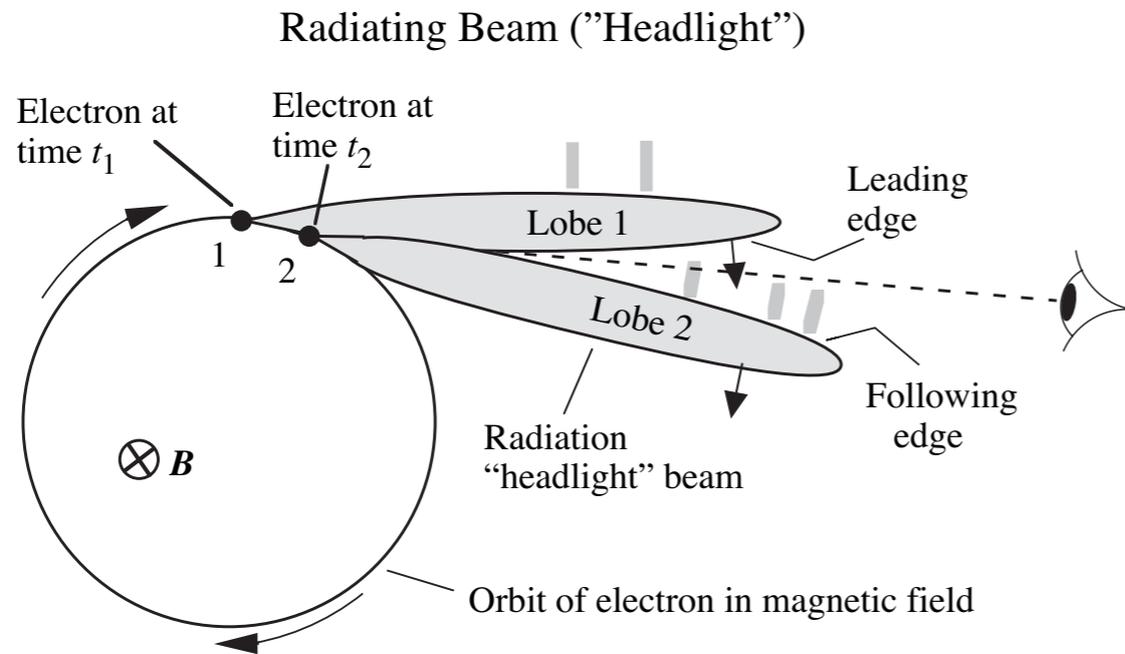


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$$\tau_0 \approx \frac{2 \theta_b}{\omega_r} = \frac{2m}{qB} = \frac{2}{\omega}$$

$$\Delta t_3 = (1 - \beta \cos \theta) \Delta t$$

$$\frac{\tau_{\text{syn}}}{\tau_0} \approx (1 - \beta \cos 0^\circ) = (1 - \beta) \frac{1 + \beta}{1 + \beta} = \frac{1 - \beta^2}{1 + \beta} \approx \frac{1}{2 \gamma^2}$$

$$\rightarrow \tau_{\text{syn}} \approx \frac{\tau_0}{2 \gamma^2} = \frac{1}{\omega} \frac{1}{\gamma^2}$$

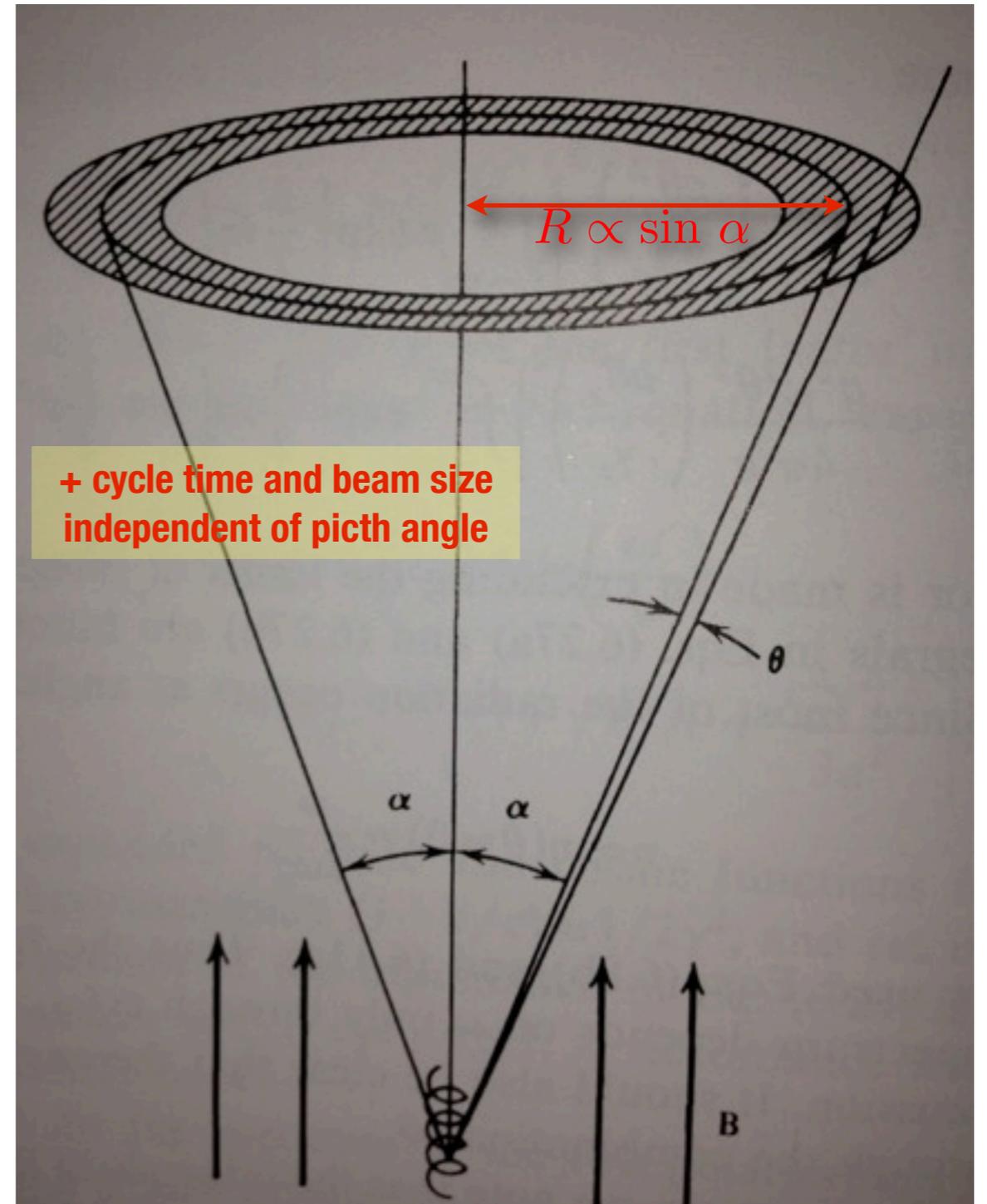
# OBSERVED FREQUENCY

$$\tau_{\text{syn}} \approx \frac{m}{\gamma^2 q B}$$

$$\omega_{\text{syn}} \approx \frac{1}{\tau_{\text{syn}}}$$

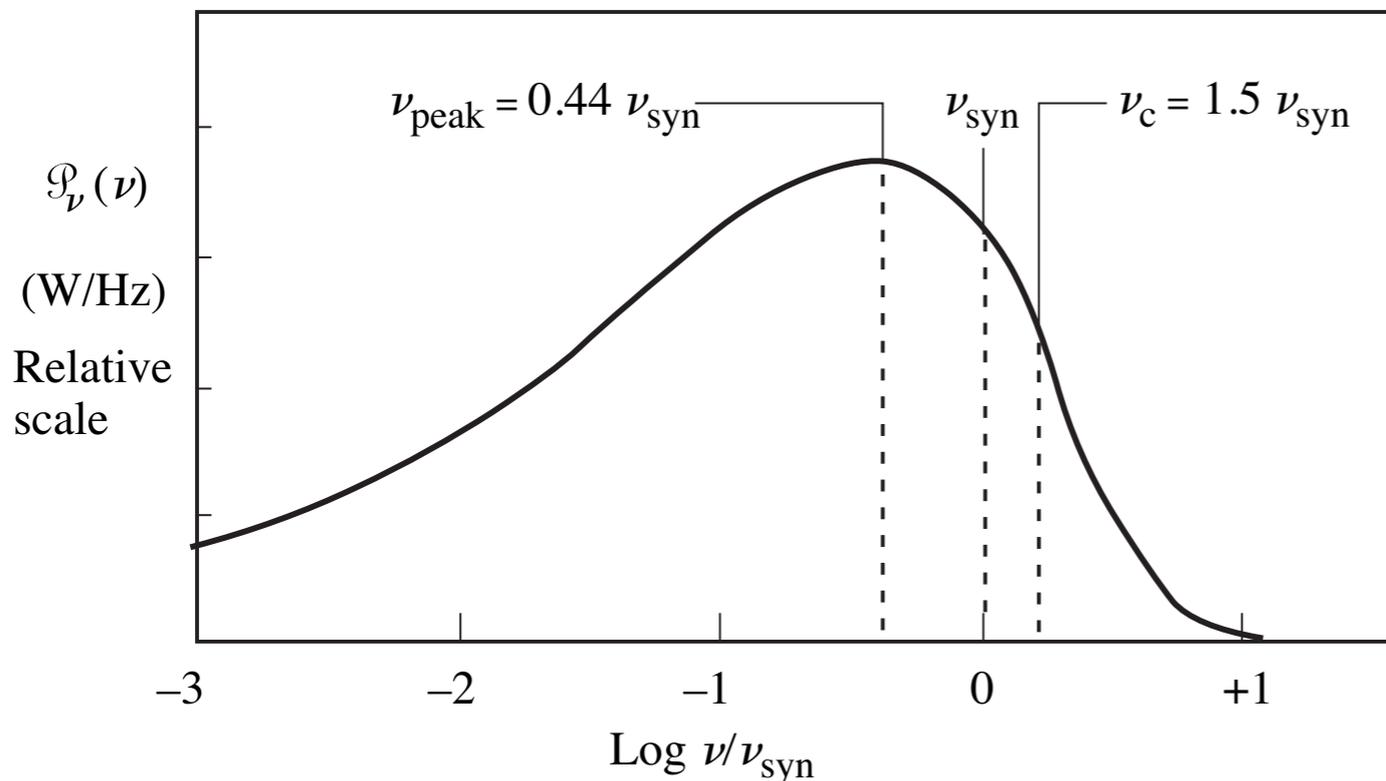
$$\nu_{\text{syn}} \equiv \frac{\omega_{\text{syn}}}{2\pi} \approx \frac{1}{2\pi} \gamma^2 \frac{q B}{m} \sin\phi$$

$$\gamma^2 = \left( \frac{U}{m c^2} \right)^2$$



# POWER SPECTRUM SHAPE

Radiated Power; Single Electron



$$P(\nu) = \frac{\sqrt{3} e^3 B \sin \phi}{m c^2} \left( \frac{\nu}{\nu_c} \right) \int_{\nu/\nu_c}^{\infty} K_{5/3}(\eta) d\eta$$

See Pacholczyk's "Astrophysics Processes"

Fig. 8.7: Astrophysics Processes (CUP), H. Bradt 2008  
 After B. Rossi & S. Olbert, *Intro. to the Physics of Space* (McGraw Hill) 1970

$$\nu_c \equiv \frac{3}{2} \nu_{\text{syn}} = \frac{3}{2} \gamma^2 \frac{\omega_{\text{syn}}}{2\pi} \sin \phi$$

# POWER RADIATED BY THE ELECTRON

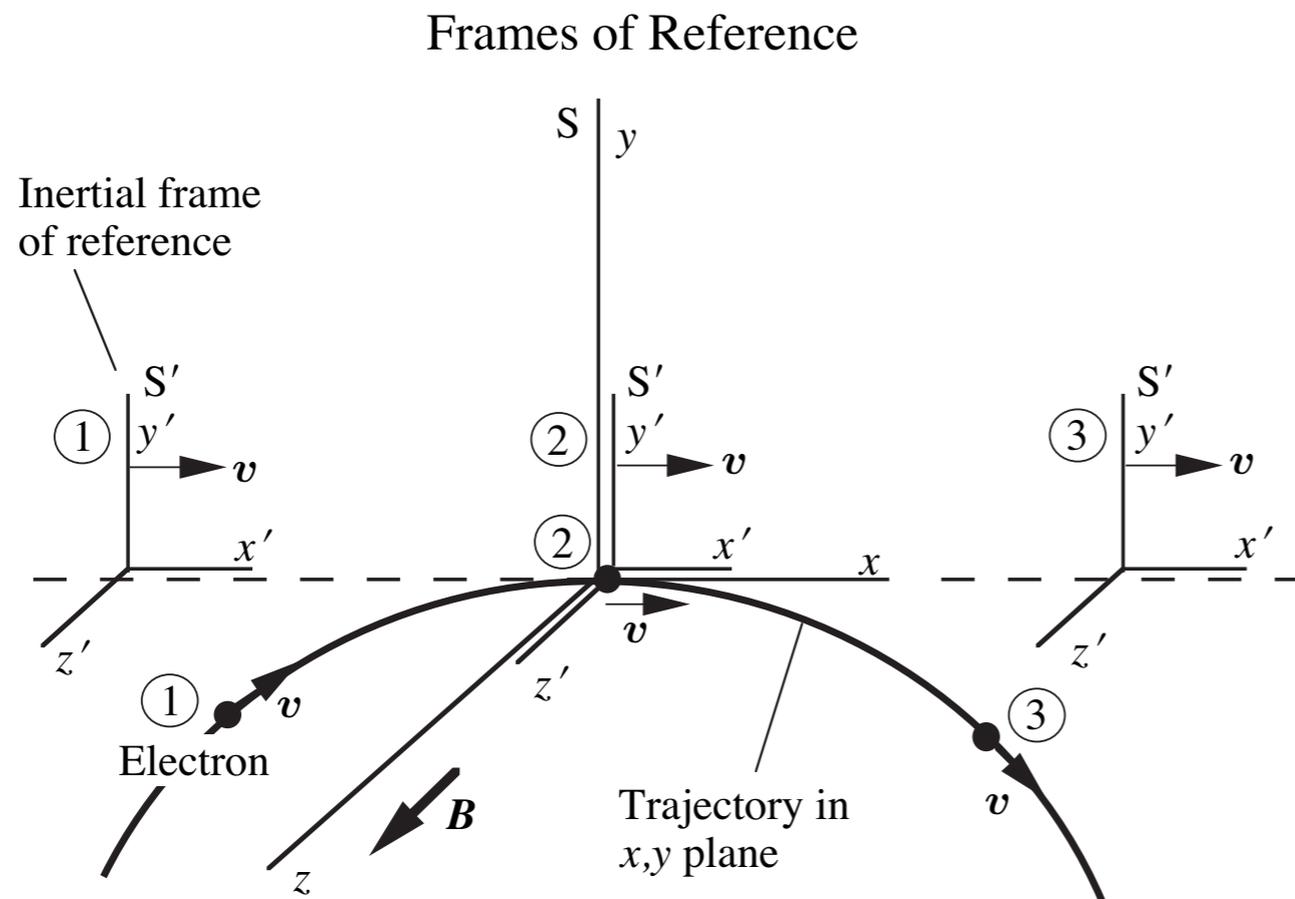


Fig. 8.8: Astrophysics Processes (CUP), © H. Bradt 2008

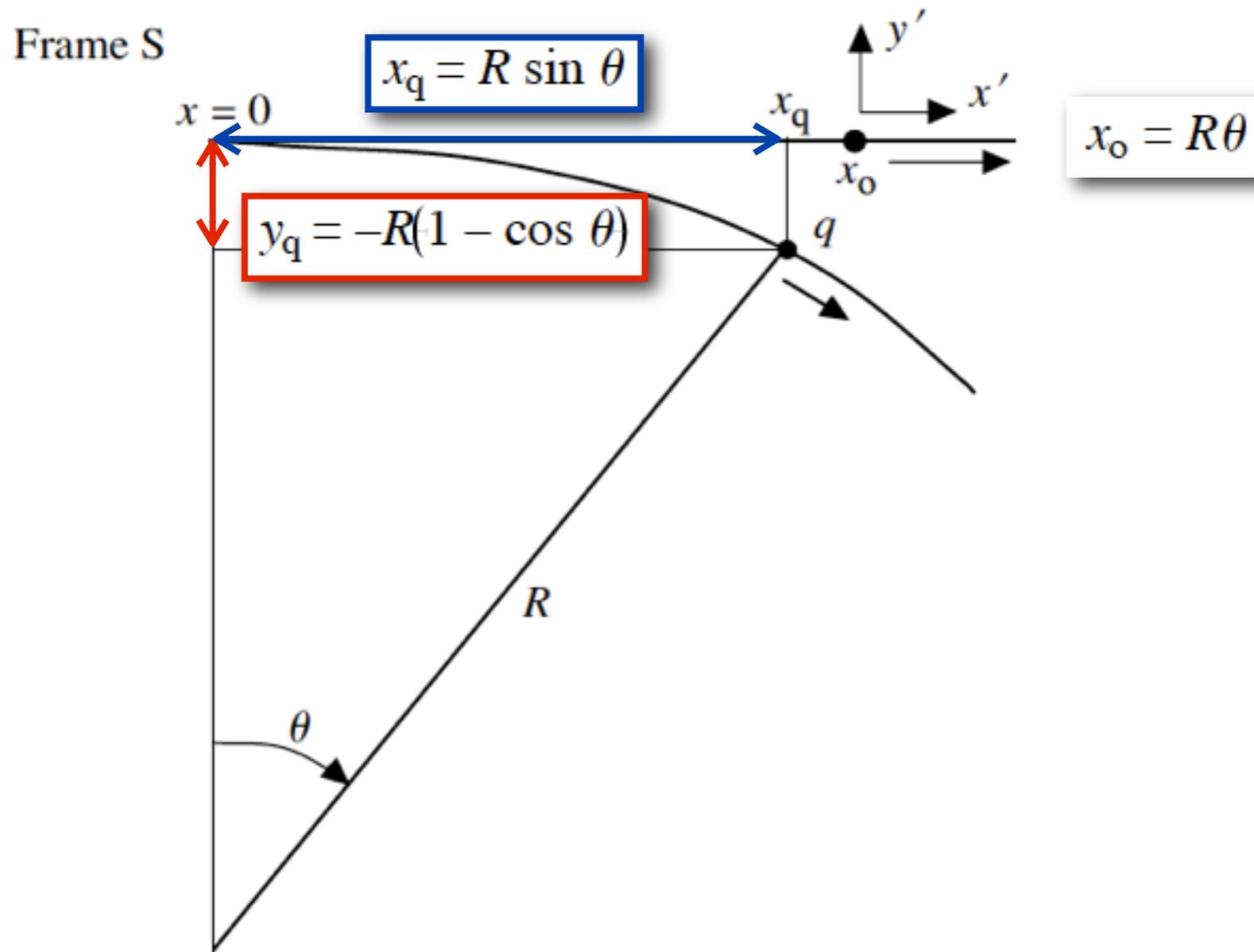
$$\mathbf{F} = q(\mathbf{v} \times \mathbf{B})$$

$$F_y = -qvB_z$$

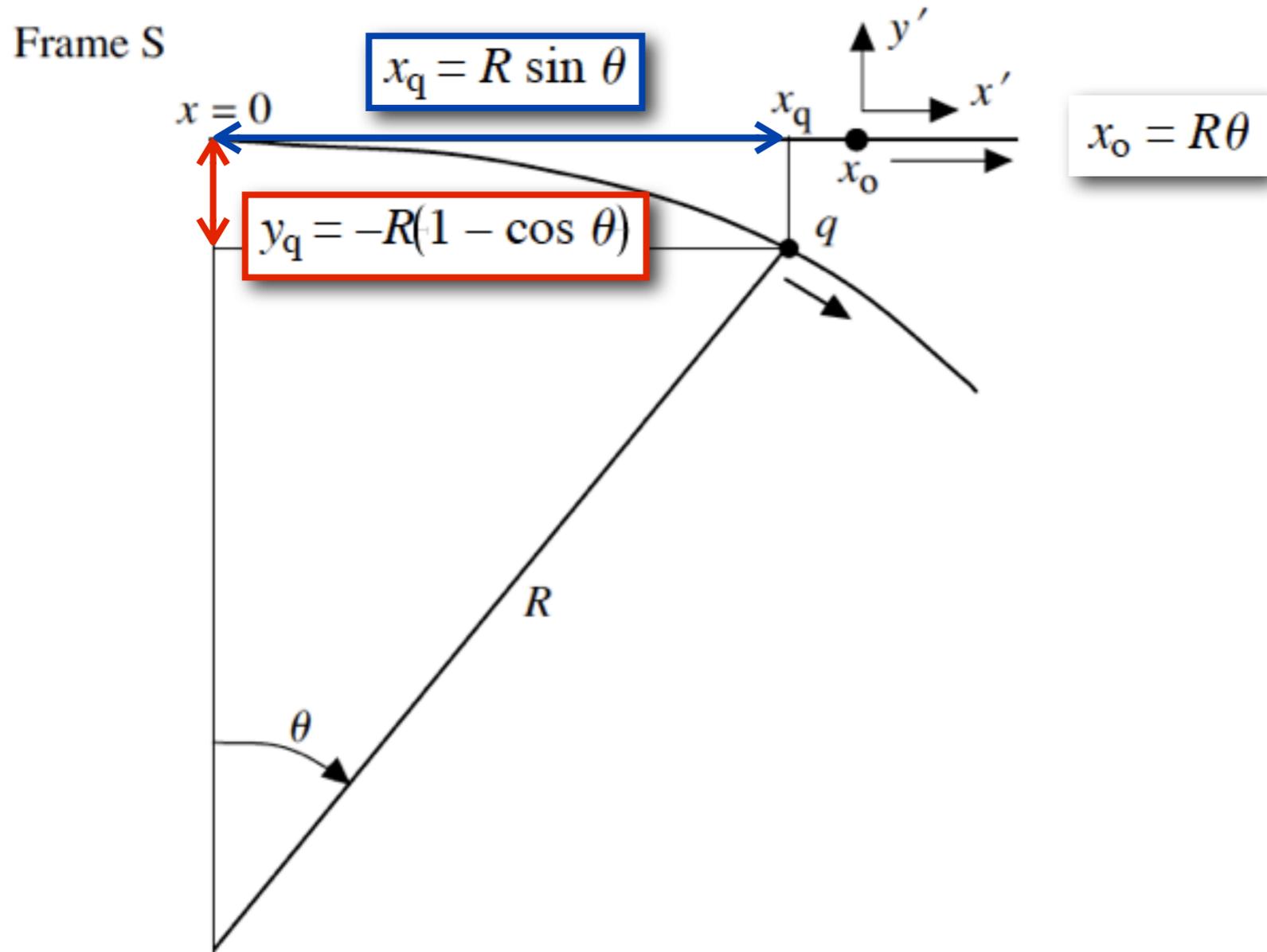
$$F_y/q = -vB_z$$

# POWER RADIATED BY THE ELECTRON

---



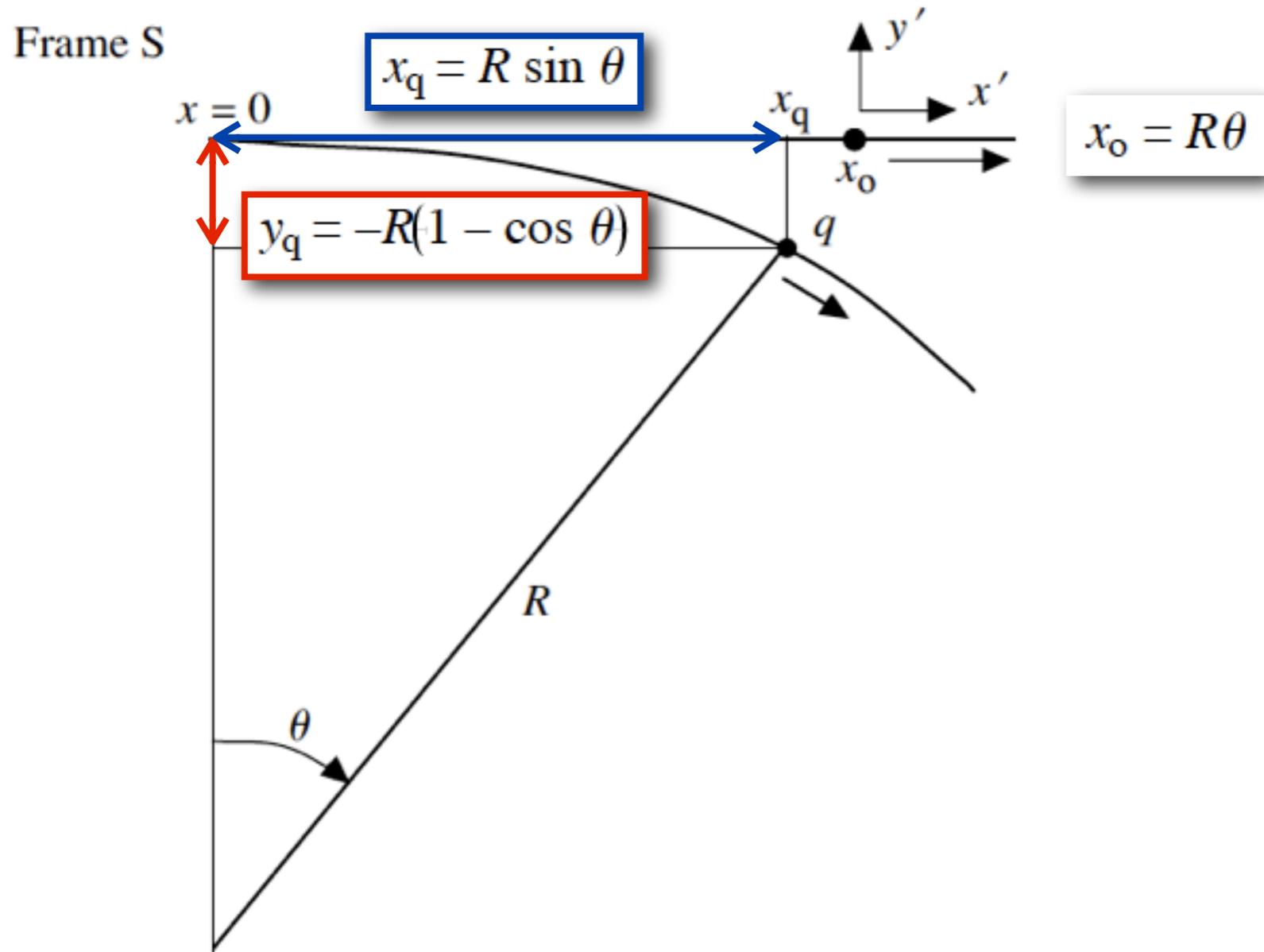
# POWER RADIATED BY THE ELECTRON



$$x' = x_q - x_o = R(\sin \theta - \theta)$$

$$y' = y_q - 0 = -R(1 - \cos \theta)$$

# POWER RADIATED BY THE ELECTRON



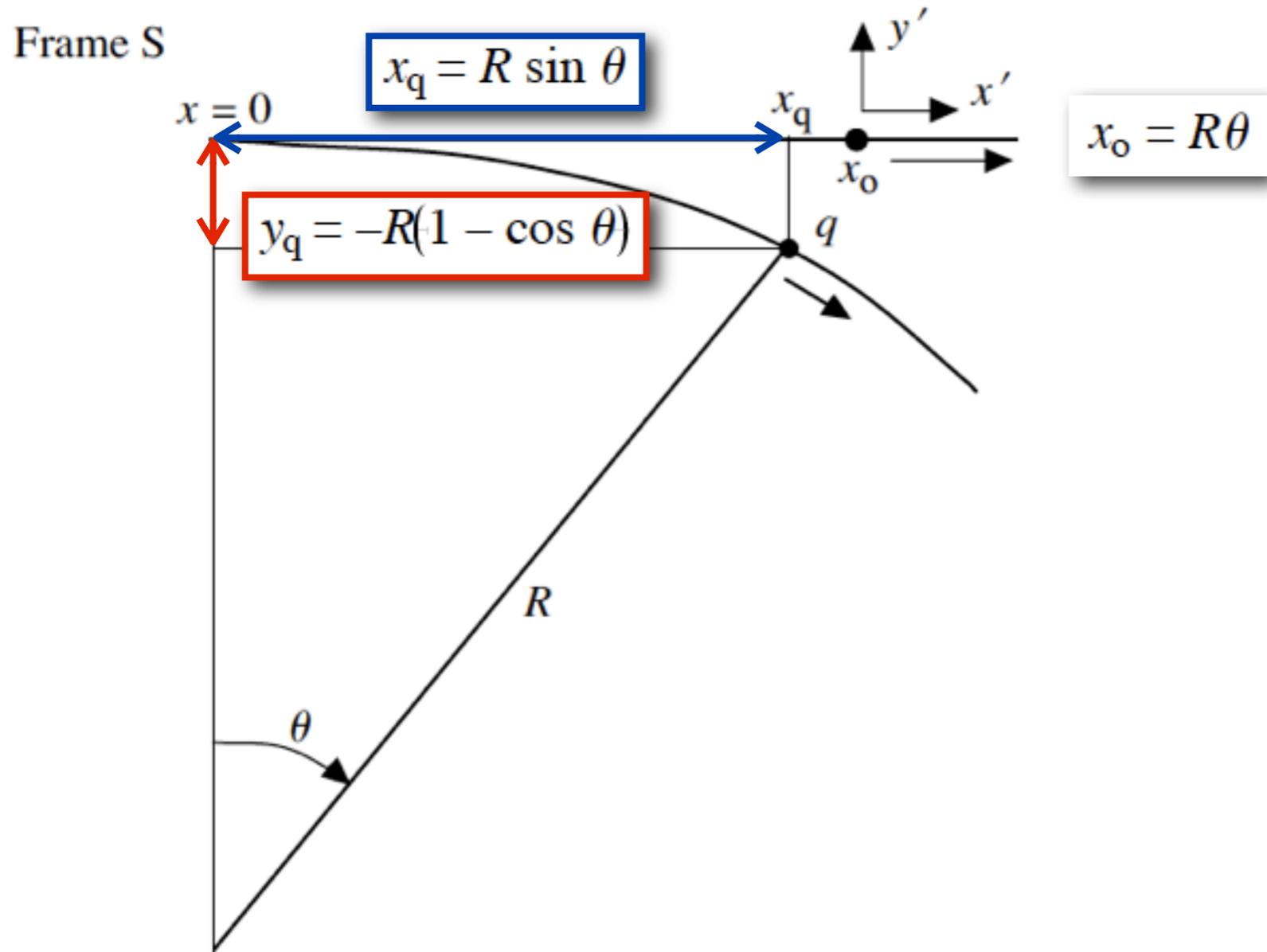
$$x' = x_q - x_o = R(\sin \theta - \theta)$$

$$y' = y_q - 0 = -R(1 - \cos \theta)$$

$$\cos \theta \approx 1 - \theta^2/2$$

$$\sin \theta \approx \theta - \theta^3/3$$

# POWER RADIATED BY THE ELECTRON



$$x' = x_q - x_o = R(\sin \theta - \theta)$$

$$y' = y_q - 0 = -R(1 - \cos \theta)$$

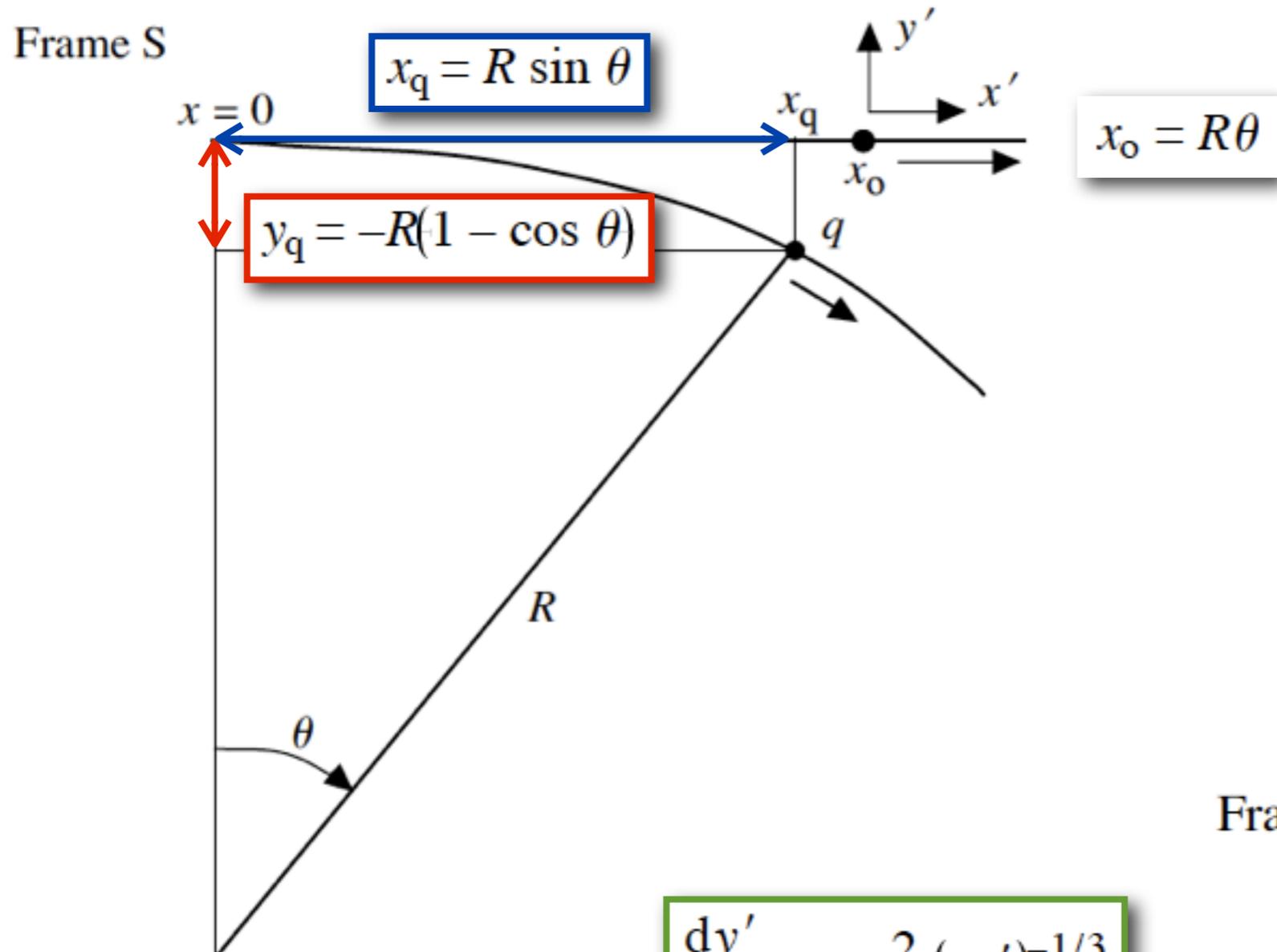
$$\cos \theta \approx 1 - \theta^2/2$$

$$\sin \theta \approx \theta - \theta^3/3$$

$$x' \approx -R\theta^3/3!$$

$$y' \approx -R\theta^2/2!$$

# POWER RADIATED BY THE ELECTRON



$$x' = x_q - x_o = R(\sin \theta - \theta)$$

$$y' = y_q - 0 = -R(1 - \cos \theta)$$

$$\cos \theta \approx 1 - \theta^2/2$$

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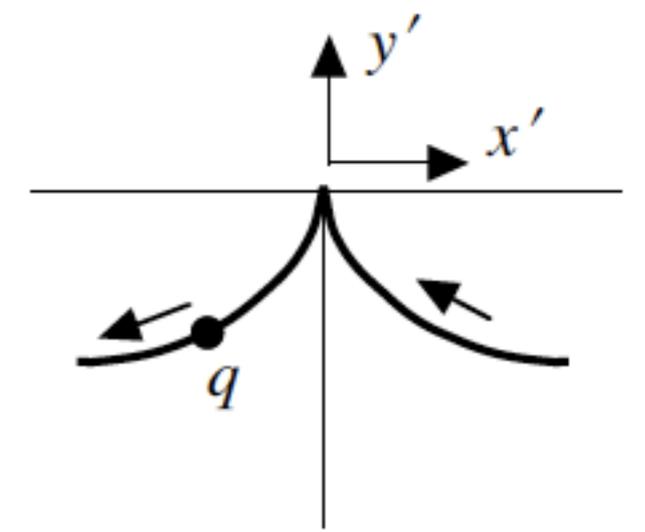
$$x' \approx -R\theta^3/3!$$

$$y' \approx -R\theta^2/2!$$

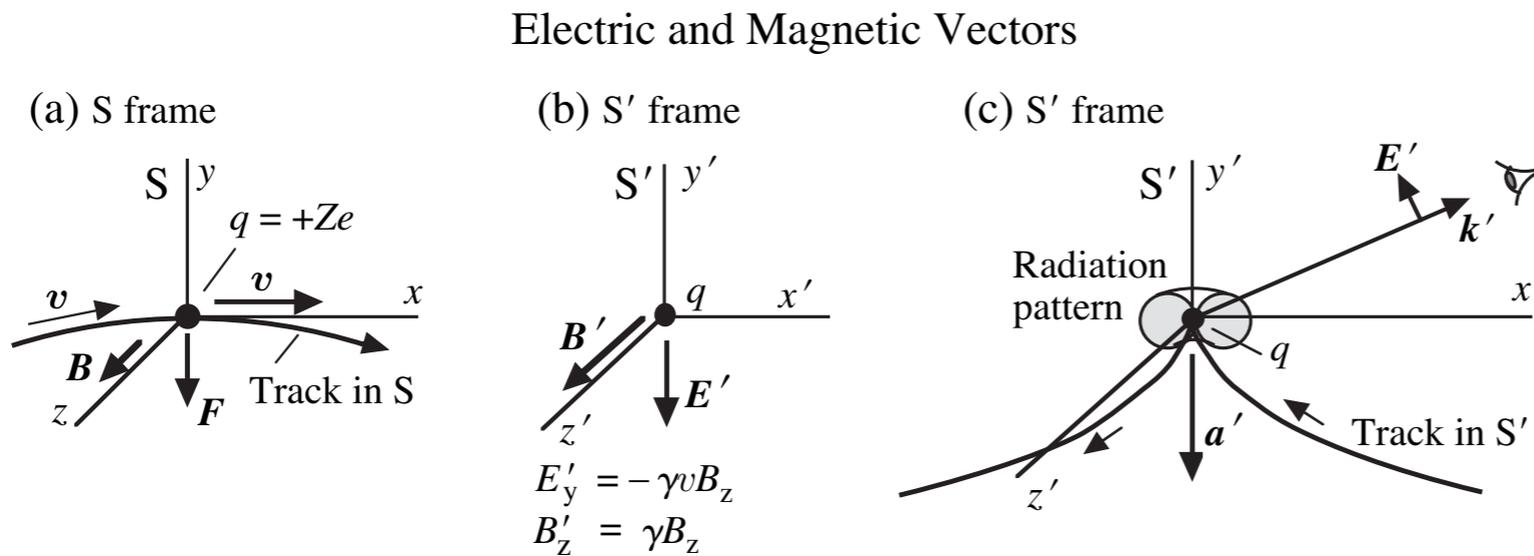
$$y' = -R^{1/3} \left(\frac{9}{2}\right)^{1/3} (-x')^{2/3}$$

$$\frac{dy'}{dx'} \propto +\frac{2}{3} (-x')^{-1/3}$$

Frame S'



# POWER RADIATED BY THE ELECTRON



$$\frac{dU'}{dt'} = -\frac{1}{6\pi\epsilon_0} \frac{q^2 a'^2}{c^3}$$

Fig. 8.9: Astrophysics Processes (CUP), © H. Bradt 2008

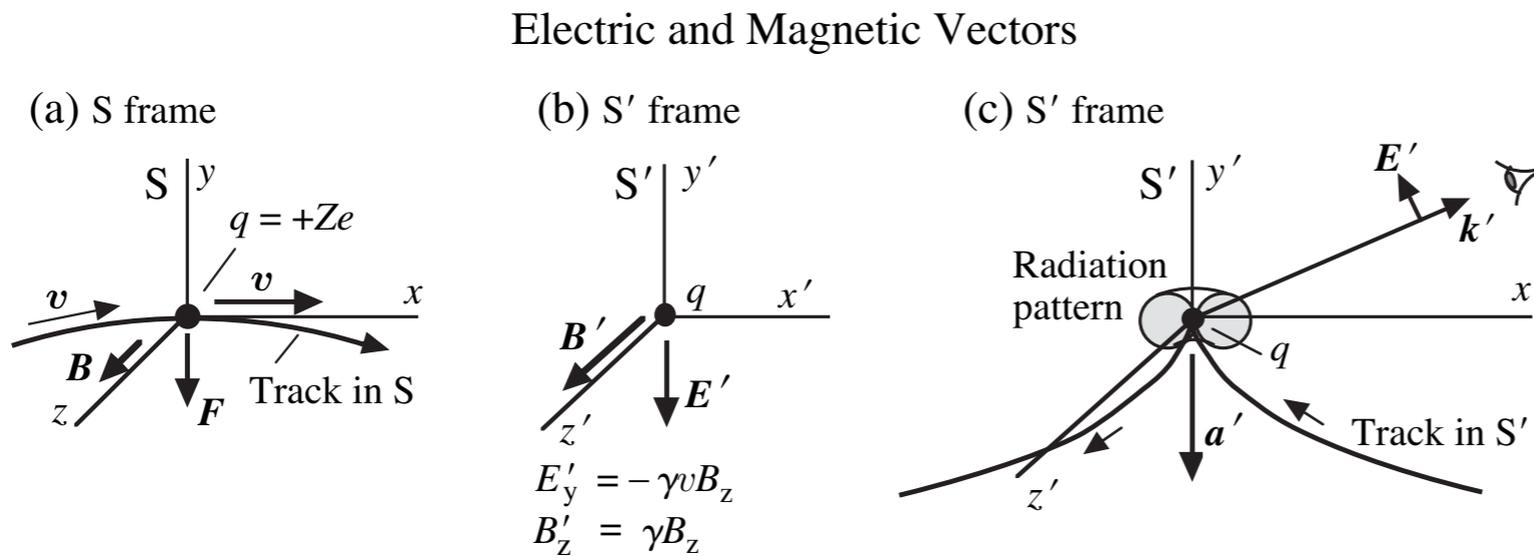
$$\mathbf{B} : 0, 0, B_z; \quad \mathbf{E} : 0, 0, 0$$

$$\mathbf{B}' : 0, 0, \gamma B_z; \quad \mathbf{E}' : 0, -\gamma\beta c B_z, 0$$

$$B'_z = -\gamma B_z$$

$$E'_y = -\gamma\beta c B_z = -\gamma v B_z$$

# POWER RADIATED BY THE ELECTRON



$$\frac{dU'}{dt'} = -\frac{1}{6\pi\epsilon_0} \frac{q^2 a'^2}{c^3}$$

Fig. 8.9: Astrophysics Processes (CUP), © H. Bradt 2008

$$\mathbf{B} : 0, 0, B_z; \quad \mathbf{E} : 0, 0, 0$$

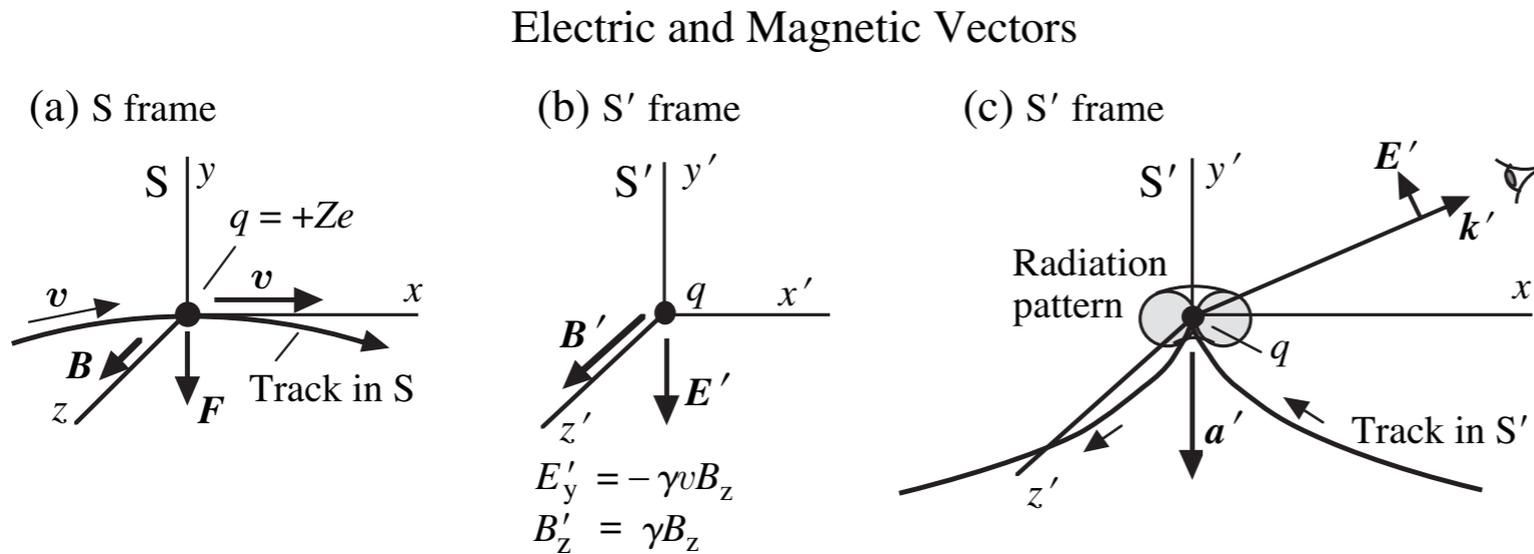
$$\mathbf{B}' : 0, 0, \gamma B_z; \quad \mathbf{E}' : 0, -\gamma\beta c B_z, 0$$

$$B'_z = -\gamma B_z$$

$$F_y/q = -v B_z$$

$$E'_y = -\gamma\beta c B_z = -\gamma v B_z$$

# POWER RADIATED BY THE ELECTRON



$$\frac{dU'}{dt'} = -\frac{1}{6\pi\epsilon_0} \frac{q^2 a'^2}{c^3}$$

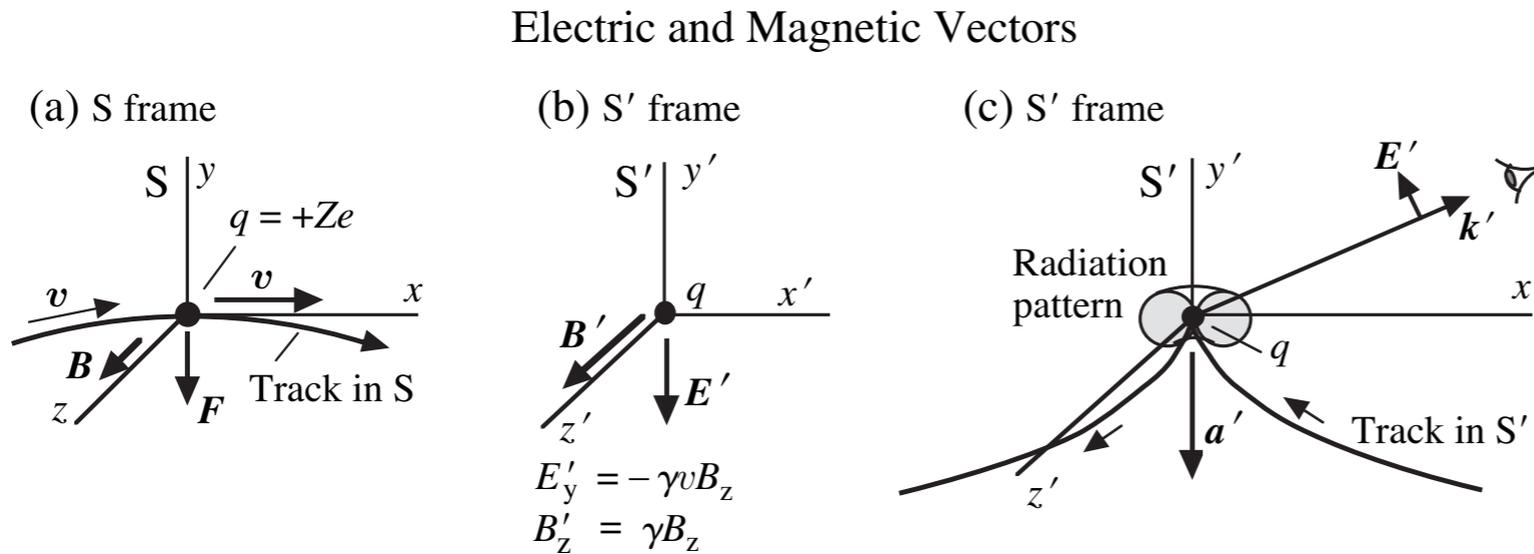
Fig. 8.9: Astrophysics Processes (CUP), © H. Bradt 2008

$$E'_y = -\gamma\beta c B_z = -\gamma v B_z \quad a'_y = \frac{F'_y}{m} = \frac{q E'_y}{m} = -\frac{Ze\gamma v B_z}{m}$$

$$B'_z = -\gamma B_z \quad a' \equiv a'_y \quad B_z \equiv B$$

$$a' = -Ze \frac{U}{mc^2} \frac{v B}{m} \approx -\frac{ZeBU}{m^2 c}$$

# POWER RADIATED BY THE ELECTRON



$$\frac{dU'}{dt'} = -\frac{1}{6\pi\epsilon_0} \frac{q^2 a'^2}{c^3}$$

Fig. 8.9: Astrophysics Processes (CUP), © H. Bradt 2008

$$E'_y = -\gamma\beta c B_z = -\gamma v B_z \quad a'_y = \frac{F'_y}{m} = \frac{q E'_y}{m} = -\frac{Ze\gamma v B_z}{m}$$

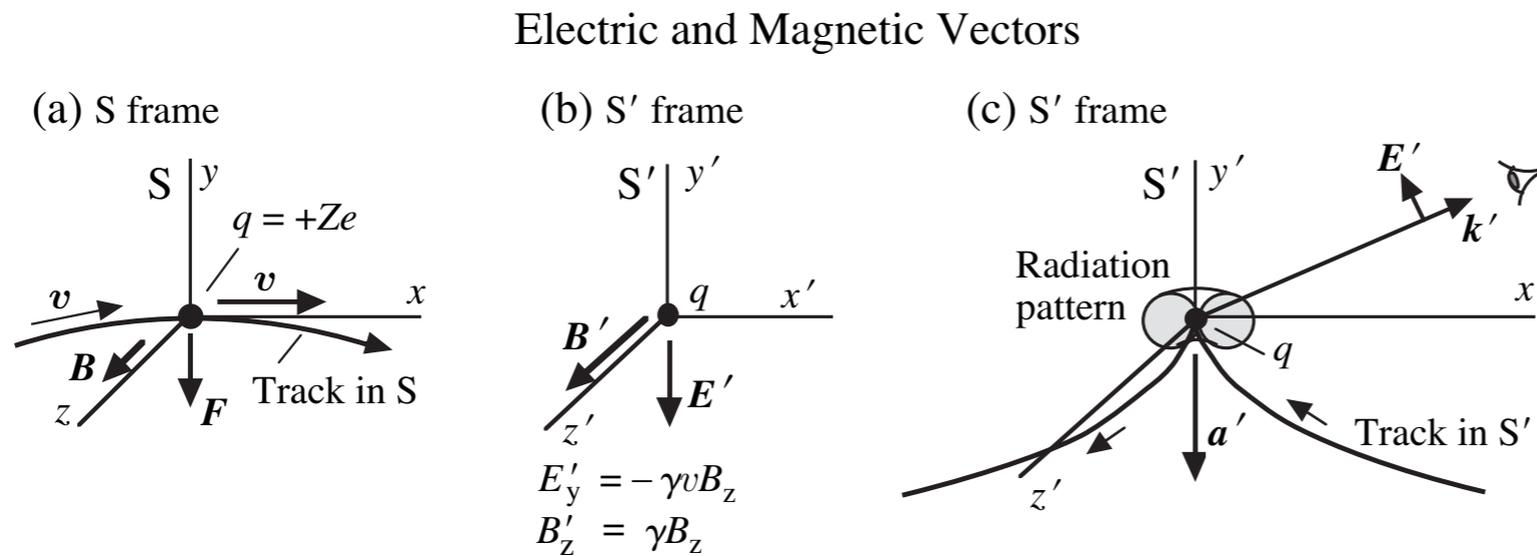
$$B'_z = -\gamma B_z \quad a' \equiv a'_y \quad B_z \equiv B$$

$$a' = -Ze \frac{U}{mc^2} \frac{v B}{m} \approx -\frac{ZeBU}{m^2 c}$$

$$v \approx c$$

$$U = \gamma mc^2$$

# POWER RADIATED BY THE ELECTRON



$$\frac{dU'}{dt'} = -\frac{1}{6\pi\epsilon_0} \frac{q^2 a'^2}{c^3}$$

$$a' = -\frac{ZeBU}{m^2 c}$$

Fig. 8.9: Astrophysics Processes (CUP), © H. Bradt 2008

$$\frac{dU'}{dt'} = -\frac{1}{6\pi\epsilon_0} \frac{(Ze)^2}{c^3} \left( \frac{ZeBU}{m^2 c} \right)^2$$

$$\frac{dU'}{dt'} = -\frac{1}{6\pi\epsilon_0} \frac{1}{c^5} \left( \frac{Ze}{m} \right)^4 U^2 B^2 \quad [\text{W}]$$

$$v \approx c$$

$$U = \gamma m c^2$$

# LORENTZ TRANSFORMATIONS

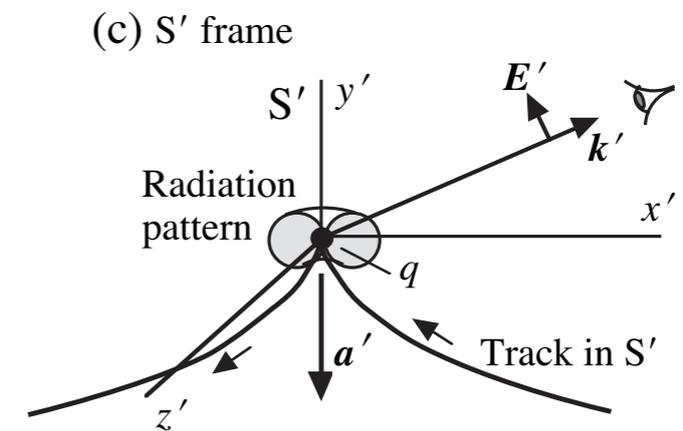
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## Space-Time

$$\begin{bmatrix} ct' \\ x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} \gamma & -\beta\gamma & 0 & 0 \\ -\beta\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} ct \\ x \\ y \\ z \end{bmatrix}$$

## Momentum-Energy

$$\begin{bmatrix} E' \\ p_x' c \\ p_y' c \\ p_z' c \end{bmatrix} = \begin{bmatrix} \gamma & -\beta\gamma & 0 & 0 \\ -\beta\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} E \\ p_x c \\ p_y c \\ p_z c \end{bmatrix}$$



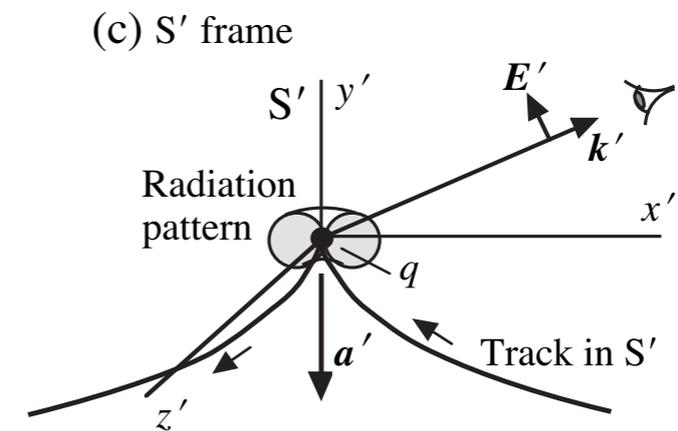
# LORENTZ TRANSFORMATIONS

## Space-Time

$$\begin{bmatrix} ct' \\ x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} \gamma & -\beta\gamma & 0 & 0 \\ -\beta\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} ct \\ x \\ y \\ z \end{bmatrix}$$

## Momentum-Energy

$$\begin{bmatrix} E' \\ p_x' c \\ p_y' c \\ p_z' c \end{bmatrix} = \begin{bmatrix} \gamma & -\beta\gamma & 0 & 0 \\ -\beta\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} E \\ p_x c \\ p_y c \\ p_z c \end{bmatrix}$$



$$dU = \gamma dU'$$

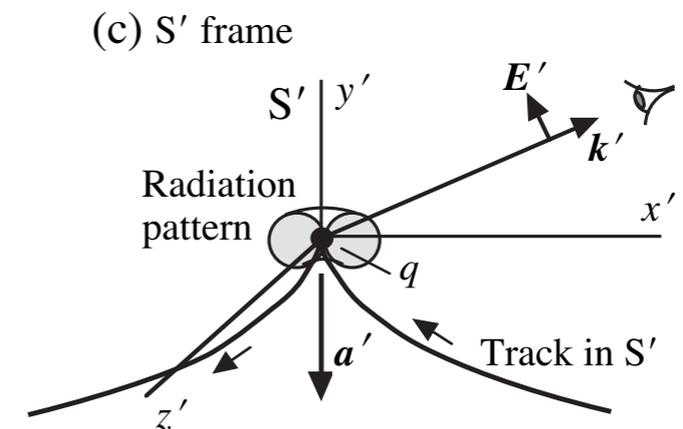
# LORENTZ TRANSFORMATIONS

## Space-Time

$$\begin{bmatrix} ct' \\ x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} \gamma & -\beta\gamma & 0 & 0 \\ -\beta\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} ct \\ x \\ y \\ z \end{bmatrix}$$

## Momentum-Energy

$$\begin{bmatrix} E' \\ p_x' c \\ p_y' c \\ p_z' c \end{bmatrix} = \begin{bmatrix} \gamma & -\beta\gamma & 0 & 0 \\ -\beta\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} E \\ p_x c \\ p_y c \\ p_z c \end{bmatrix}$$



$$dU = \gamma dU'$$

$$dt = \gamma dt'$$

# LORENTZ TRANSFORMATIONS

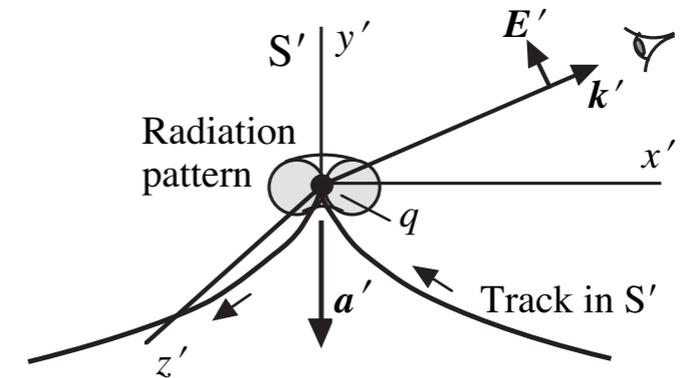
## Space-Time

$$\begin{bmatrix} ct' \\ x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} \gamma & -\beta\gamma & 0 & 0 \\ -\beta\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} ct \\ x \\ y \\ z \end{bmatrix}$$

## Momentum-Energy

$$\begin{bmatrix} E' \\ p_x' c \\ p_y' c \\ p_z' c \end{bmatrix} = \begin{bmatrix} \gamma & -\beta\gamma & 0 & 0 \\ -\beta\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} E \\ p_x c \\ p_y c \\ p_z c \end{bmatrix}$$

(c) S' frame



$$dU = \gamma dU'$$

$$dt = \gamma dt'$$

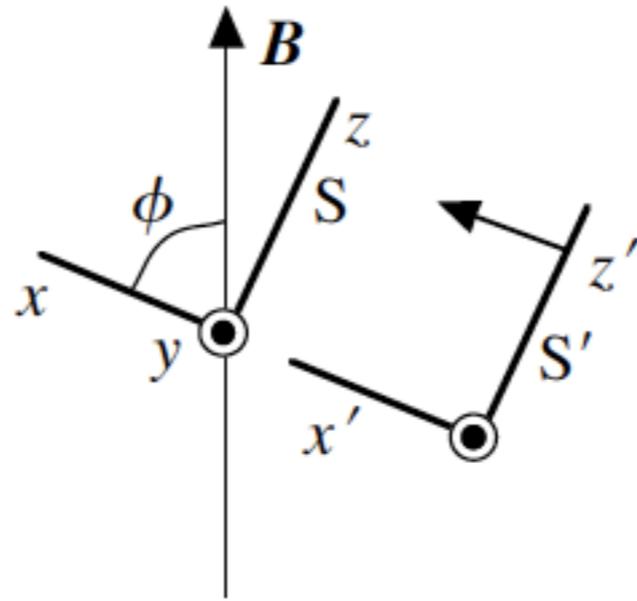
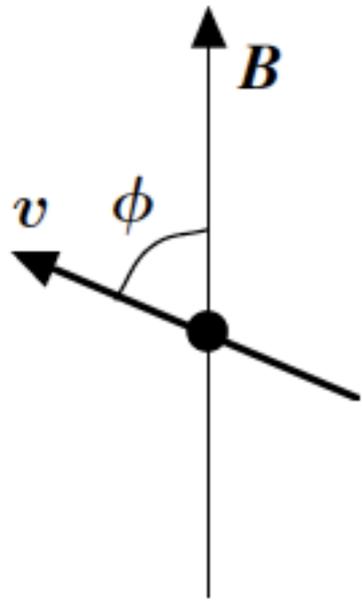


$$\frac{dU}{dt} = \frac{dU'}{dt'}$$



$$\frac{dU'}{dt'} = -\frac{1}{6\pi\epsilon_0} \frac{1}{c^5} \left( \frac{Ze}{m} \right)^4 U^2 B^2 \quad [\text{W}]$$

# POWER RADIATED BY THE ELECTRON



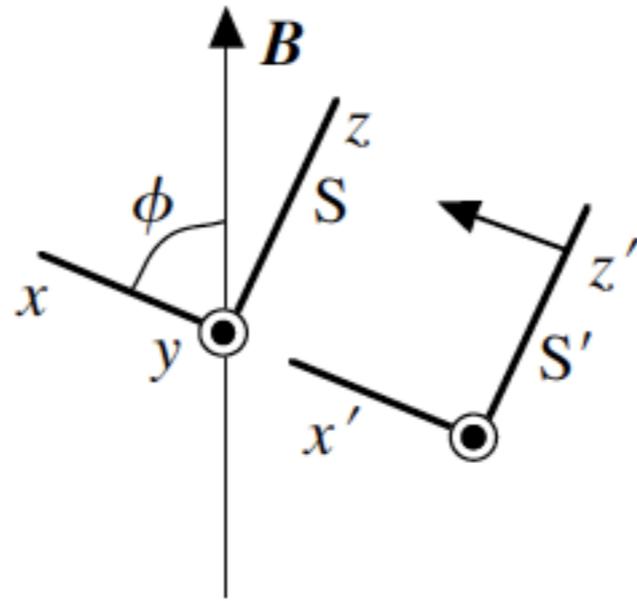
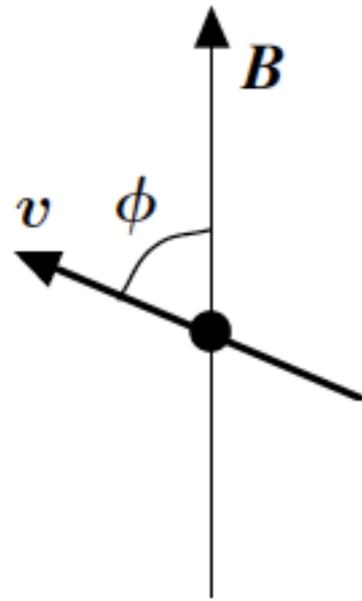
$$B_z = B \sin \phi$$

$$\frac{dU'}{dt'} = -\frac{1}{6\pi\epsilon_0} \frac{q^2 a'^2}{c^3}$$

$$a' = -Ze \frac{U}{mc^2} \frac{v B}{m} \sin \phi$$

$$\frac{dU}{dt} = -\frac{1}{6\pi\epsilon_0} \frac{1}{c^5} \left( \frac{Ze}{m} \right)^4 U^2 \beta^2 B^2 \sin^2 \phi \quad [\text{W}]$$

# POWER RADIATED BY THE ELECTRON



$$B_z = B \sin \phi$$

$$\frac{dU'}{dt'} = -\frac{1}{6\pi\epsilon_0} \frac{q^2 a'^2}{c^3}$$

$$a' = -Ze \frac{U}{mc^2} \frac{v B}{m} \sin \phi$$

**$\sim 10^{13}$  times greater for electrons than for protons !**

$$\frac{dU}{dt} = -\frac{1}{6\pi\epsilon_0} \frac{1}{c^5} \left( \frac{Ze}{m} \right)^4 U^2 \beta^2 B^2 \sin^2 \phi \quad [\text{W}]$$

# POWER RADIATED BY THE ELECTRON

---

$$\frac{dU}{dt} = -\frac{1}{6\pi\epsilon_0} \frac{1}{c^5} \left(\frac{Ze}{m}\right)^4 U^2 \beta^2 B^2 \sin^2 \phi \quad [\text{W}]$$

$$u_B = \frac{B^2}{2\mu_0} \quad [\text{J/m}^3]$$

$$\sigma_T = \frac{\mathcal{P}}{\mathcal{F}_P} = \frac{8\pi}{3} r_e^2 = 6.6525 \times 10^{-29} \text{ m}^2$$

$$\langle \sin^2 \phi \rangle_{\text{av}} = \frac{1}{4\pi} \int_{\text{sphere}} \sin^2 \phi \, d\Omega = \frac{2}{3}$$

$$r_e = \frac{1}{4\pi\epsilon_0} \frac{e^2}{m_e c^2} = 2.8179 \times 10^{-15} \text{ m}$$

$$U = \gamma m c^2 \quad c^2 = (\mu_0 \epsilon_0)^{-1}$$

$$\frac{dU}{dt} = -\frac{4}{3} \sigma_T c \beta^2 \gamma^2 u_B$$

# ELECTRON ENERGY LIFETIME

---

$$\frac{dU}{dt} \propto -U^2 B^2$$

$$\tau = -\frac{U}{dU/dt} \propto \frac{U}{U^2 B^2} \propto \frac{1}{UB^2} \propto \frac{1}{\gamma B^2}$$

$$\tau = \frac{5.16}{B^2} \frac{1}{\gamma}$$

**Relativistic electron decay time in sec; B in Teslas (=10 kGauss)**

# INVERSE COMPTON

## FULL DERIVATION POWER FOR SINGLE SCATTERING

---

$$\frac{dE_{\text{rad}}}{dt} = c \sigma_T U_{\text{rad}} \left[ \gamma^2 + \frac{1}{3} \gamma^2 \beta^2 - 1 \right] = c \sigma_T U_{\text{rad}} \left[ (\gamma^2 - 1) + \frac{1}{3} \gamma^2 \beta^2 \right] = c \sigma_T U_{\text{rad}} \left[ \gamma^2 \beta^2 + \frac{1}{3} \gamma^2 \beta^2 \right]$$

$$\frac{dE_{\text{rad}}}{dt} = P_{\text{compt}} = \frac{4}{3} \sigma_T c \gamma^2 \beta^2 U_{\text{rad}}$$

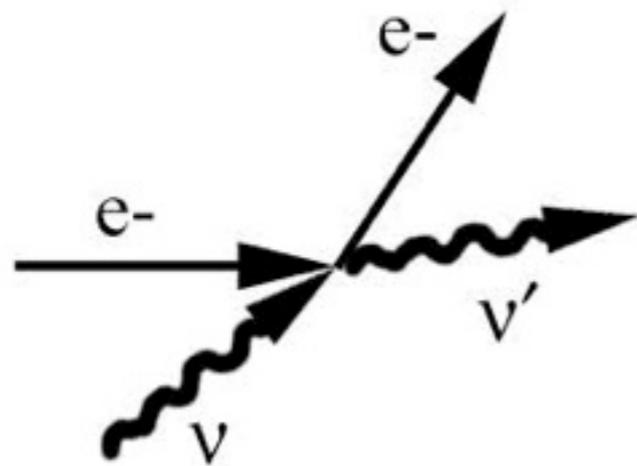
**Net power lost by the relativistic electron  
and converted into increased radiation**

$$\frac{dE_{\text{rad}}}{dt} = \frac{4}{3} \sigma_T c \gamma^2 \int \epsilon v d\epsilon \quad (\beta \approx 1)$$

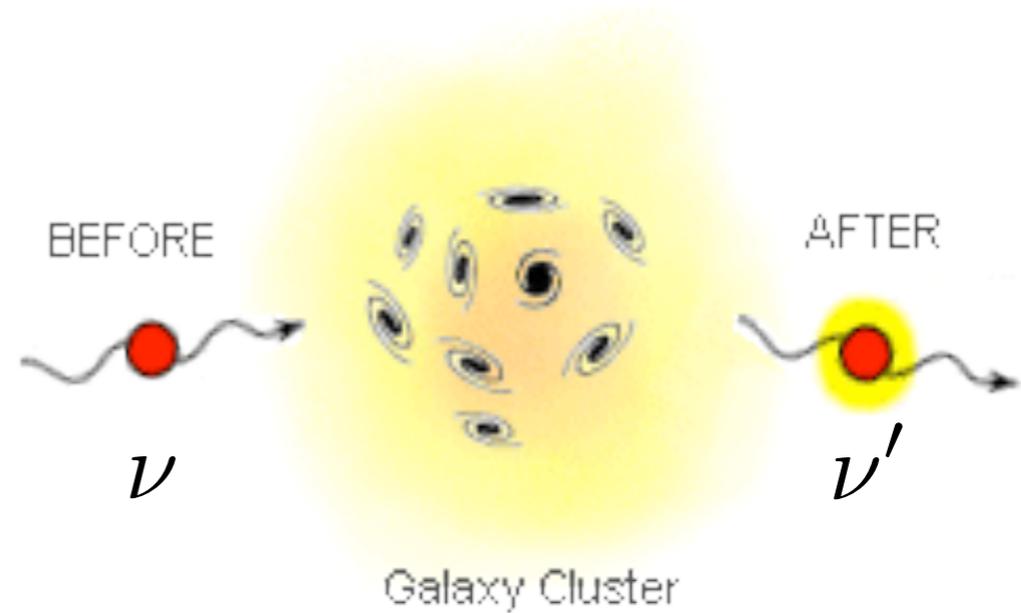
$$\frac{dE_{\text{rad}}}{dt} = \frac{4}{3} \sigma_T c \gamma^2 \langle \epsilon \rangle \int v d\epsilon = \sigma_T c \langle \epsilon_1 \rangle \int v d\epsilon$$

# ELECTRON ENERGY LIFETIME

Inverse Compton scattering



$\nu' > \nu$   
High energy e- initially  
e- loses energy

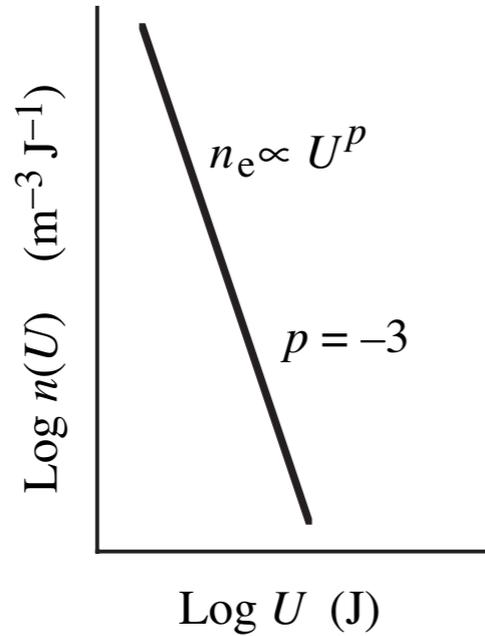


$$\frac{\langle \nu' \rangle}{\nu} = \frac{4}{3} \gamma_{el}^2$$

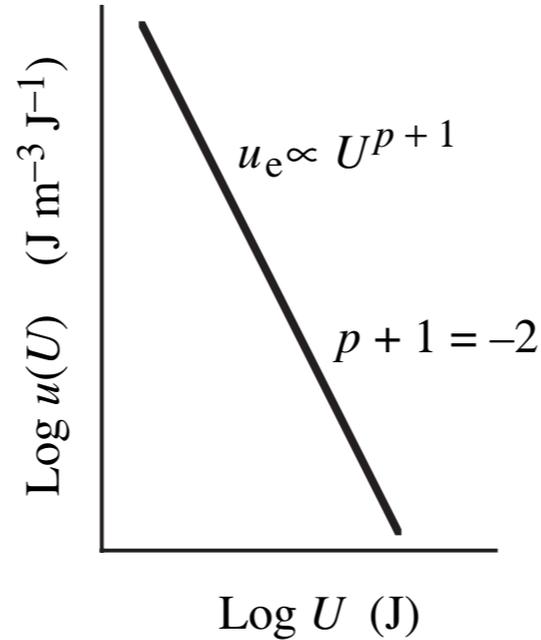
$$\tau \approx 2 \times 10^3 \gamma^{-1} \left[ (1+z)^4 + \left( \frac{B}{3.3 \mu\text{G}} \right)^2 \right]^{-1} \text{Gyr} \begin{cases} E_{e^-} \approx \text{GeV} \rightarrow \tau \lesssim \text{Gyr} \\ E_{e^-} \approx \text{TeV} \rightarrow \tau \lesssim \text{Myr} \end{cases}$$

# ENSEMBLE OF RADIATING PARTICLES

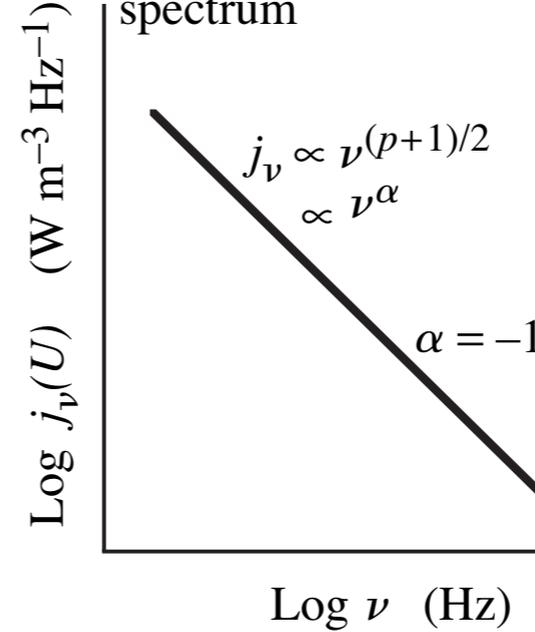
(a) Electron number-density spectrum



(b) Electron energy-density spectrum



(c) Photon energy-density (volume-emissivity) spectrum



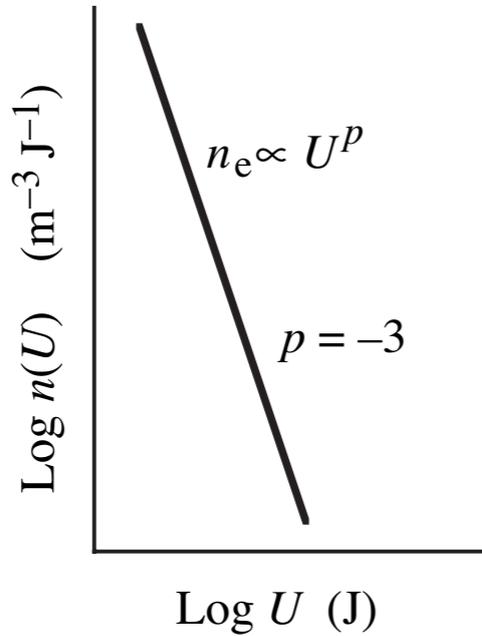
$$n(U)dU \propto U^p dU = \frac{n_0}{U_0^p} U^p dU = n_0 \left( \frac{U}{U_0} \right)^p dU \quad (\text{Particles}/\text{m}^3 \text{ at } U \text{ in } dU)$$

$$u(U)dU = U n(U) dU = U n_0 \left( \frac{U}{U_0} \right)^p dU = u_0 \left( \frac{U}{U_0} \right)^{p+1} dU \quad (\text{Energy density in } dU \text{ at } U)$$

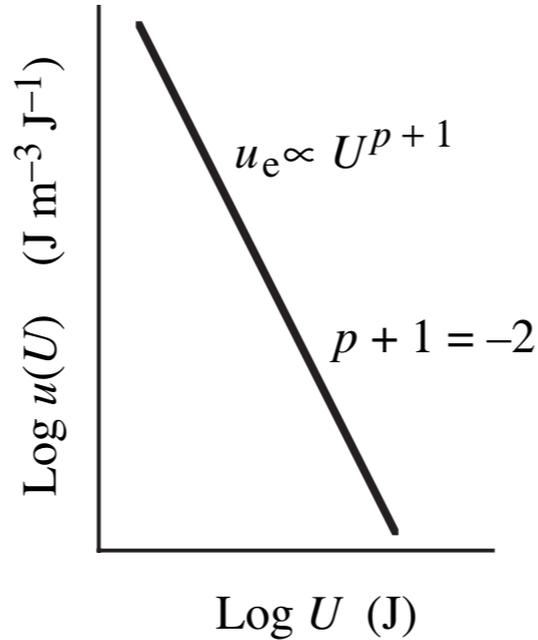
$\times U_0/U_0; u_0 \equiv U_0 n_0$

# ENSEMBLE OF RADIATING PARTICLES

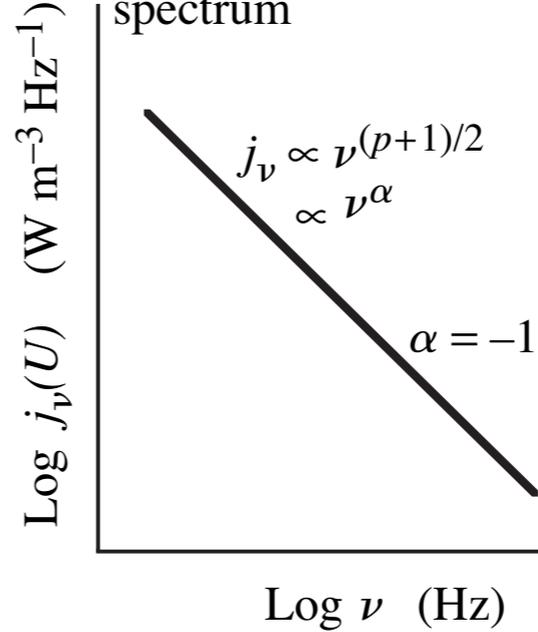
(a) Electron number-density spectrum



(b) Electron energy-density spectrum



(c) Photon energy-density spectrum (volume-emissivity) spectrum



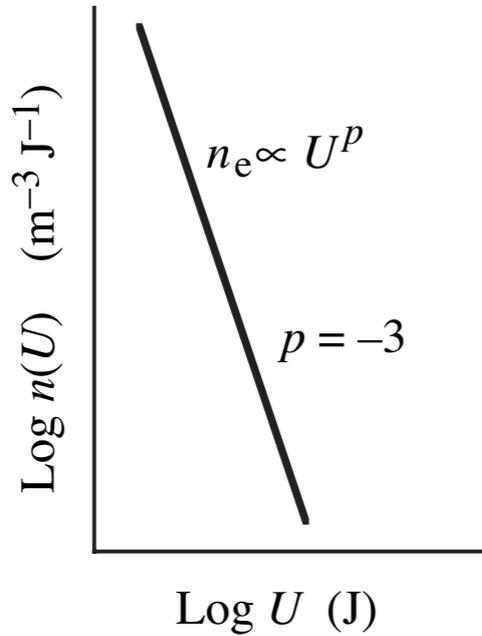
$$n(U)dU \propto U^p dU = \frac{n_0}{U_0^p} U^p dU = n_0 \left( \frac{U}{U_0} \right)^p dU \quad (\text{Particles}/\text{m}^3 \text{ at } U \text{ in } dU)$$

$$u(U)dU = U n(U) dU = U n_0 \left( \frac{U}{U_0} \right)^p dU = u_0 \left( \frac{U}{U_0} \right)^{p+1} dU \quad (\text{Energy density in } dU \text{ at } U)$$

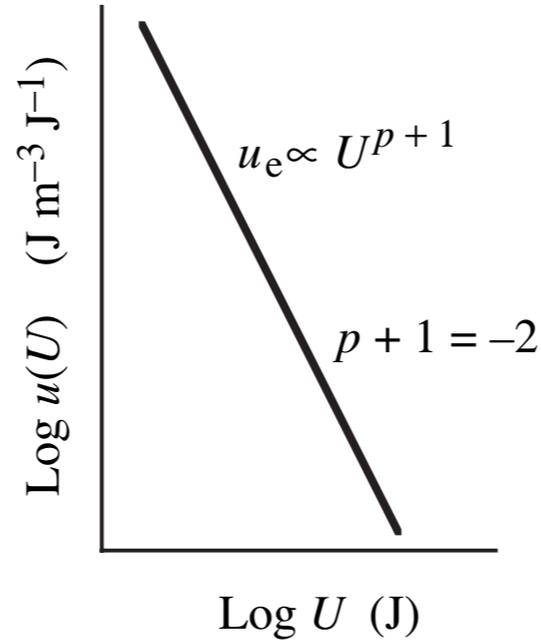
$$j_u(U)dU = -(dU/dt) \times n(U)dU \quad (\text{W}/\text{m}^3 \text{ at } U \text{ in } dU)$$

# ENSEMBLE OF RADIATING PARTICLES

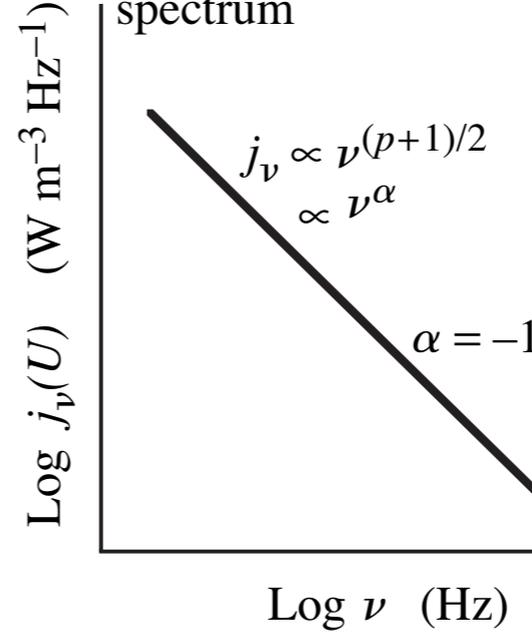
(a) Electron number-density spectrum



(b) Electron energy-density spectrum



(c) Photon energy-density (volume-emissivity) spectrum



$$n(U)dU \propto U^p dU = \frac{n_0}{U_0^p} U^p dU = n_0 \left( \frac{U}{U_0} \right)^p dU \quad (\text{Particles}/\text{m}^3 \text{ at } U \text{ in } dU)$$

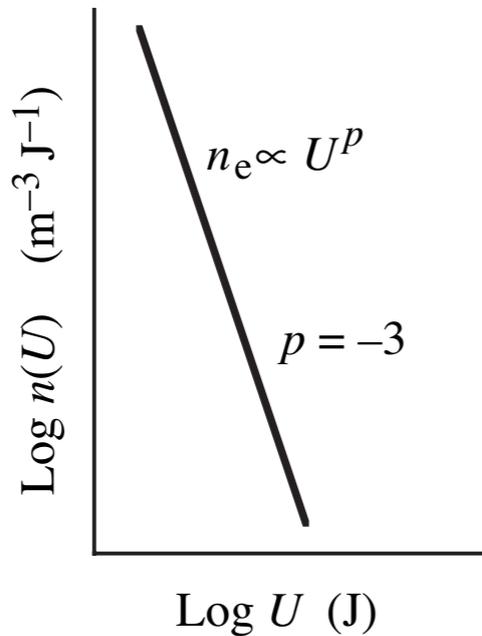
$$u(U)dU = U n(U) dU = U n_0 \left( \frac{U}{U_0} \right)^p dU = u_0 \left( \frac{U}{U_0} \right)^{p+1} dU \quad (\text{Energy density in } dU \text{ at } U)$$

$$j_u(U)dU = -(dU/dt) \times n(U)dU \quad (\text{W}/\text{m}^3 \text{ at } U \text{ in } dU)$$

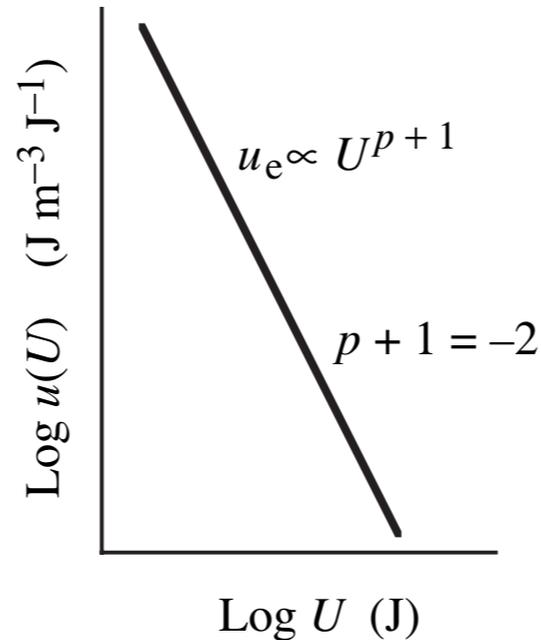
$$j_u(U)dU \propto U^2 B^2 U^p dU \rightarrow j_u(U) \propto B^2 U^{p+2}$$

# ENSEMBLE OF RADIATING PARTICLES

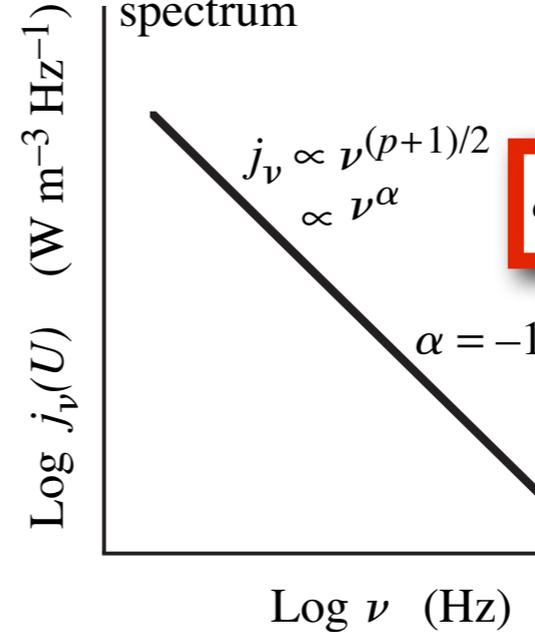
(a) Electron number-density spectrum



(b) Electron energy-density spectrum



(c) Photon energy-density (volume-emissivity) spectrum



$$\alpha \equiv \frac{p+1}{2} \quad (\text{Spectral index})$$

$$j_u(U) dU \propto U^2 B^2 U^p dU \rightarrow j_u(U) \propto B^2 U^{p+2}$$

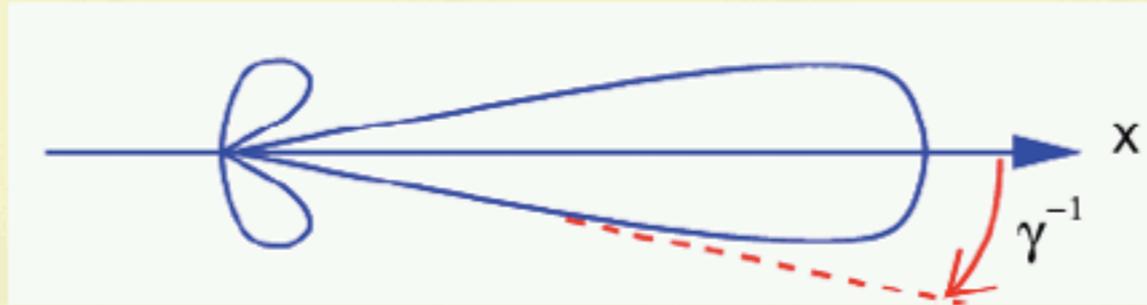
$$j_\nu(\nu) d\nu = j_u(U) dU \rightarrow j_\nu(\nu) = j_u(U) \frac{dU}{d\nu}$$

$$\nu_c \propto U^2 B \rightarrow U \propto (\nu/B)^{1/2} \rightarrow dU \propto B^{-1/2} \nu^{-1/2} d\nu$$

$$\rightarrow j_\nu(\nu) d\nu \propto B^{(1-p)/2} \nu^{(1+p)/2} d\nu \quad (\text{Synchrotron photon - energy spectrum; W/m}^3 \text{ at } \nu \text{ ind } \nu)$$

# POLARIZATION

$$\vec{E} \parallel \vec{a}$$



Elliptical polarization,  
 $a_{\max}$  perpendicular to  $\mathbf{B}$

Linear polarization,  
perpendicular to  $\mathbf{B}$

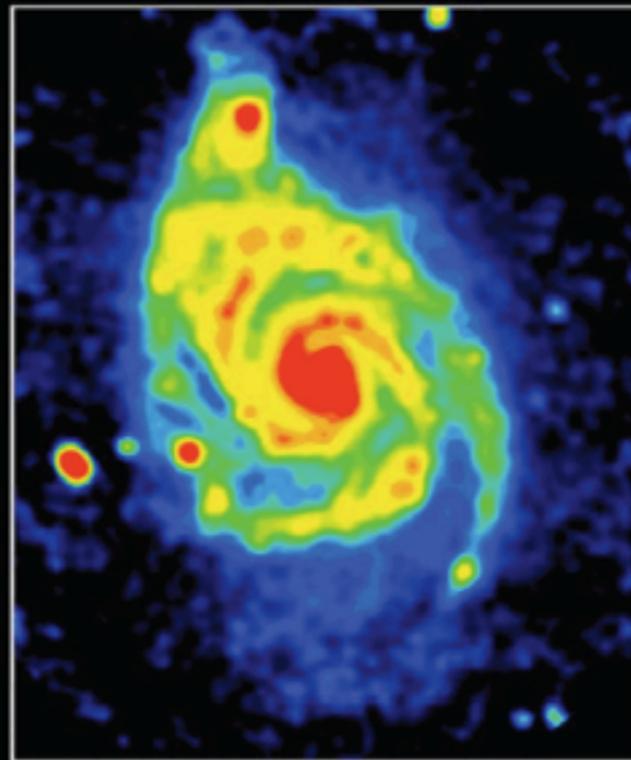
## Set of relativistic electrons:

- ▶ We receive radiation only from those with velocities within a cone of semi-aperture  $1/\gamma$  centered on the line of sight axis
- ▶ If the magnetic field is ordered the radiation is still polarized
- ▶ Since electron velocities are not ordered, the ellipses of polarization have different orientations, and they cancel each other. We will have therefore only **linear polarization**, perpendicular to the ordered magnetic field
- ▶ The **polarized signal** is thus intrinsically lower than the total one (maximum  $\sim 70\%$ )

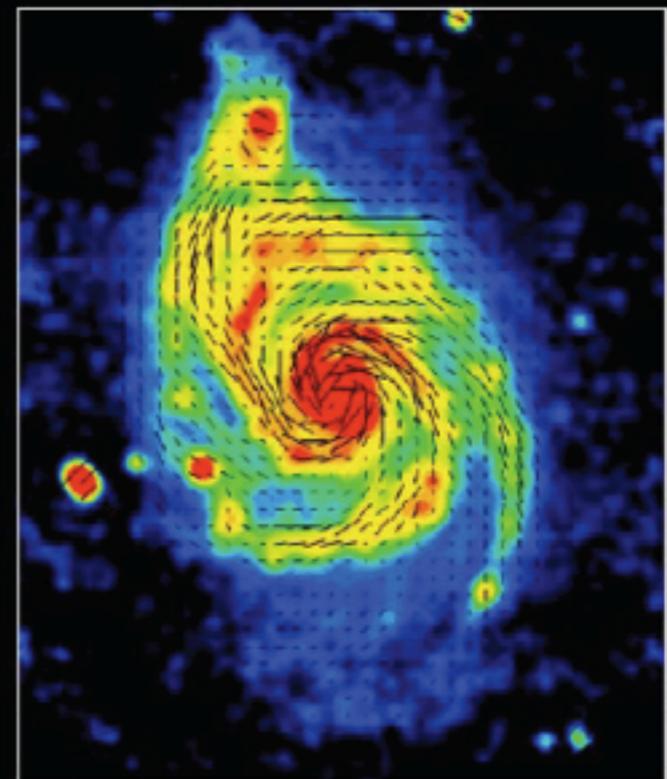
# POLARIZATION



M51 6cm VLA + Effelsberg

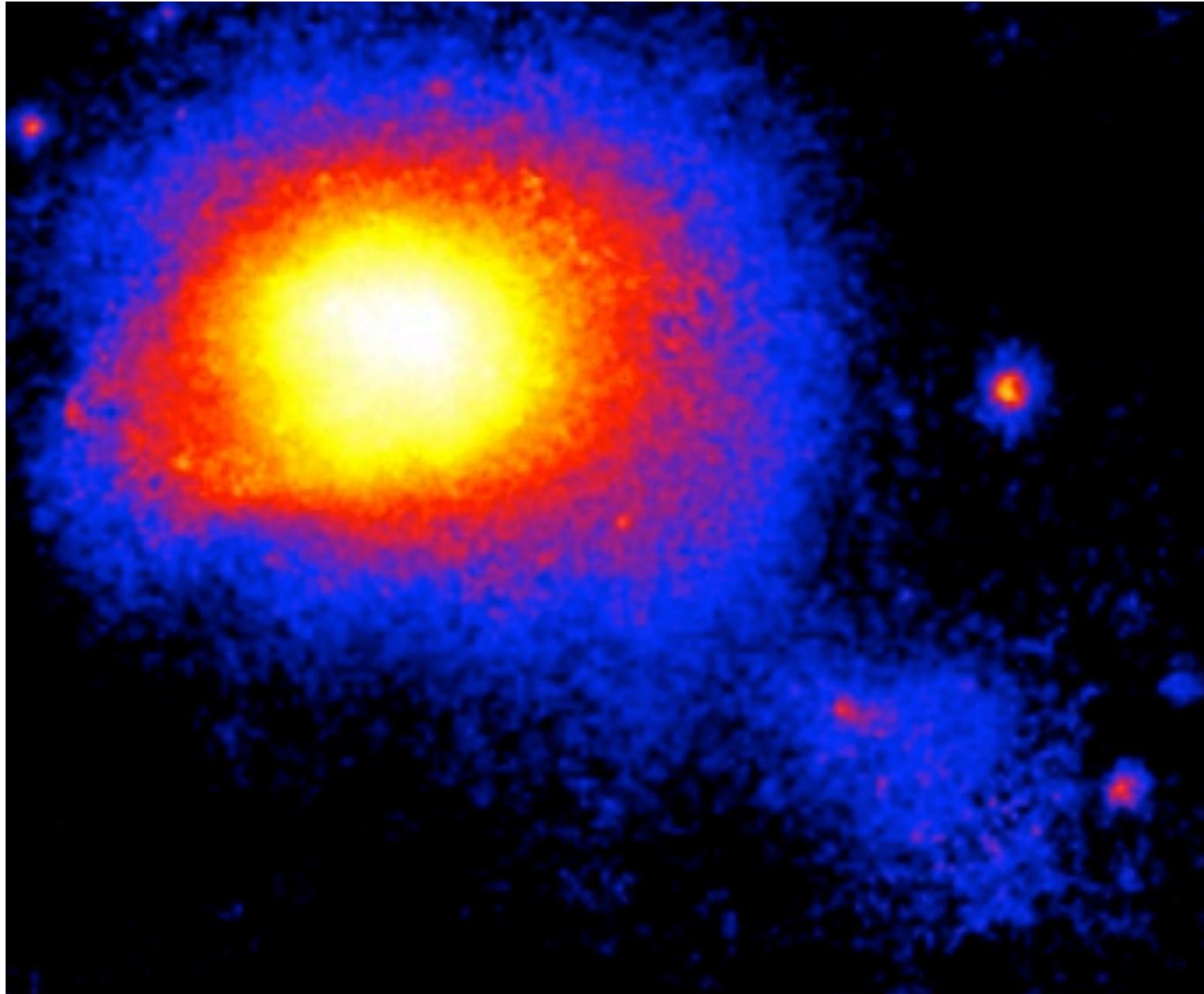


M51 6cm VLA+Effelsberg HPBW=15''



# CONTINUUM RADIO EMISSION FROM GALAXY CLUSTERS

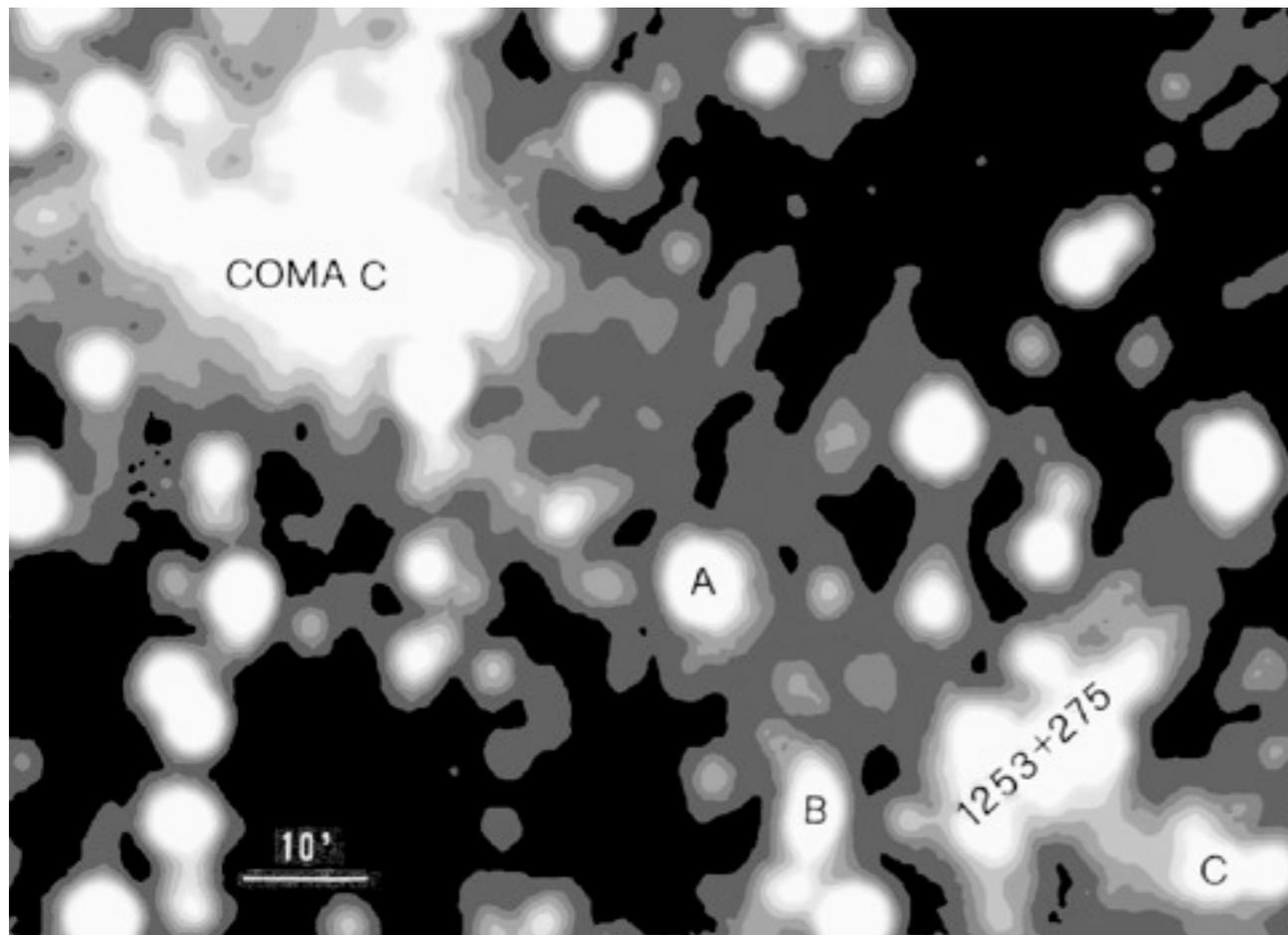
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The Coma cluster

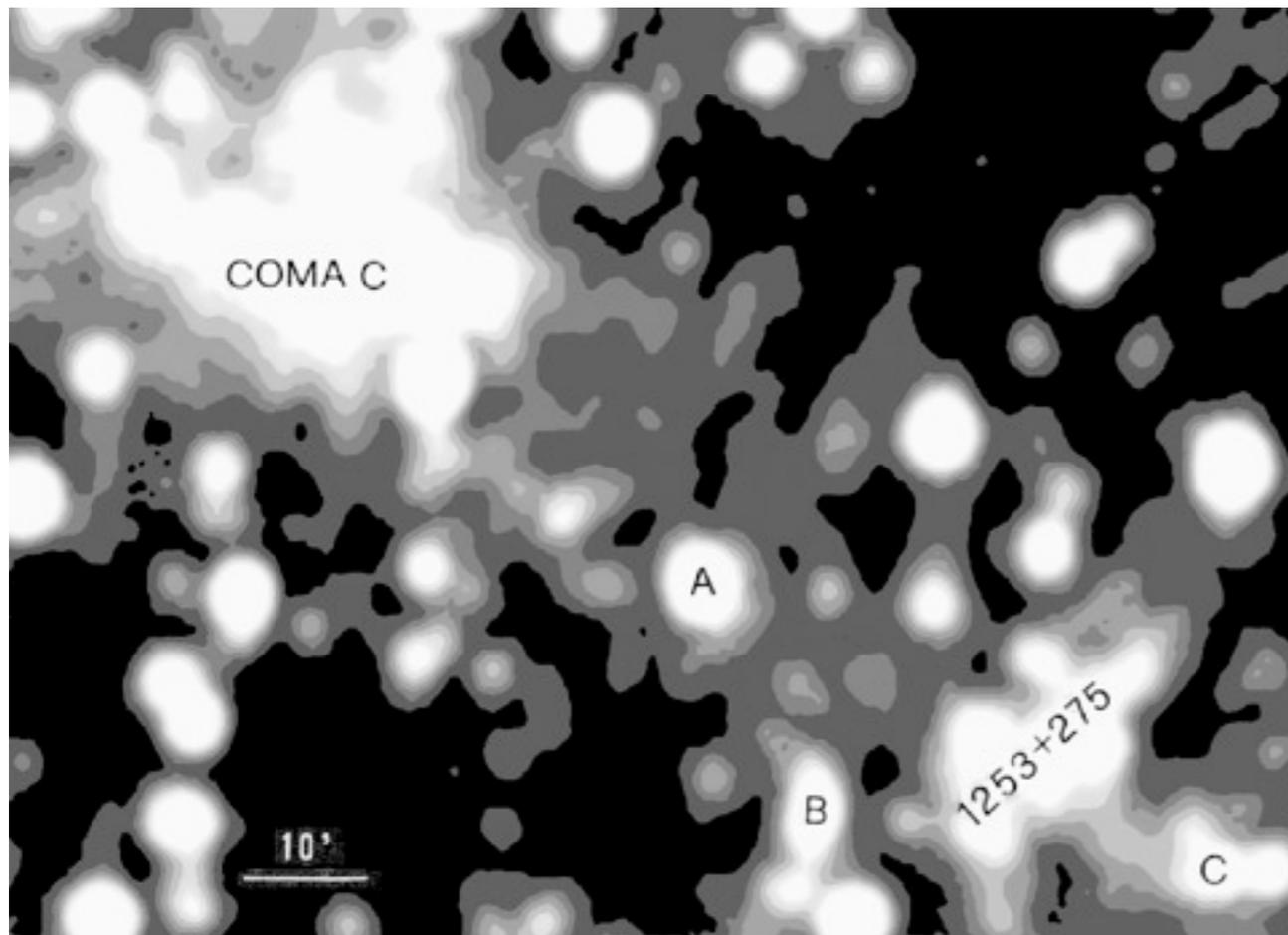
# CONTINUUM RADIO EMISSION FROM GALAXY CLUSTERS

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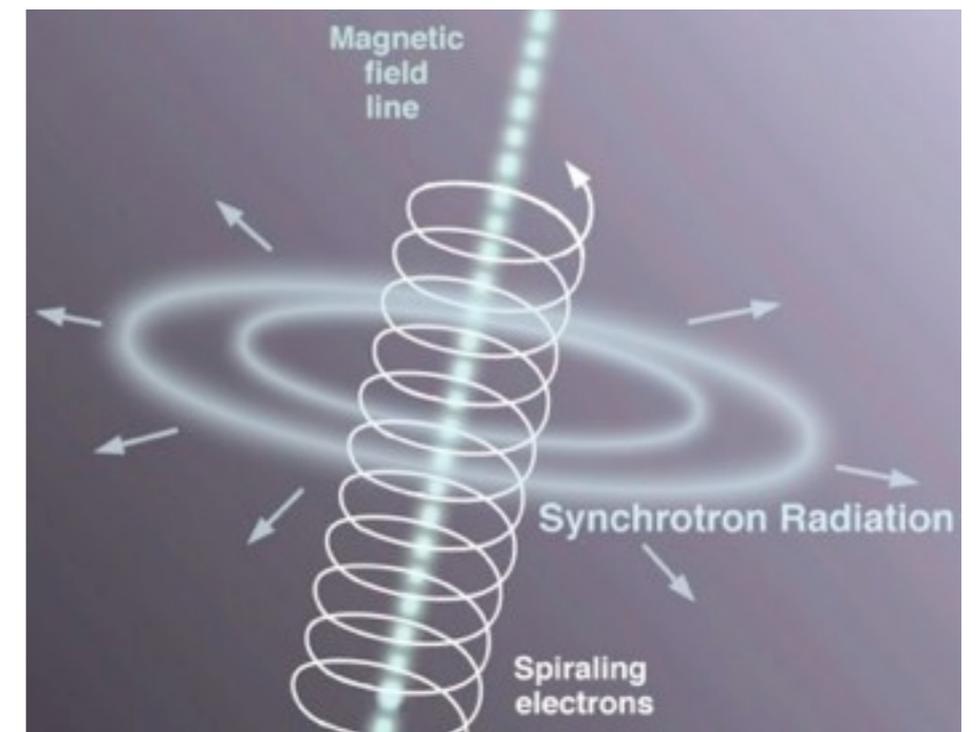
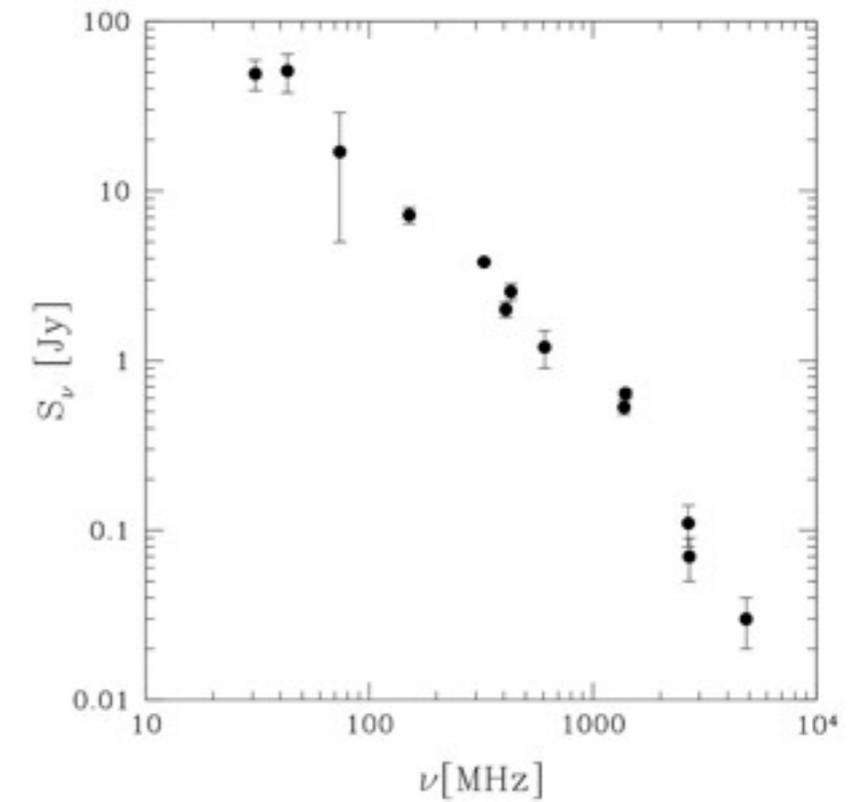


The Coma cluster

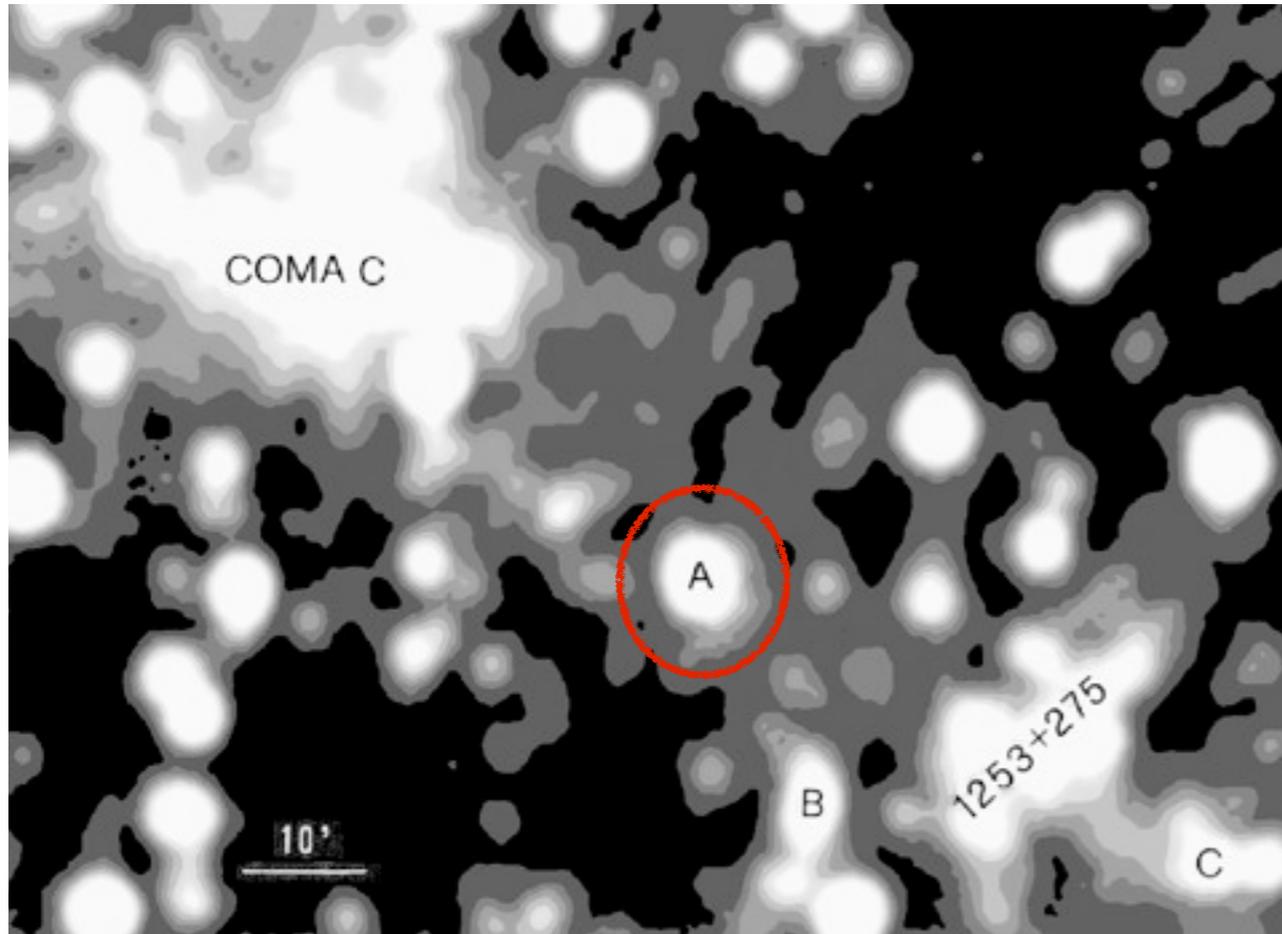
# CONTINUUM RADIO EMISSION FROM GALAXY CLUSTERS



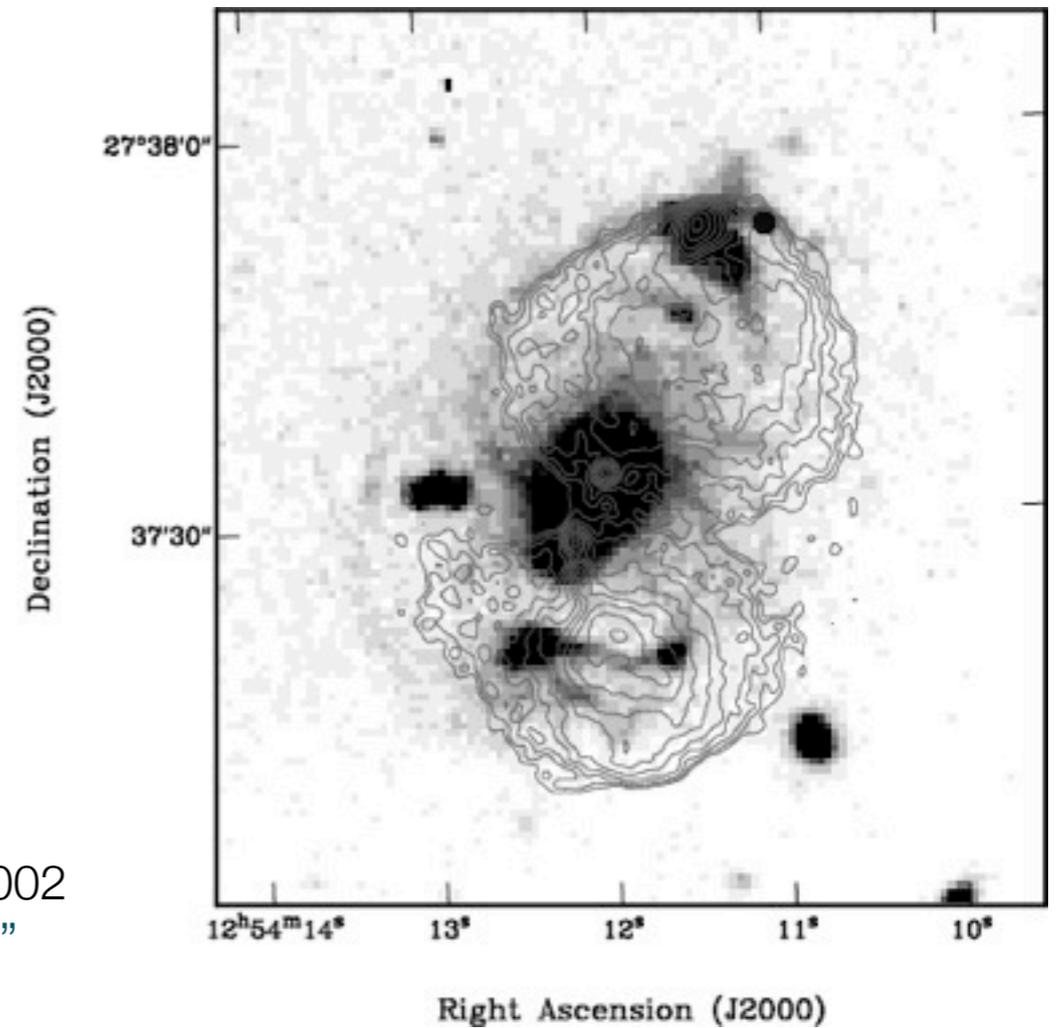
The Coma cluster



# RADIO EMISSION FROM GALAXIES



Giovannini et al. 1991  
Resolution = 120"  
 $\lambda = 91$  cm



Morganti et al. 2002  
Resolution = 1"  
 $\lambda = 20$  cm

# ACTIVE GALAXIES

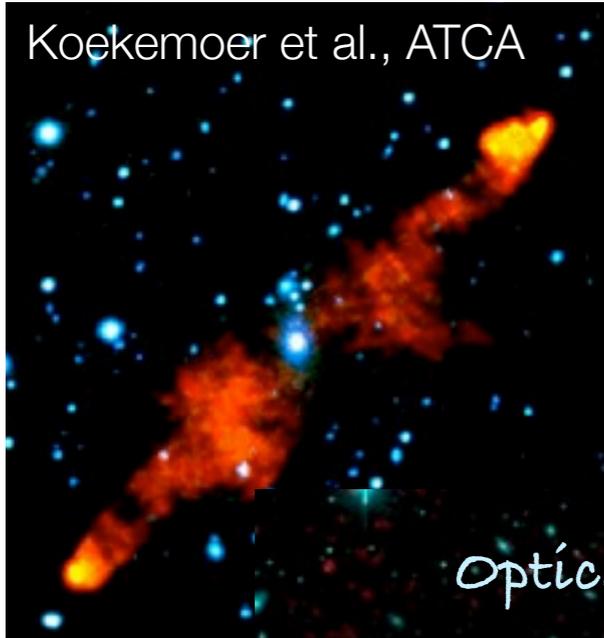
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# ACTIVE GALAXIES

---

Koekemoer et al., ATCA

Active Galactic Nucleus (AGN)



Optical + Radio

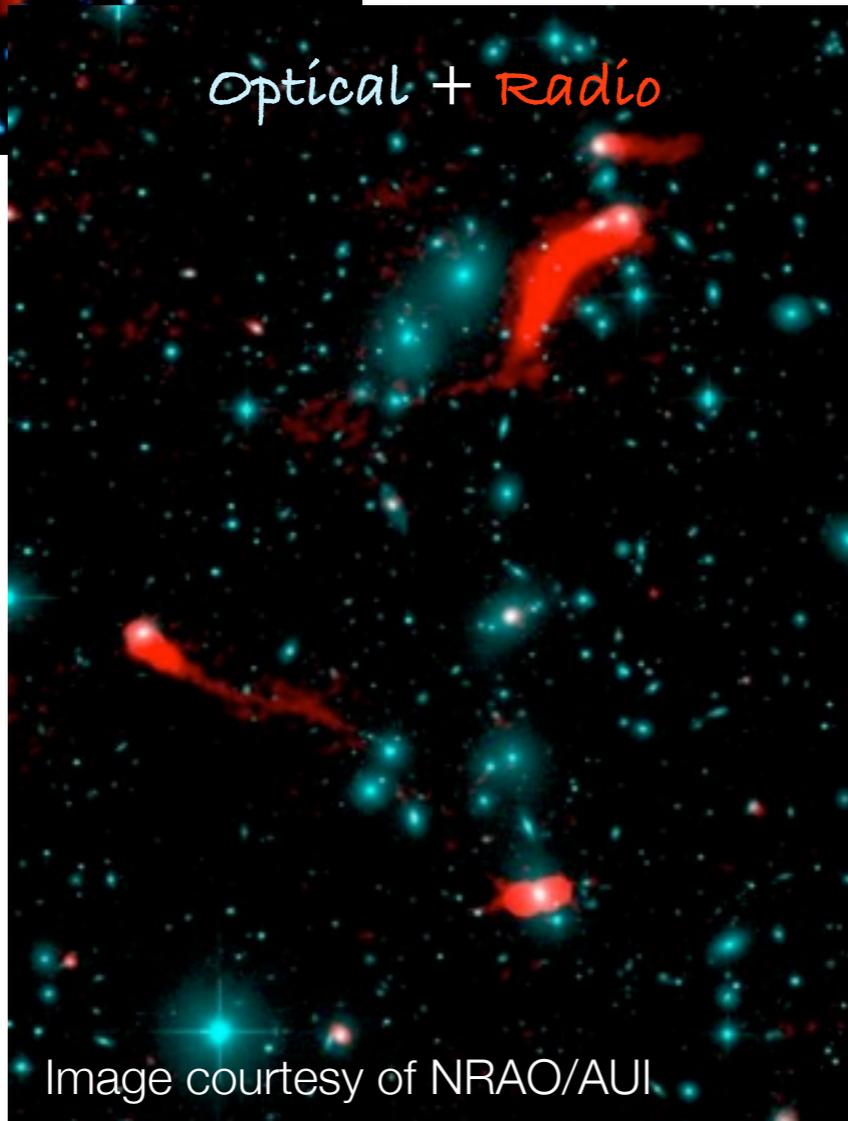


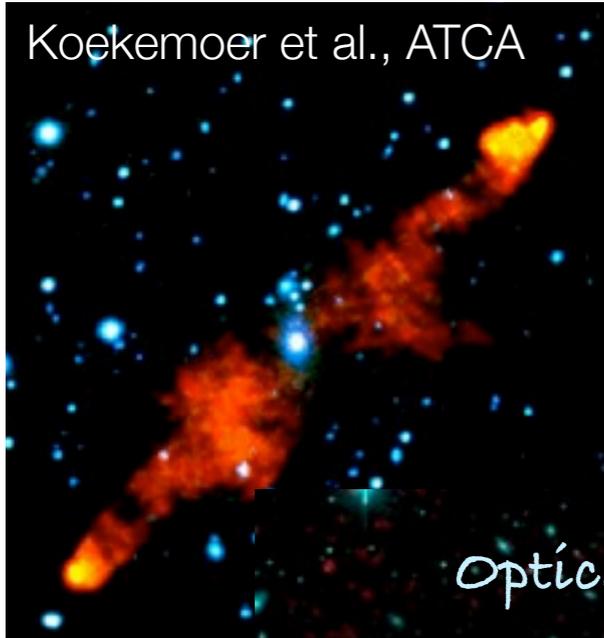
Image courtesy of NRAO/AUI

# ACTIVE GALAXIES

---

Koekemoer et al., ATCA

Active Galactic Nucleus (AGN)



Optical + Radio

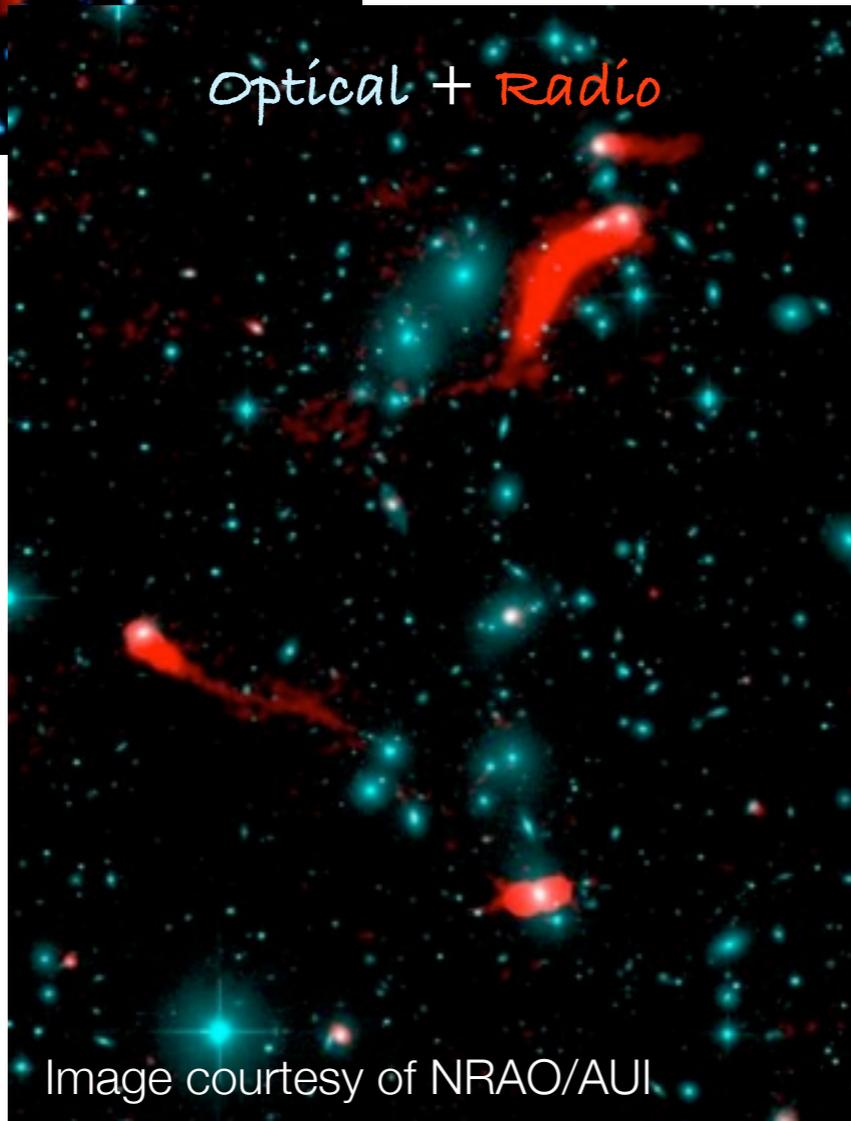
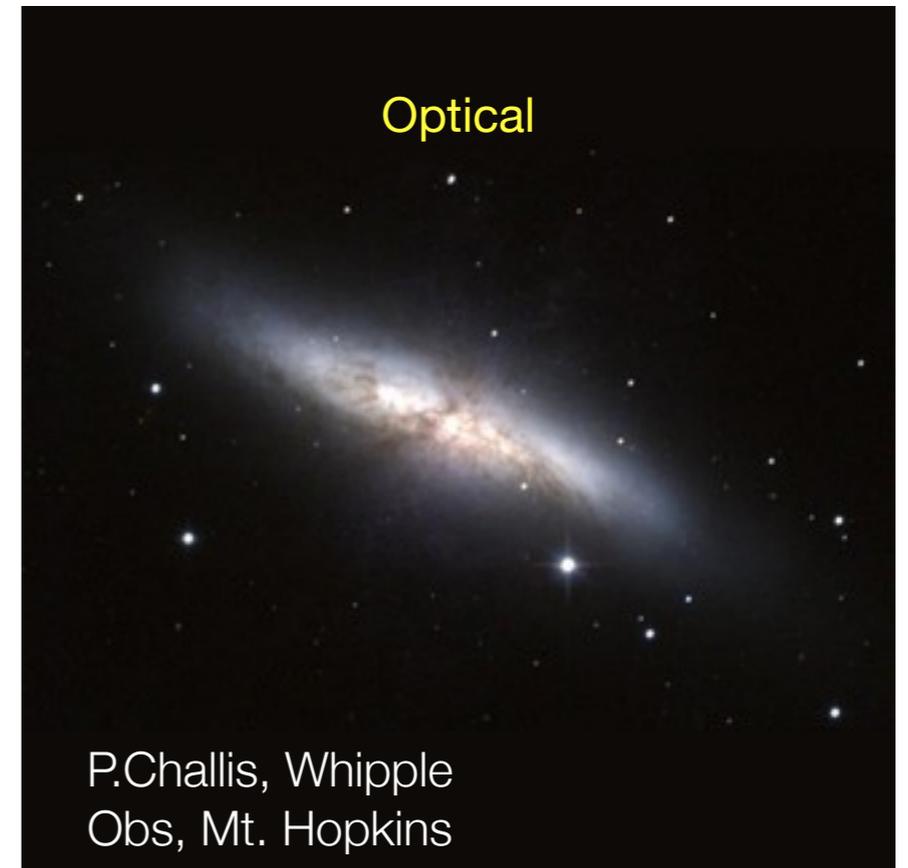


Image courtesy of NRAO/AUI

Optical



P.Challis, Whipple  
Obs, Mt. Hopkins

The starburst galaxy M82

# ACTIVE GALAXIES

Koekemoer et al., ATCA

Active Galactic Nucleus (AGN)



Optical + Radio

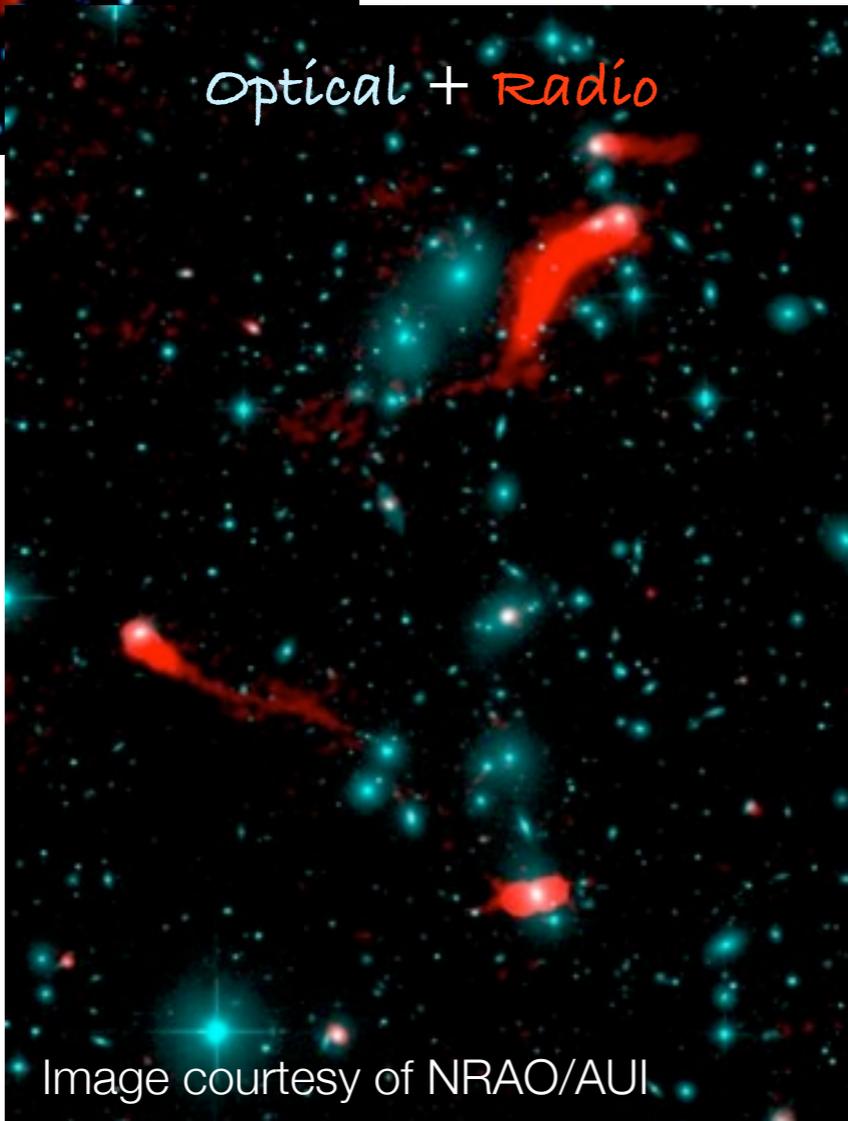
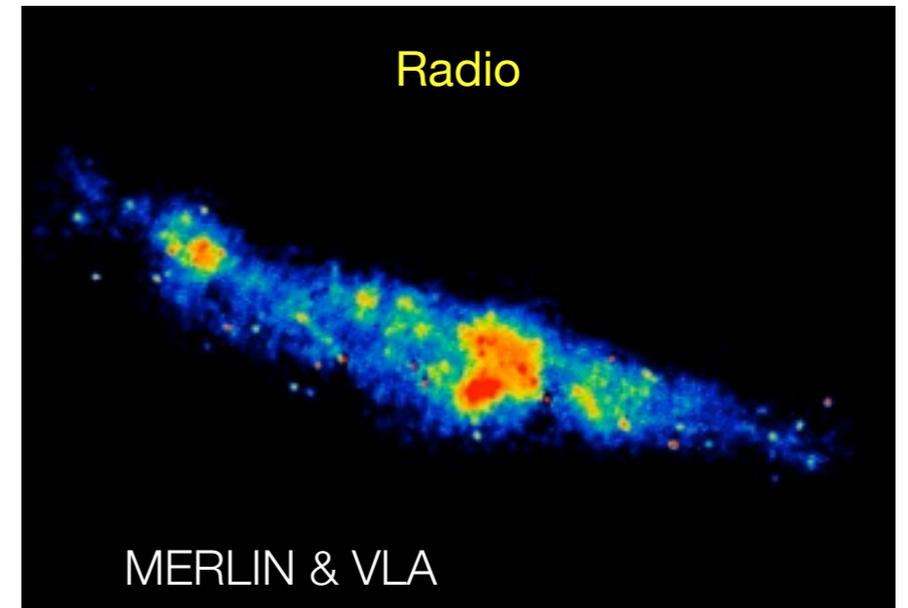
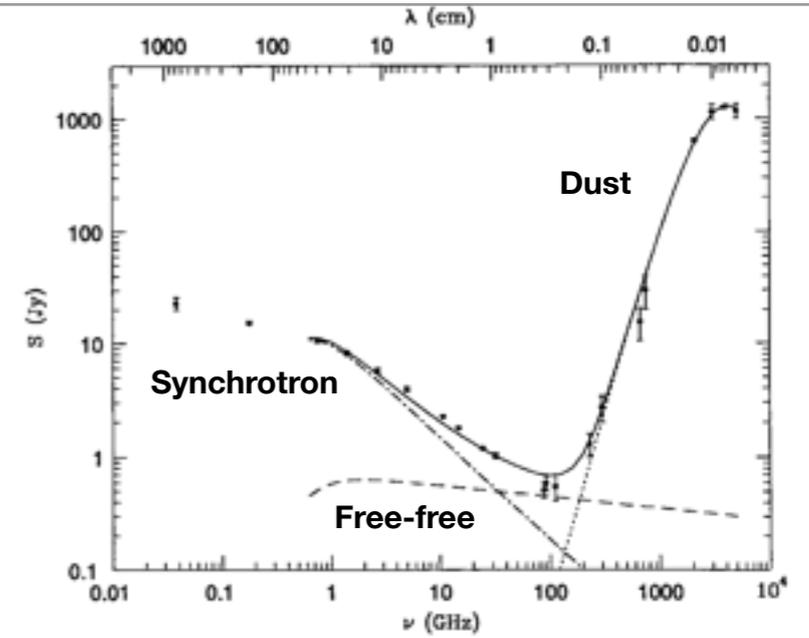


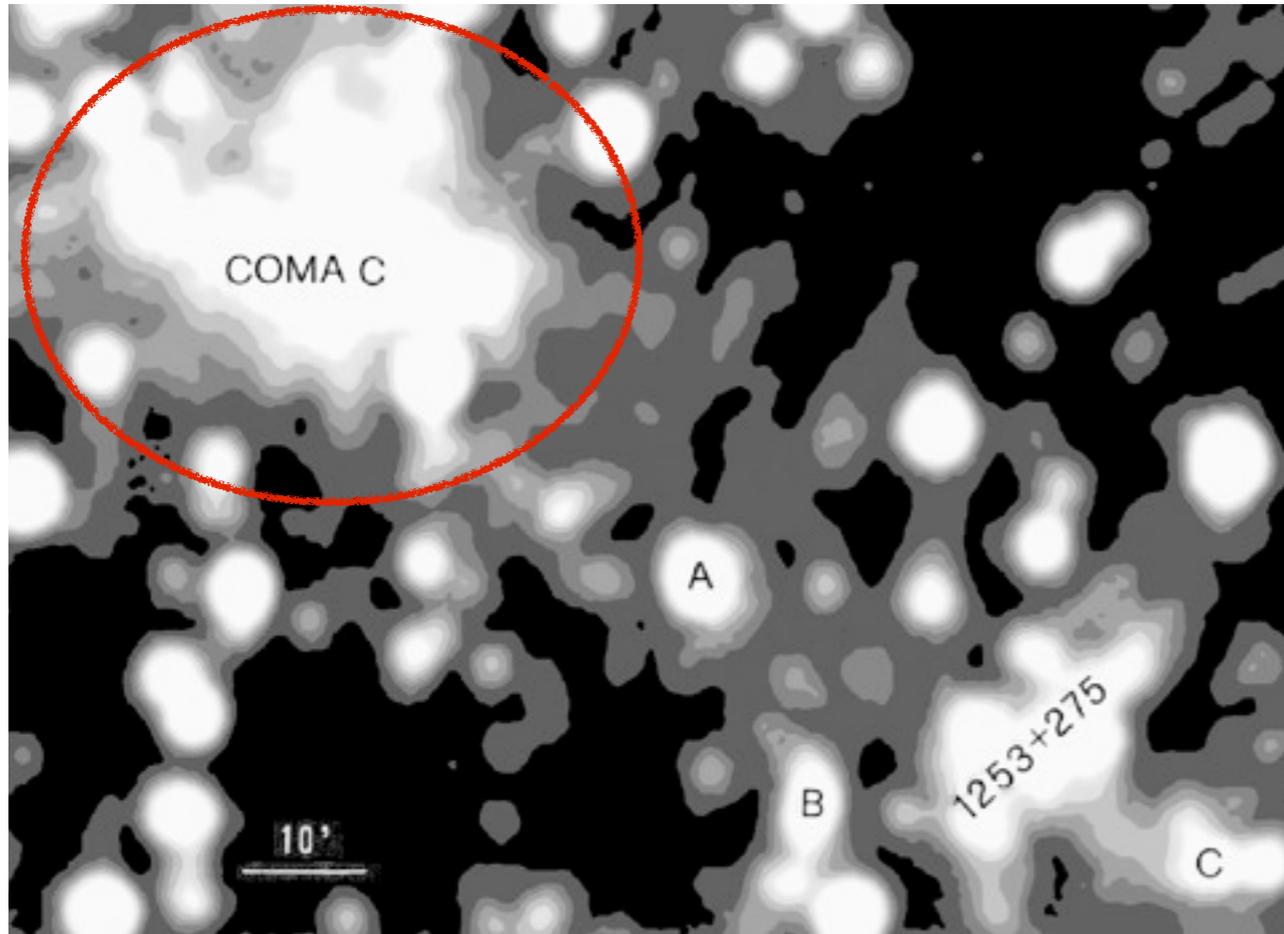
Image courtesy of NRAO/AUI



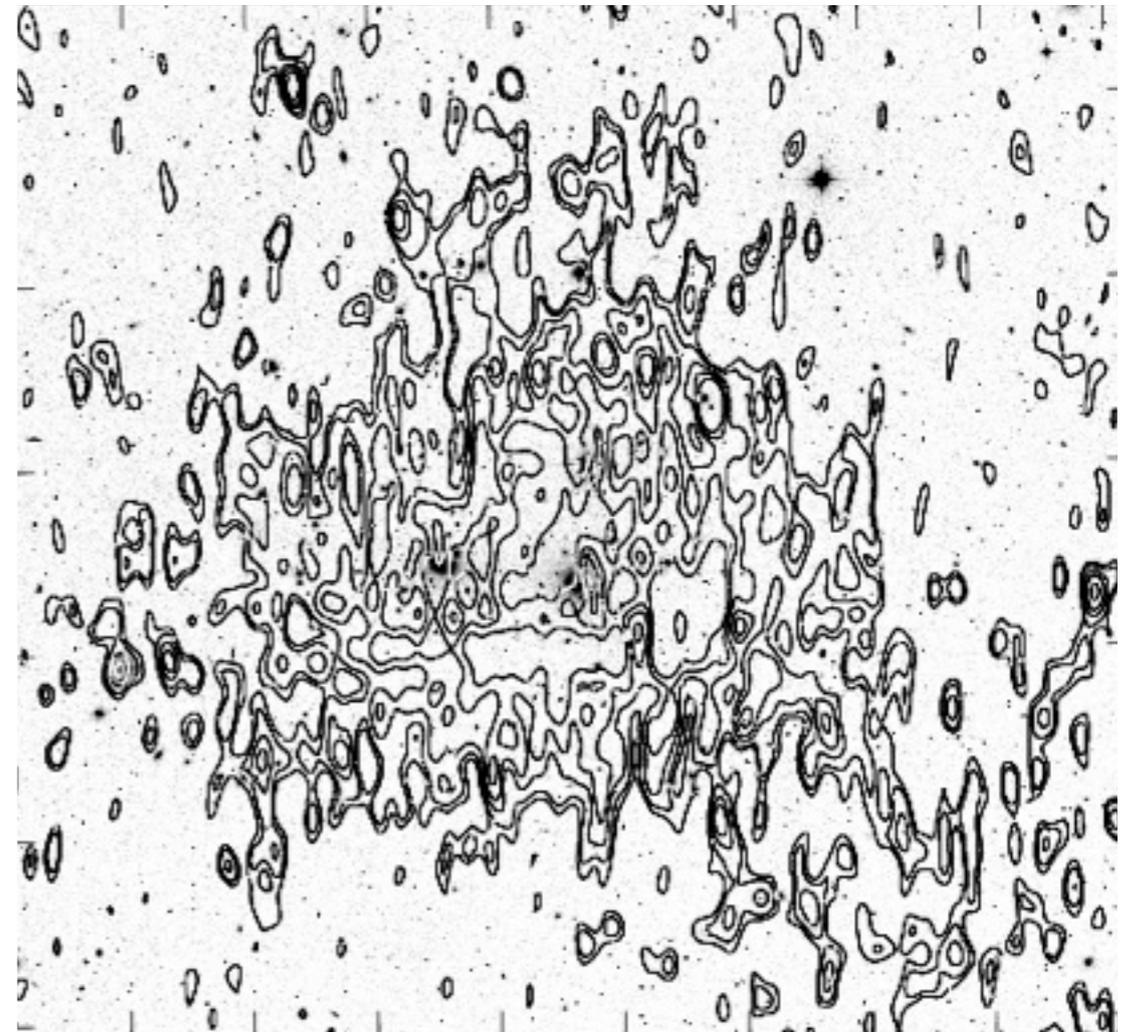
The starburst galaxy M82

# DIFFUSE RADIO SOURCES

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Giovannini et al. 1991



Feretti 2002  
Point sources subtracted

# DIFFUSE RADIO SOURCES IN CLUSTERS

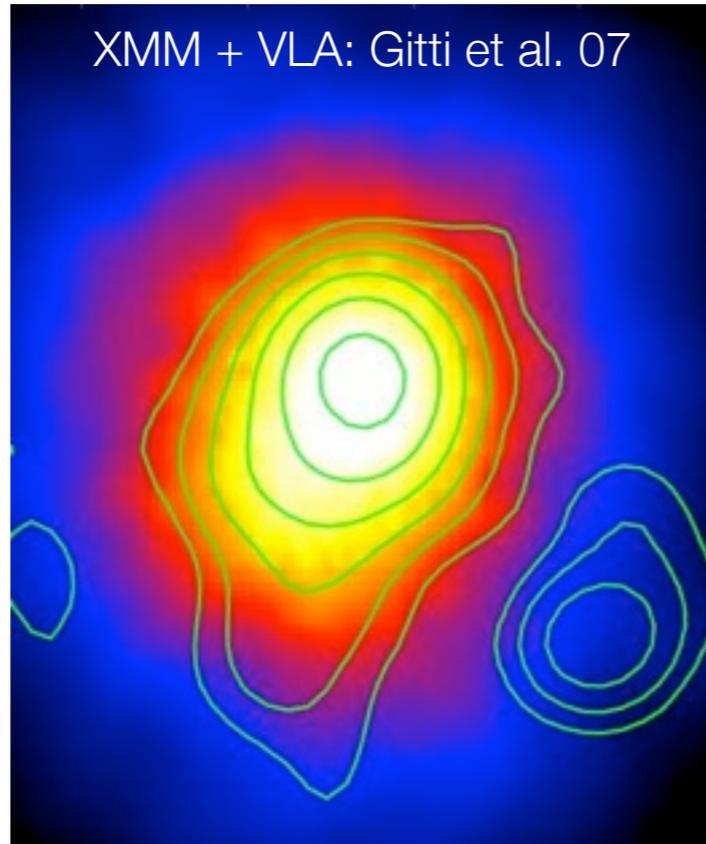
---

- Mini-halos
- Halos
- Relics

# DIFFUSE RADIO SOURCES IN CLUSTERS

intermediate extension  
( $\leq 500$  kpc)

at the centre of clusters  
with AGN & cooling-core

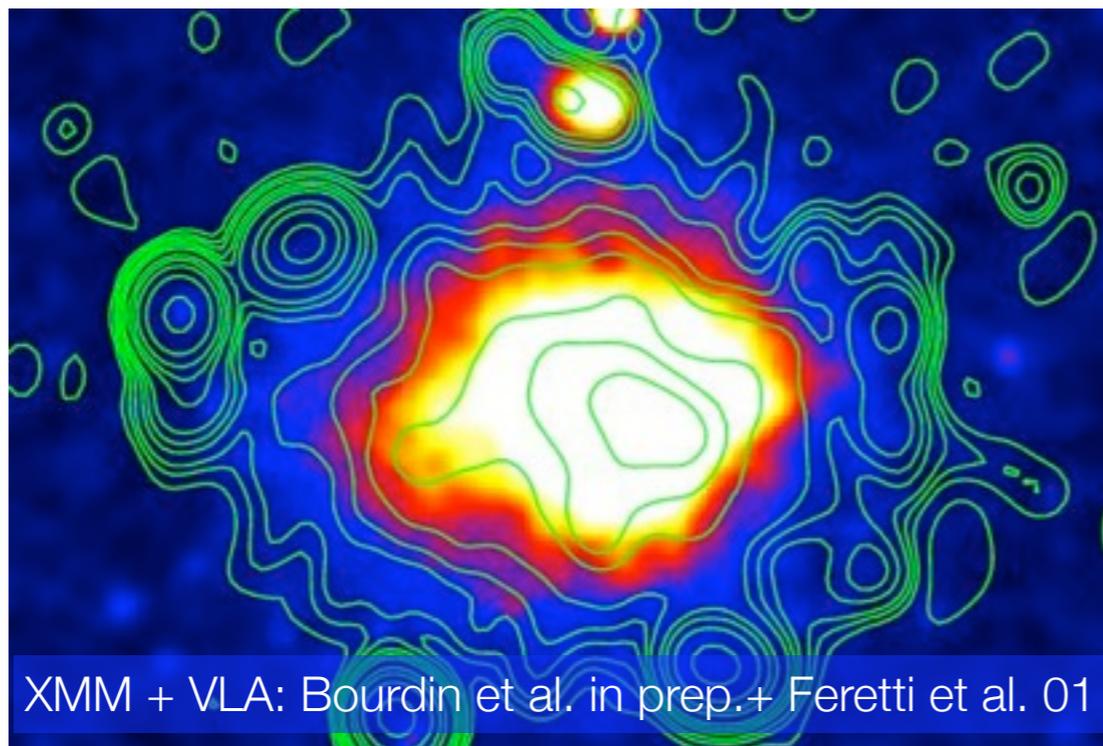


- Mini-halos
- Halos
- Relics

extended  
( $\geq 1$  Mpc)

cluster centre

regular  
morphology  
(~X-rays)

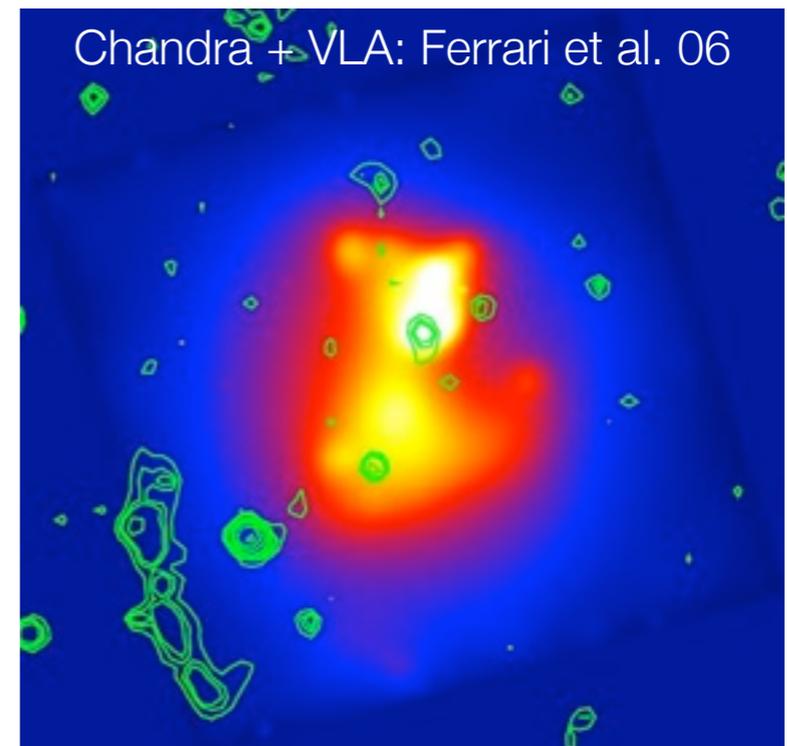


Chandra + VLA: Ferrari et al. 06

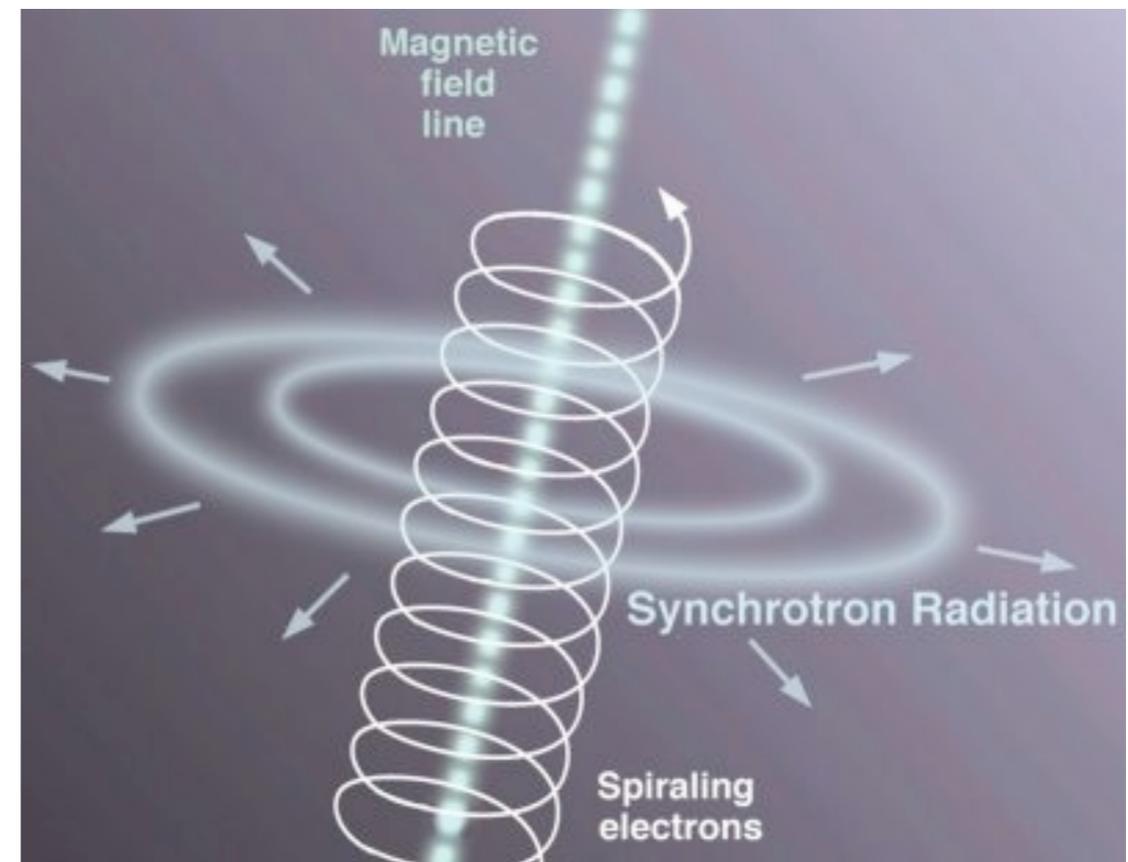
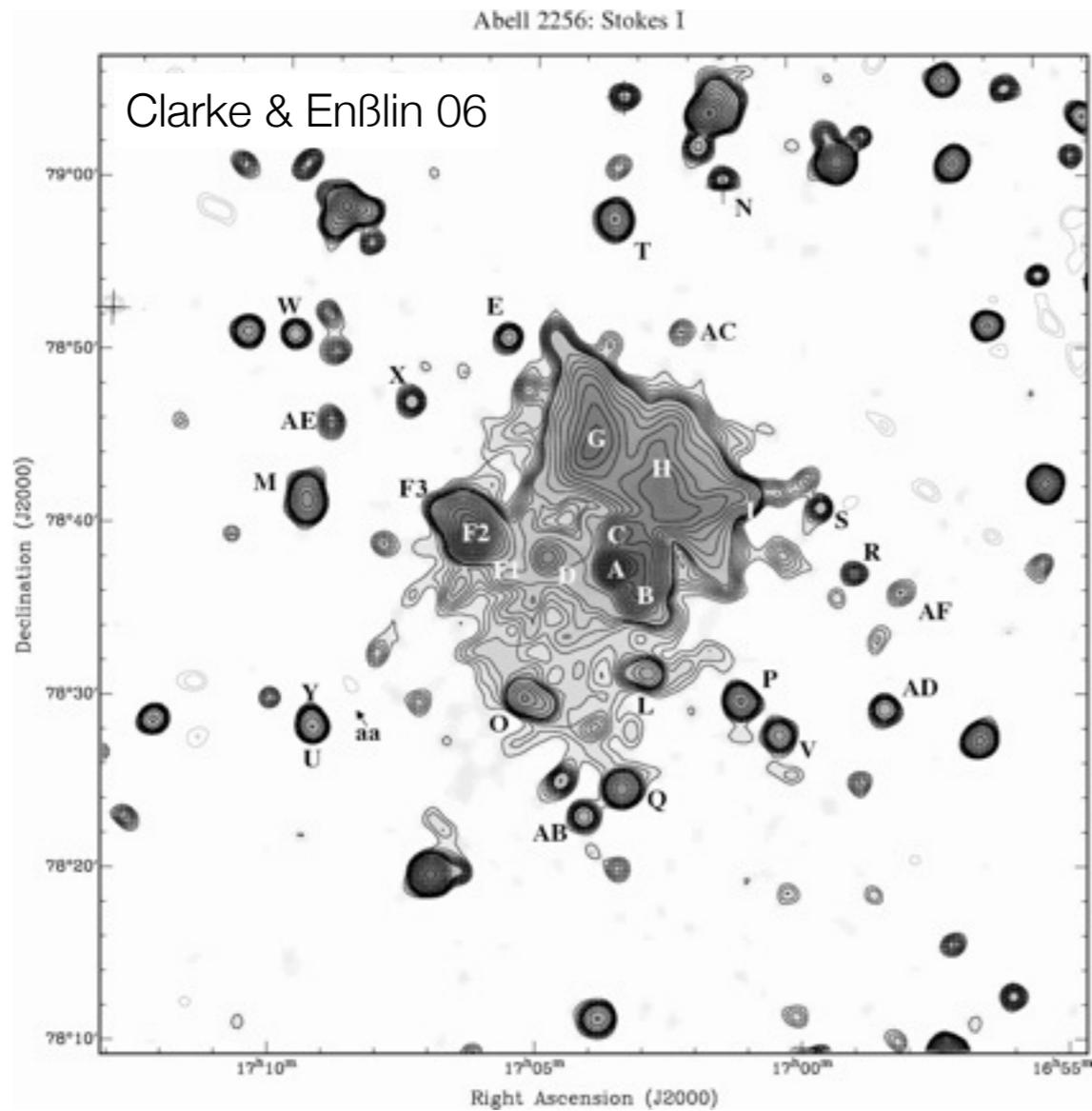
extended  
( $\geq 1$  Mpc)

cluster  
outskirts

elongated  
morphology

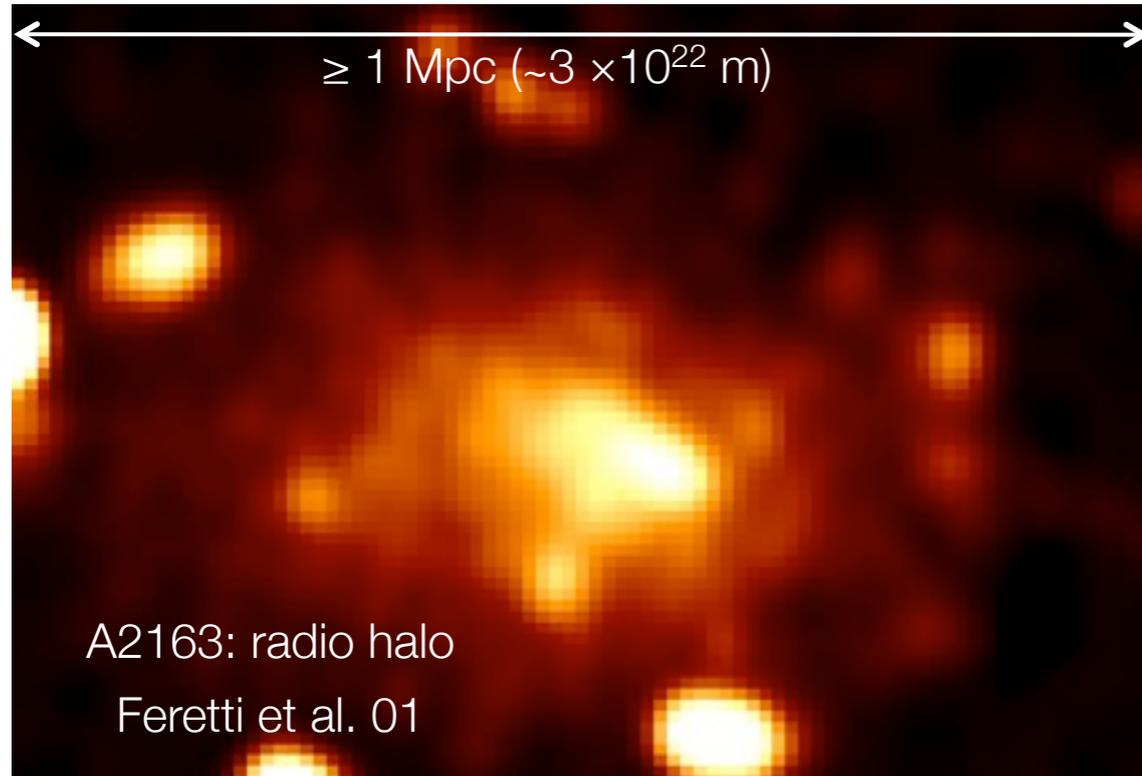


# RADIO CONTINUUM EMISSION FROM CLUSTERS - SUMMARY



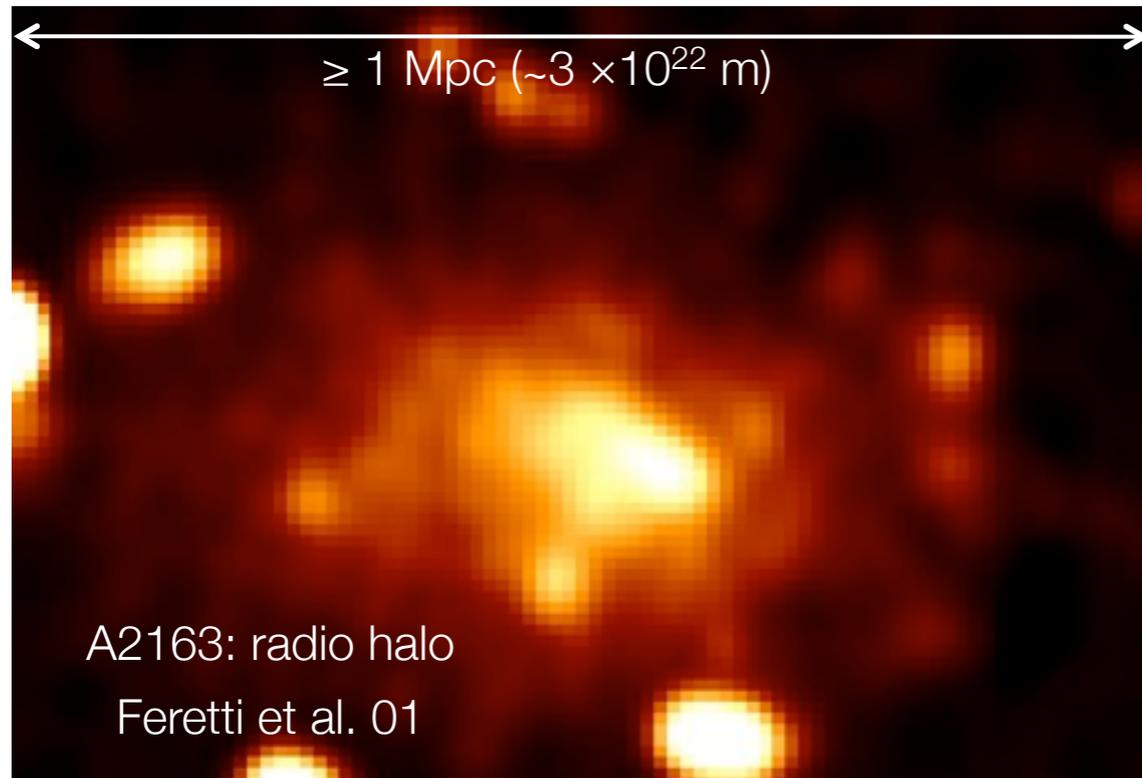
# ORIGIN OF RELATIVISTIC PARTICLES IN CLUSTERS

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# ORIGIN OF RELATIVISTIC PARTICLES IN CLUSTERS

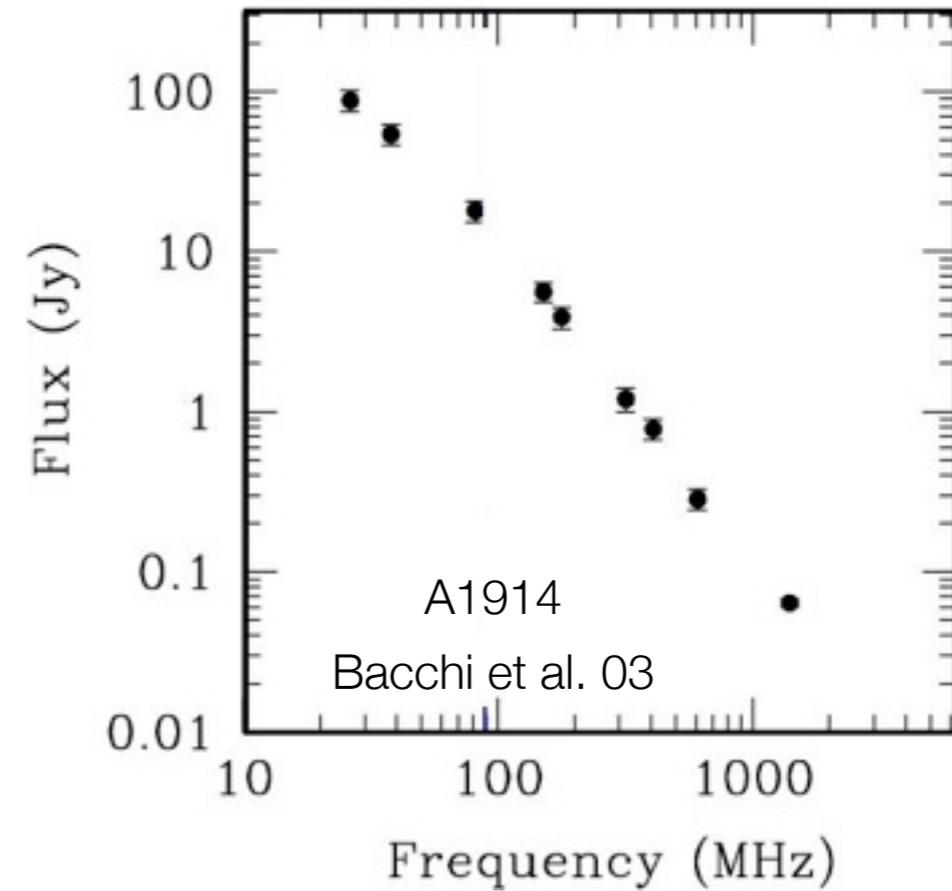
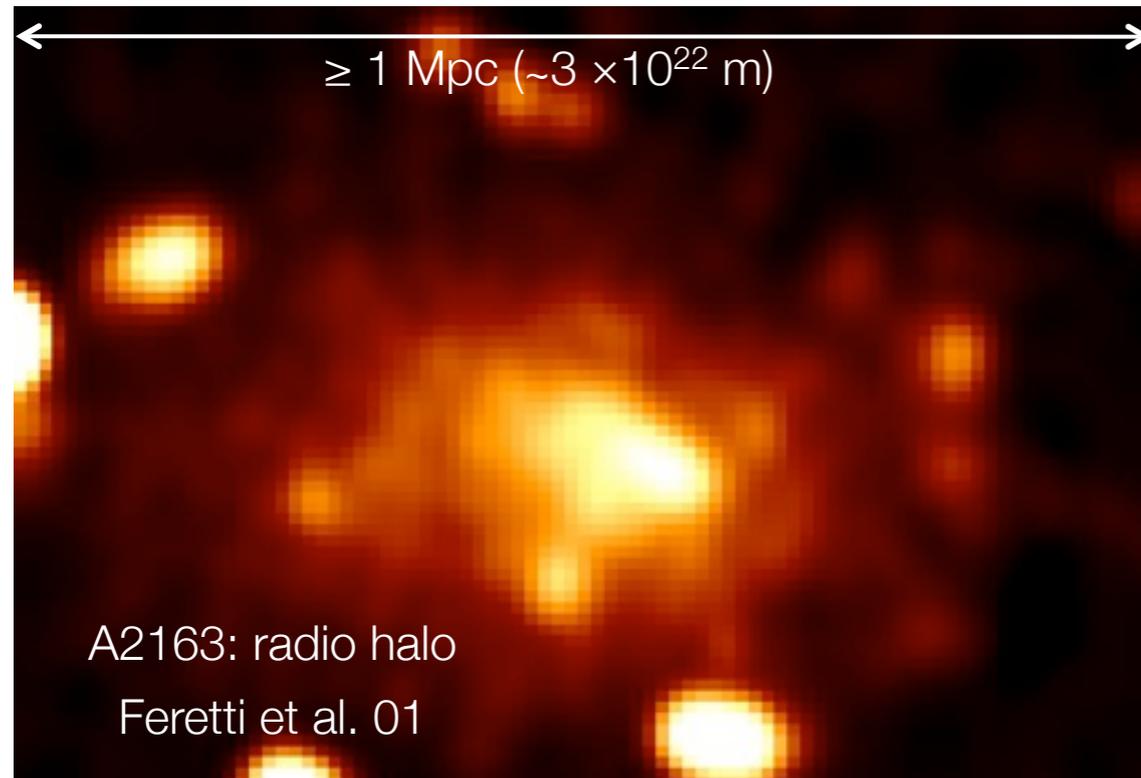
---



Dimensions:  $\sim 1$  Mpc

Crossing time of  $e^-$ :  $\sim 9.5$  Gyr

# ORIGIN OF RELATIVISTIC PARTICLES IN CLUSTERS

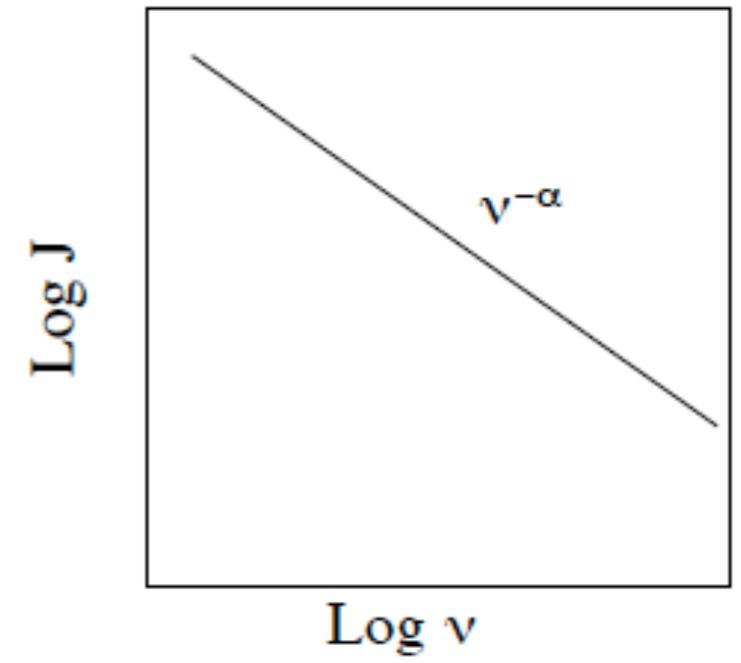


Dimensions:  $\sim 1 \text{ Mpc}$

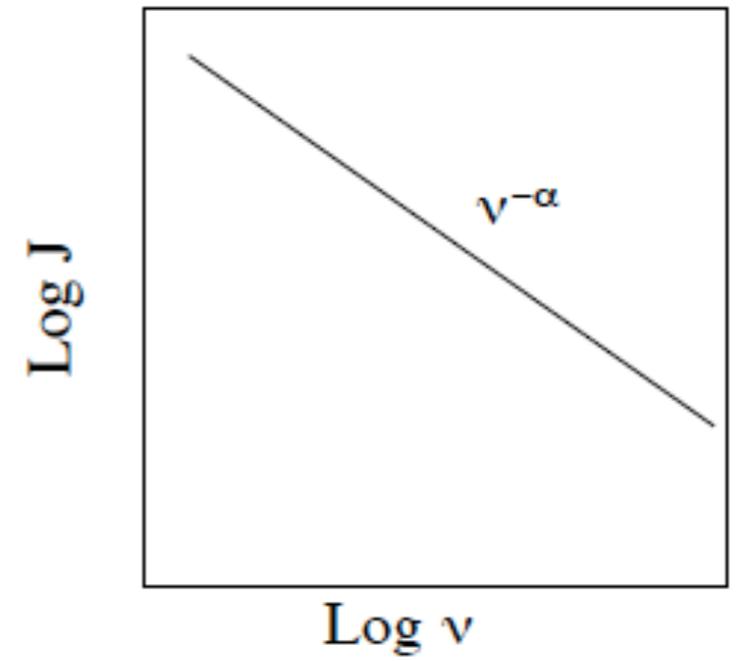
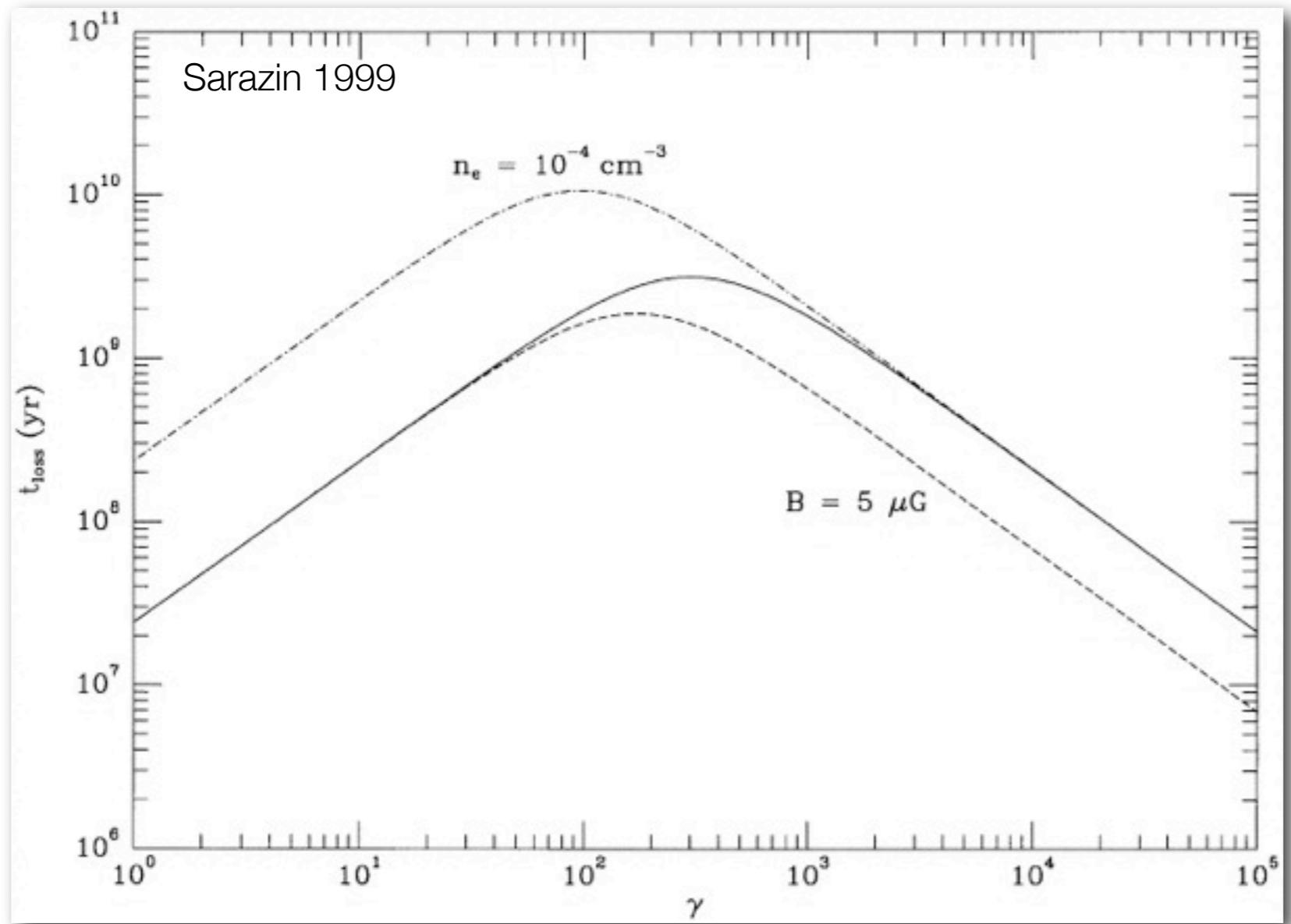
Crossing time of  $e^-$ :  $\sim 9.5 \text{ Gyr}$

# STEEP SPECTRA

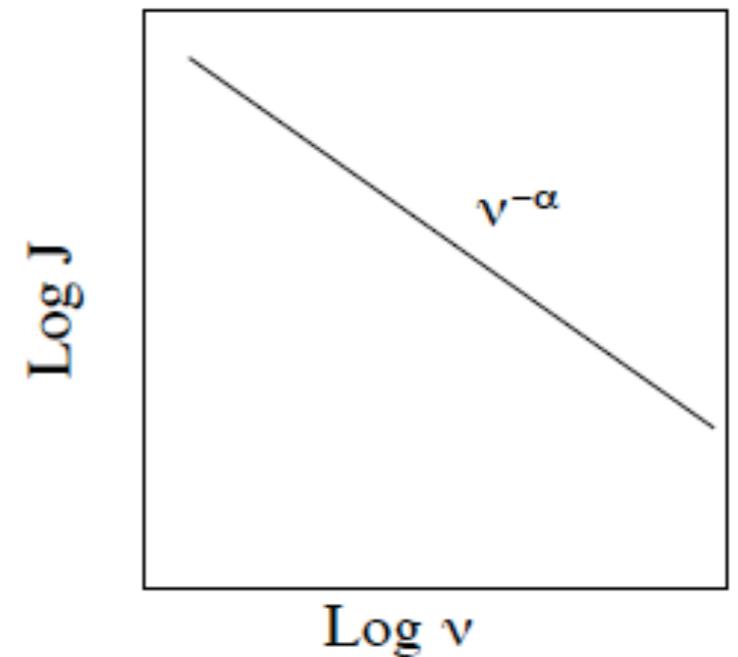
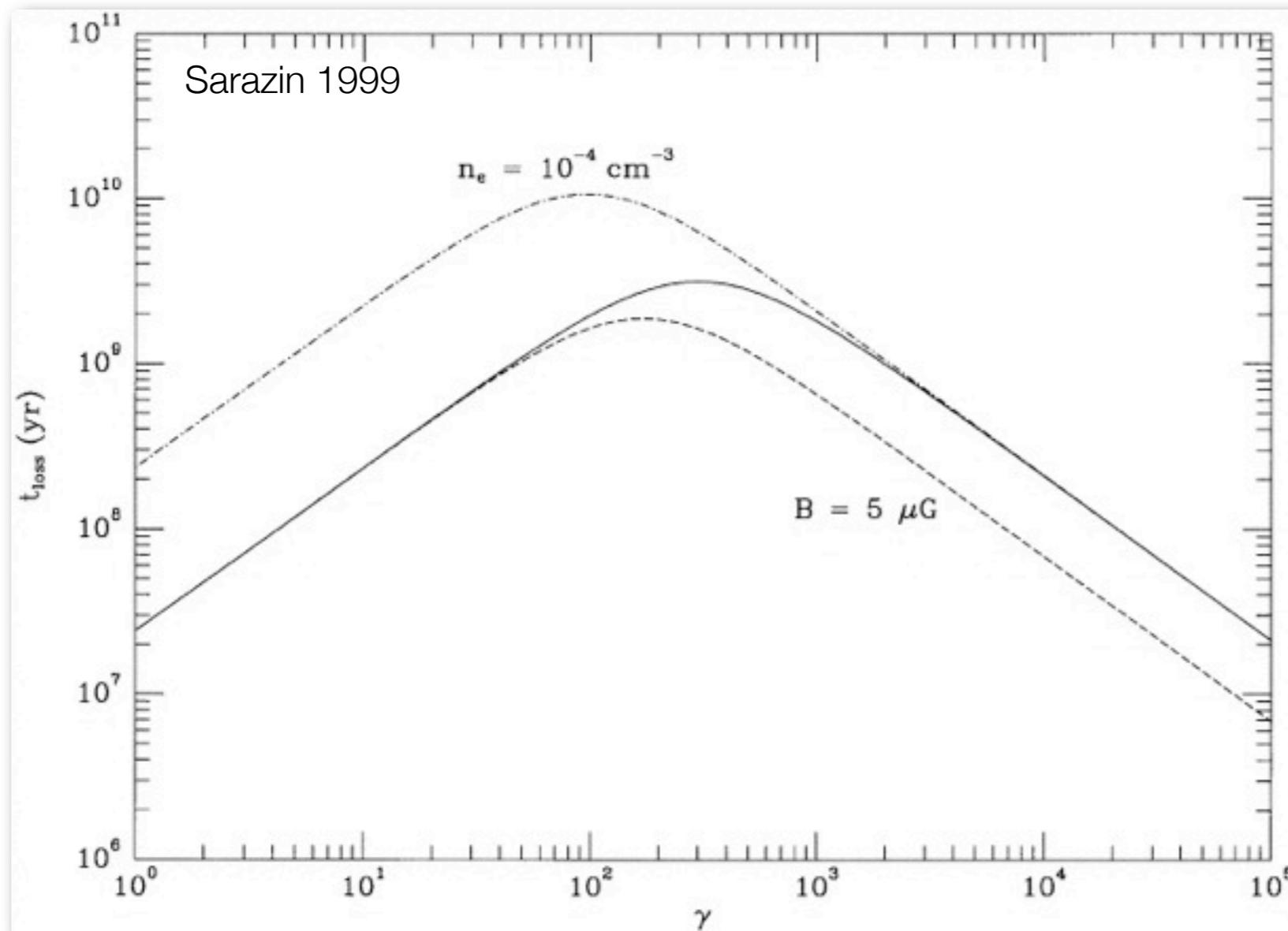
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# STEEP SPECTRA



# STEEP SPECTRA

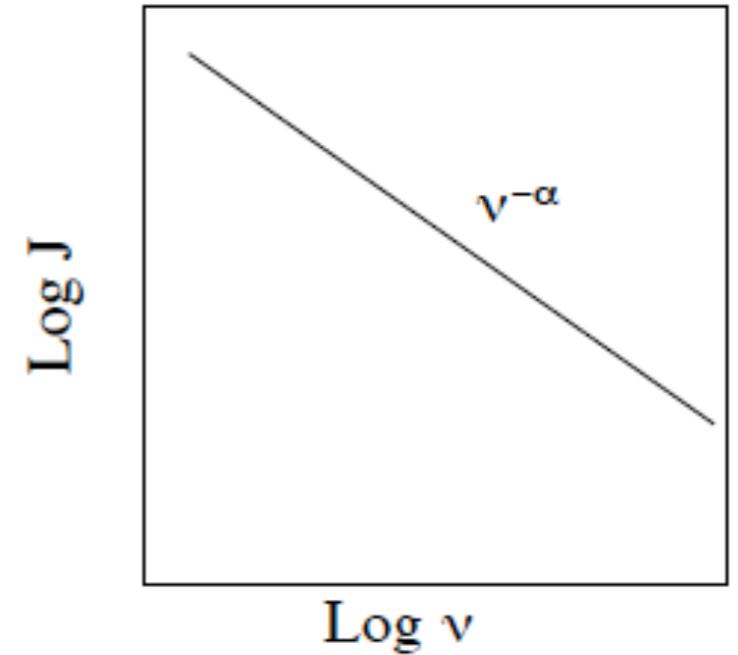
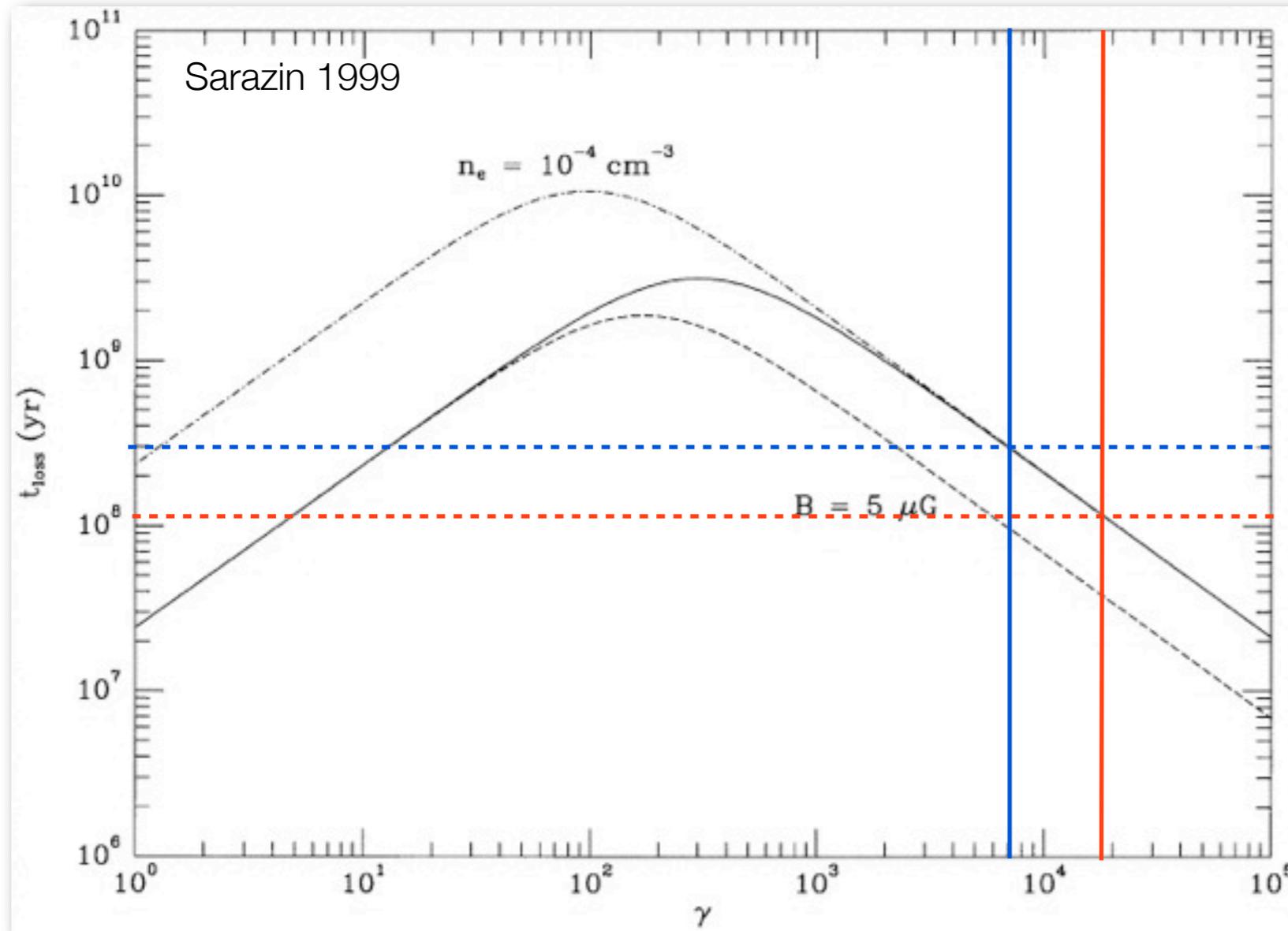


$$\nu_{\text{syn}} = 4.2 \left( \frac{B}{1 \mu\text{G}} \right) \gamma^2 \text{ Hz}$$

$$\nu_{\text{syn}} = 1.4 \text{ GHz} \rightarrow \gamma \simeq 18000$$

$$\nu_{\text{syn}} = 100 \text{ MHz} \rightarrow \gamma \simeq 5000$$

# STEEP SPECTRA

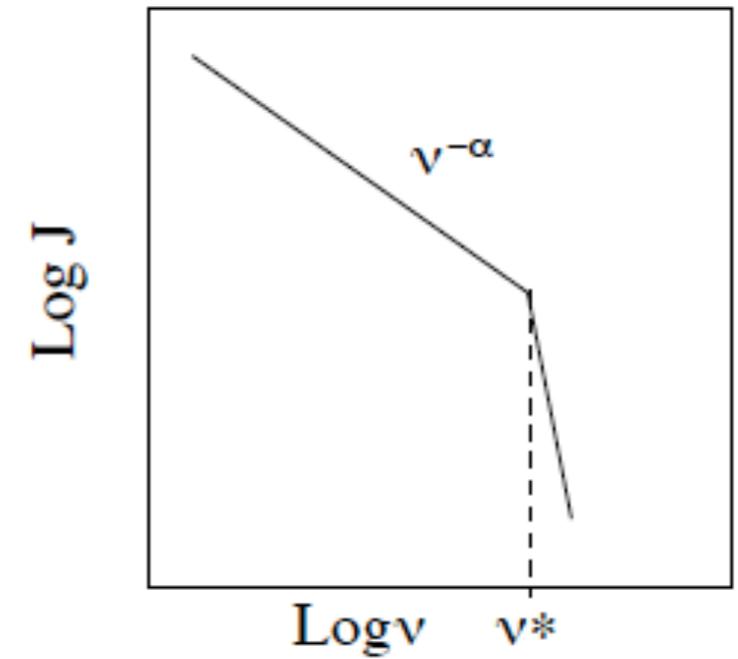
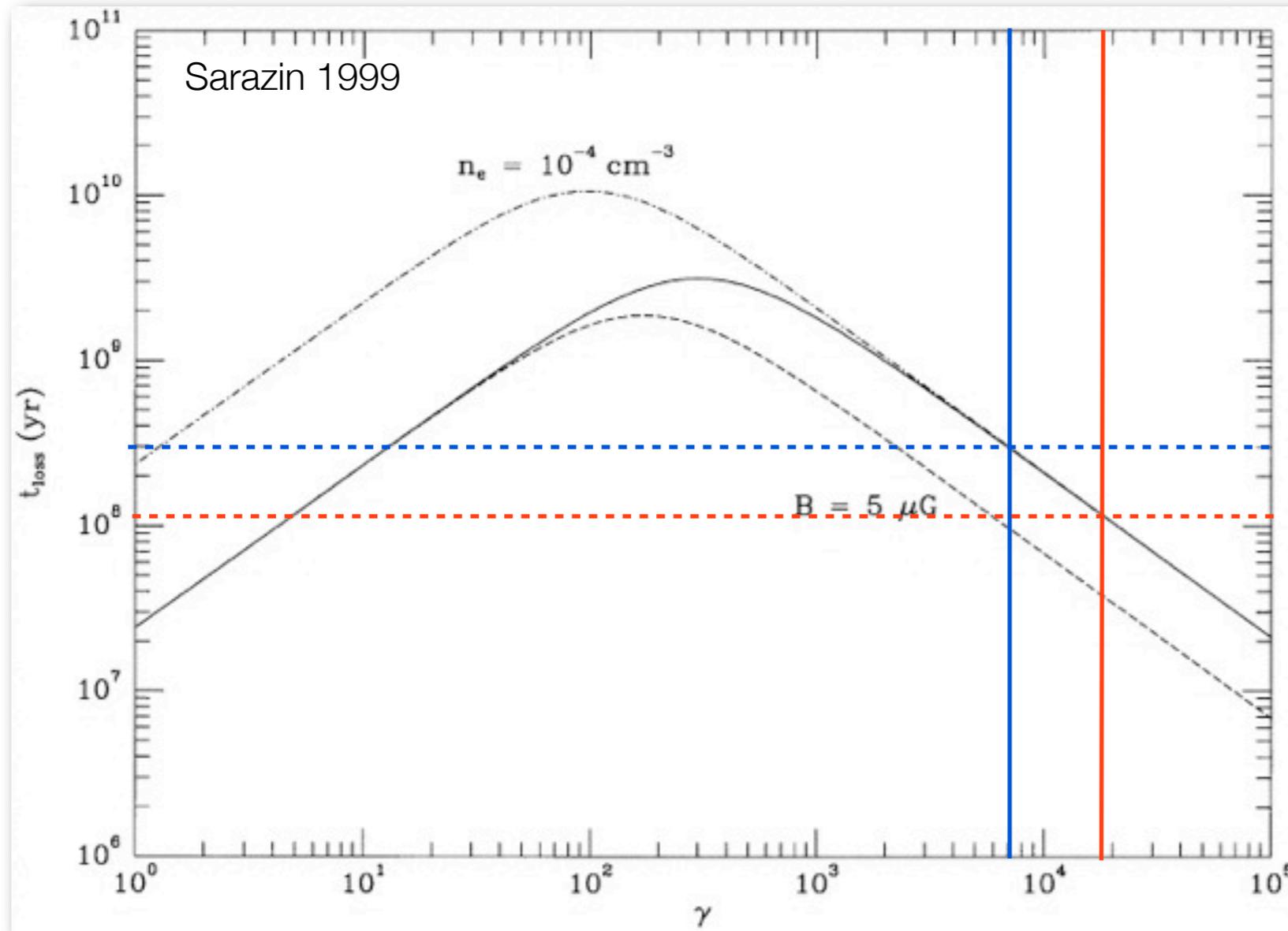


$$\nu_{\text{syn}} = 4.2 \left( \frac{B}{1 \mu\text{G}} \right) \gamma^2 \text{ Hz}$$

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# STEEP SPECTRA

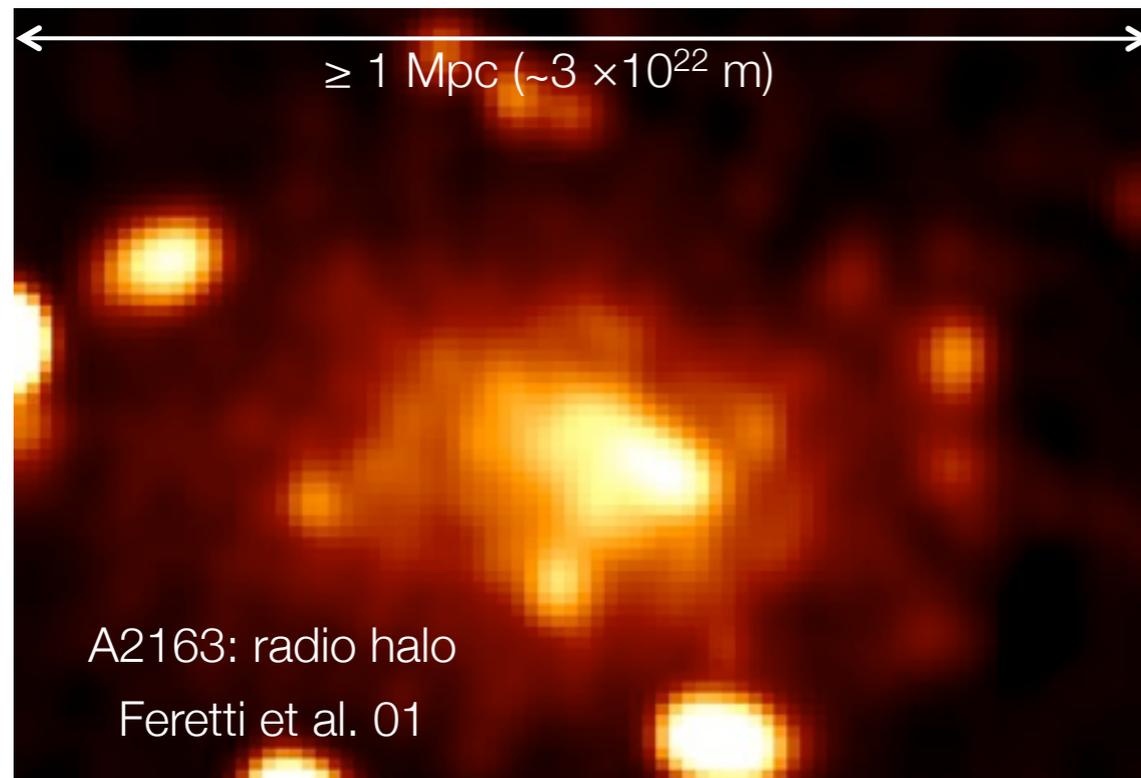


$$\nu_{\text{syn}} = 4.2 \left( \frac{B}{1 \mu\text{G}} \right) \gamma^2 \text{ Hz}$$

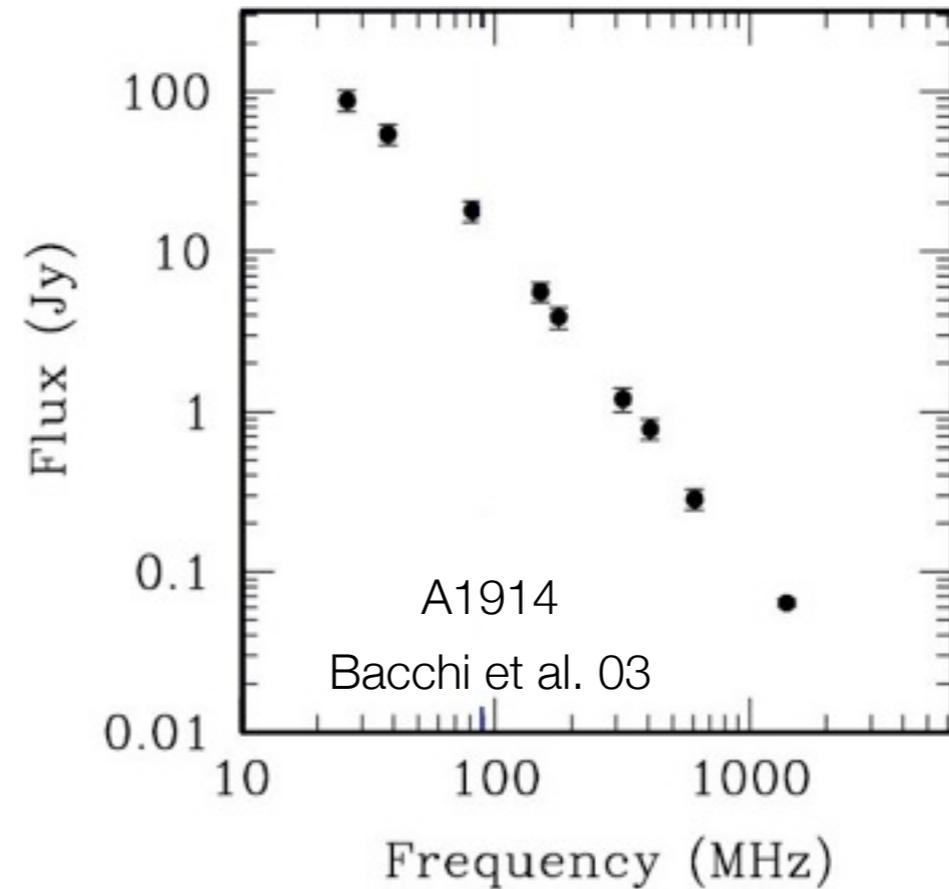
$$\nu_{\text{syn}} = 1.4 \text{ GHz} \rightarrow \gamma \simeq 18000$$

$$\nu_{\text{syn}} = 100 \text{ MHz} \rightarrow \gamma \simeq 5000$$

# ORIGIN OF RELATIVISTIC PARTICLES IN CLUSTERS

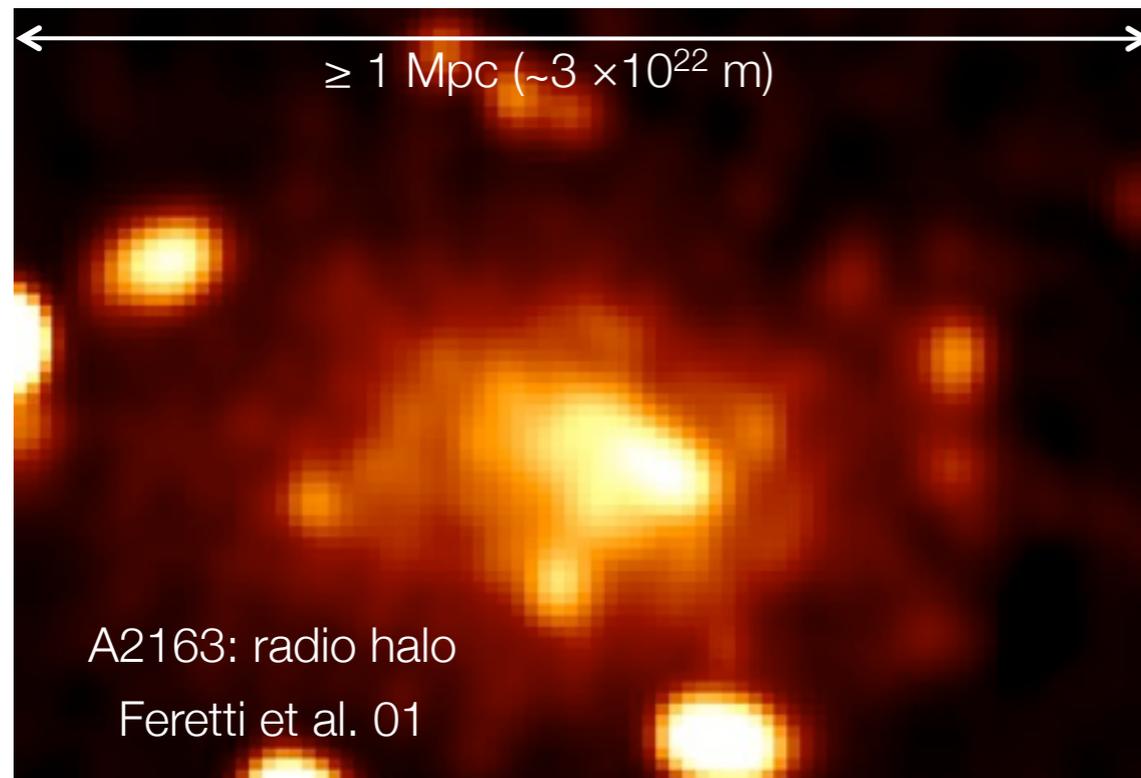


Dimensions:  $\sim 1 \text{ Mpc}$   
Crossing time of  $e^-$ :  $\sim 9.5 \text{ Gyr}$

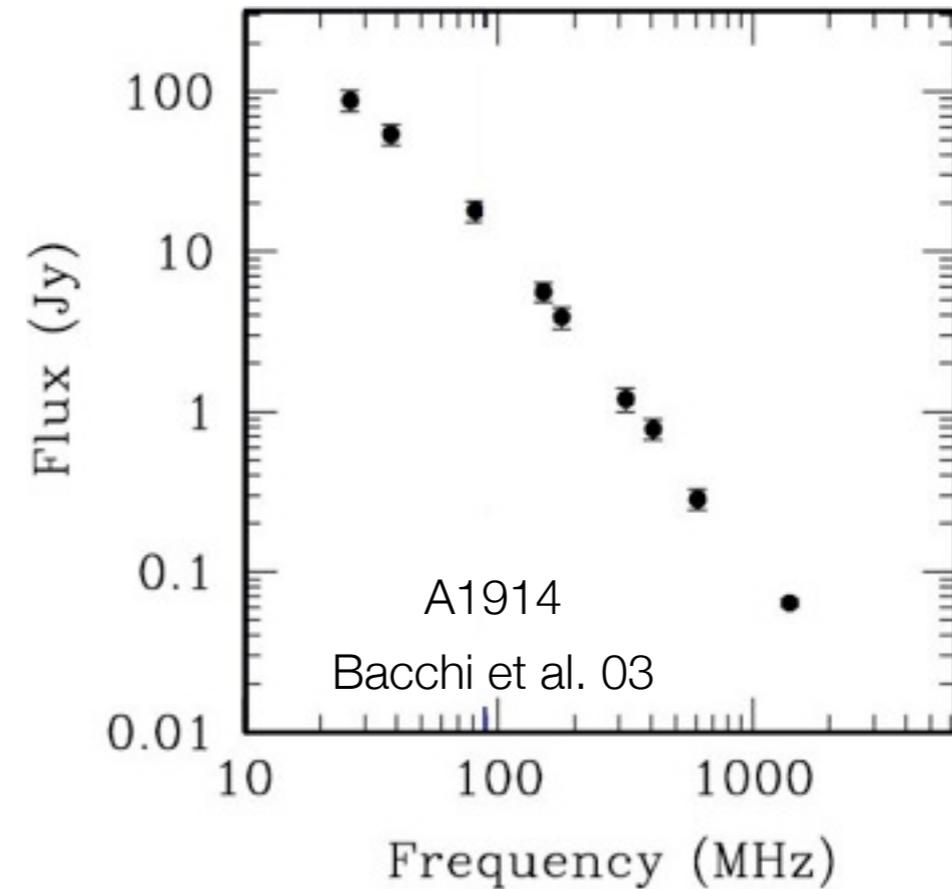


Life time of  $e^-$ :  $\sim 0.1 \text{ Gyr}$

# ORIGIN OF RELATIVISTIC PARTICLES IN CLUSTERS



Dimensions:  $\sim 1 \text{ Mpc}$   
Crossing time of  $e^-$ :  $\sim 9.5 \text{ Gyr}$



Life time of  $e^-$ :  $\sim 0.1 \text{ Gyr}$

→ *In situ acceleration of relativistic electrons*

# ORIGIN OF INTRACLUSTER PRIMARY COSMIC RAYS

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## **Primary cosmic rays**

protons (CRPs) & electrons (CREs)

# ORIGIN OF INTRACLUSTER PRIMARY COSMIC RAYS

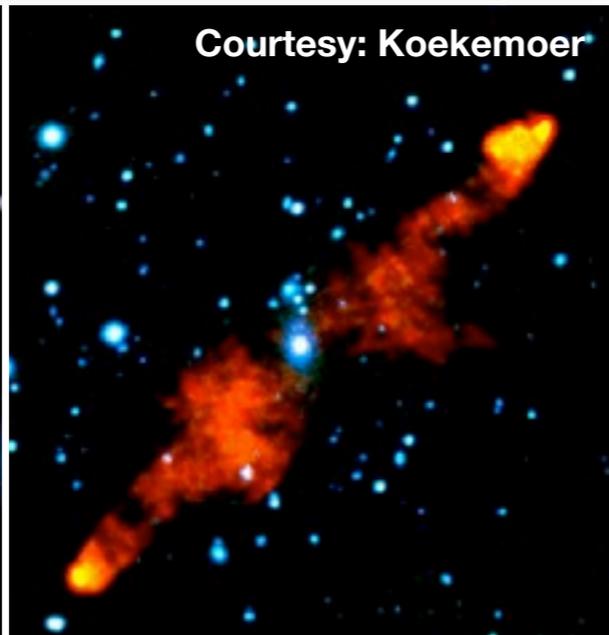
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**Primary cosmic rays**  
protons (CRPs) & electrons (CREs)

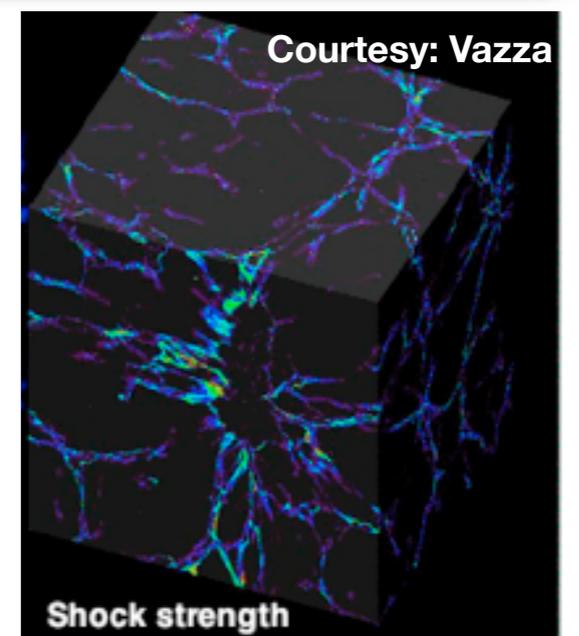
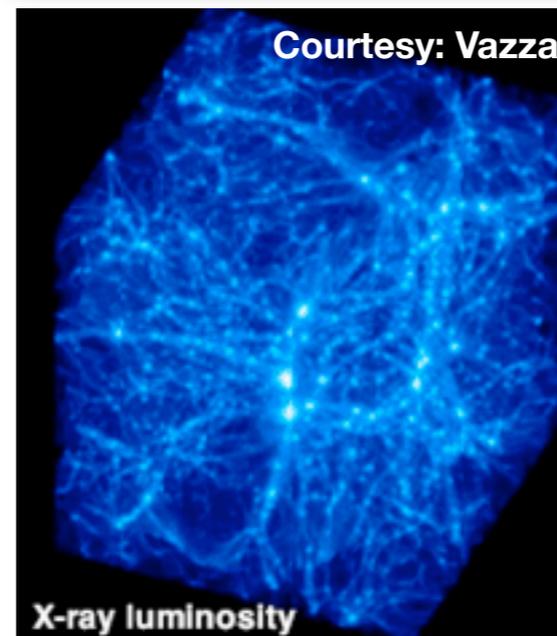
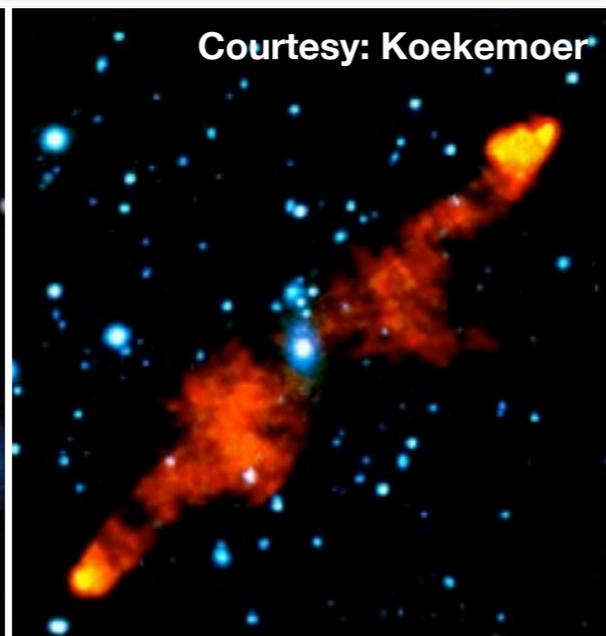
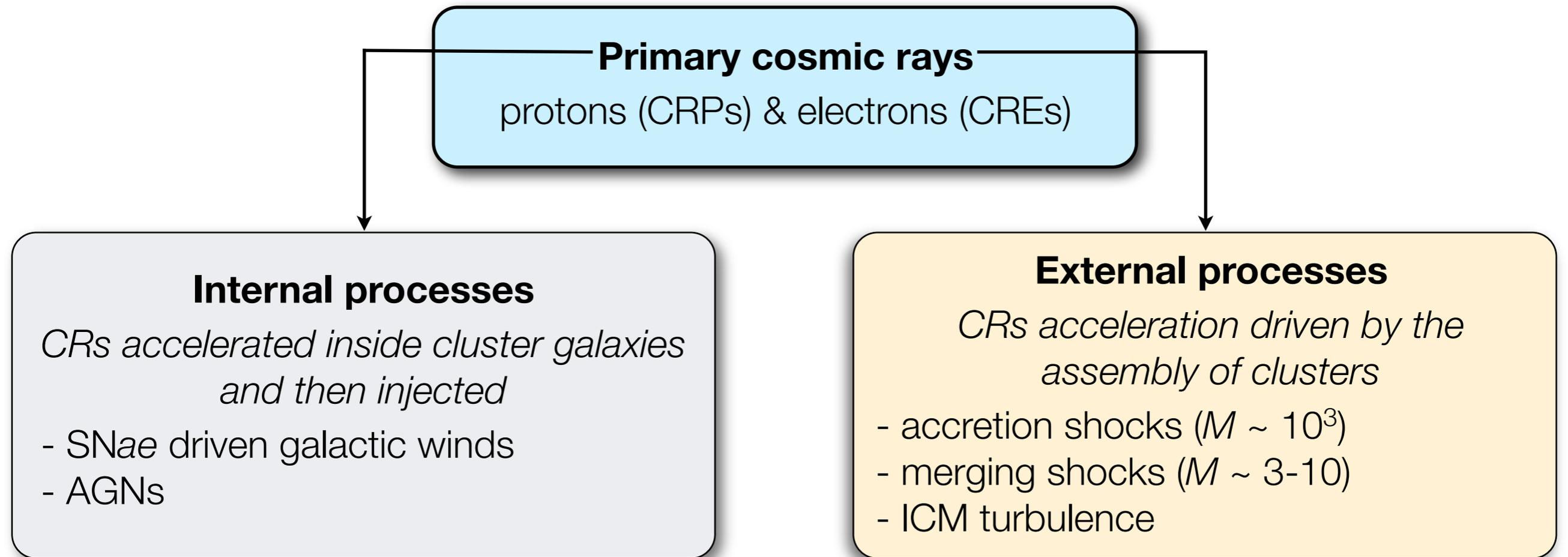


**Internal processes**  
*CRs accelerated inside cluster galaxies  
and then injected*

- SNaE driven galactic winds
- AGNs



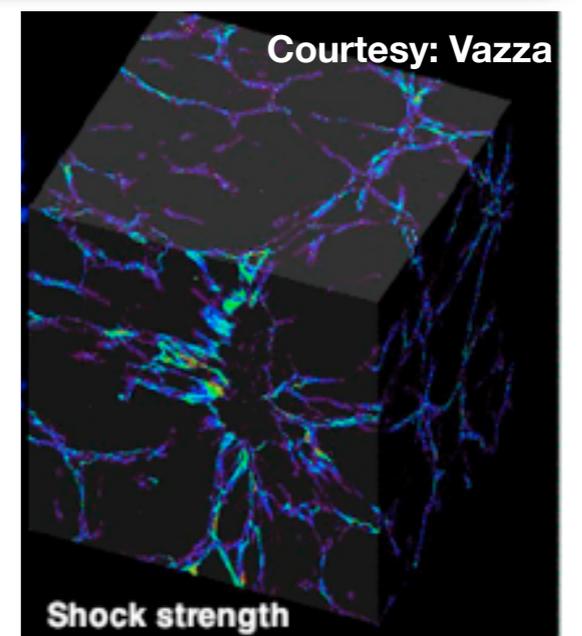
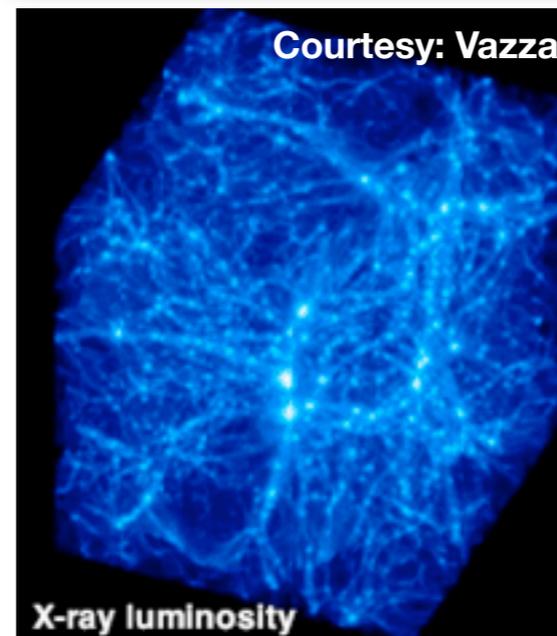
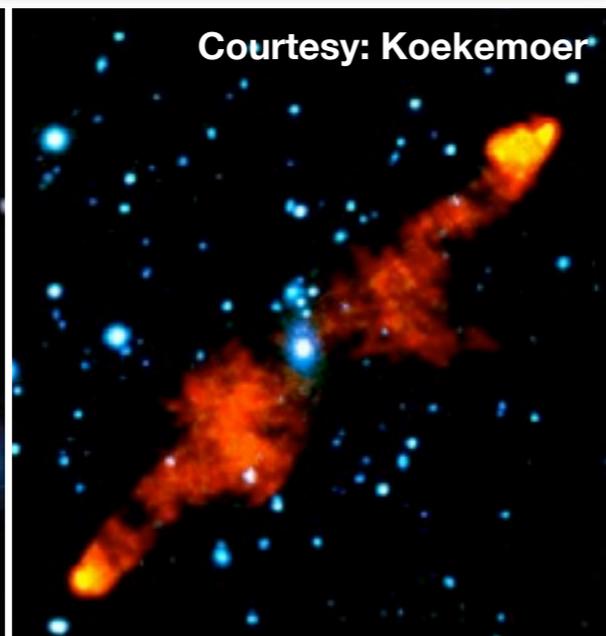
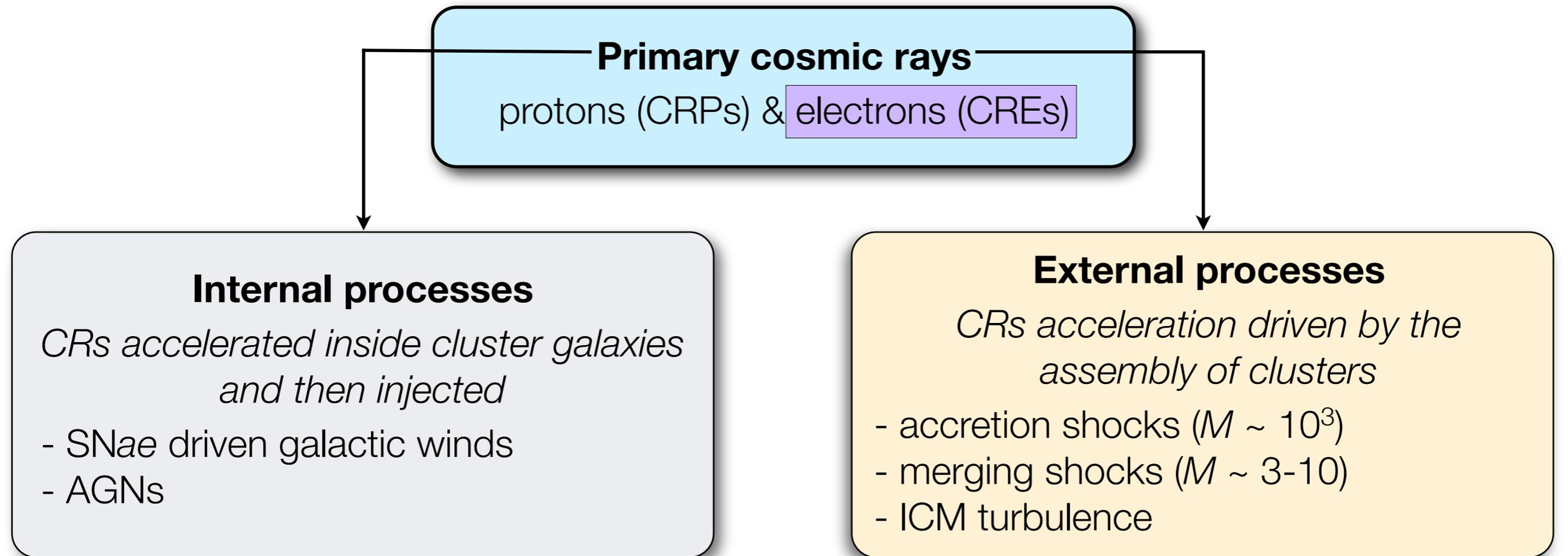
# ORIGIN OF INTRACLUSTER PRIMARY COSMIC RAYS



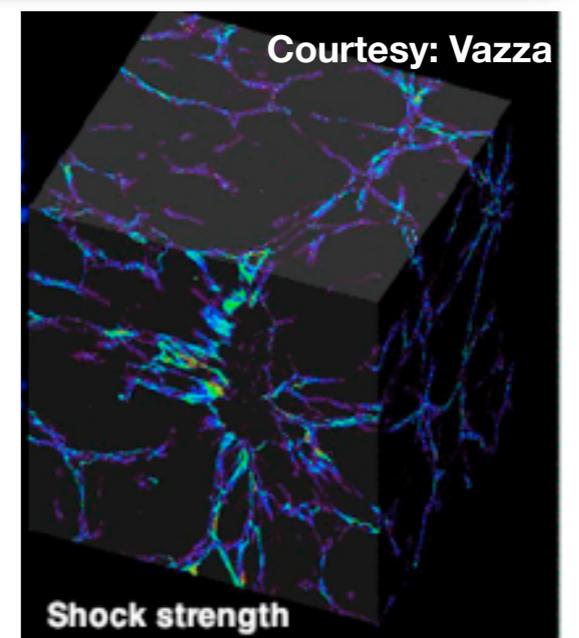
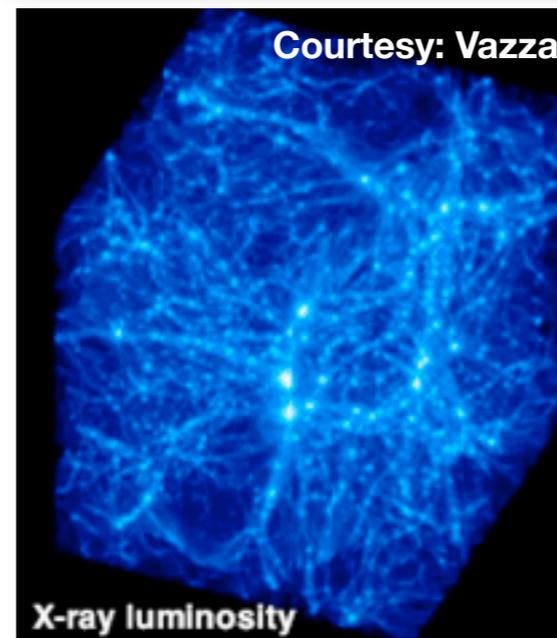
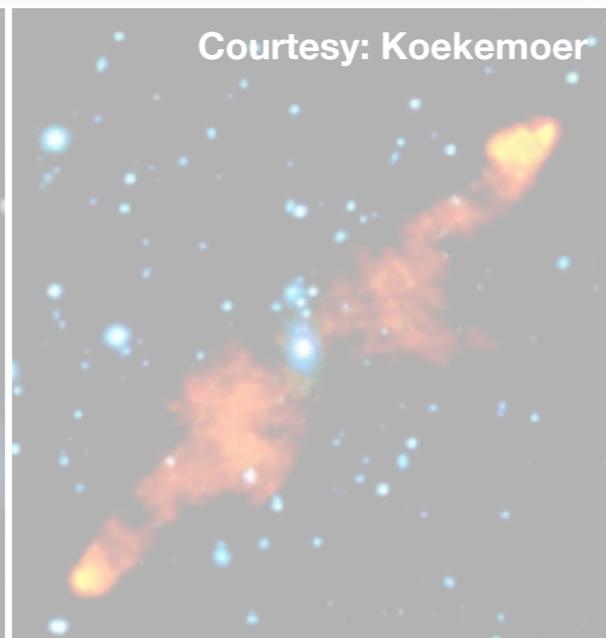
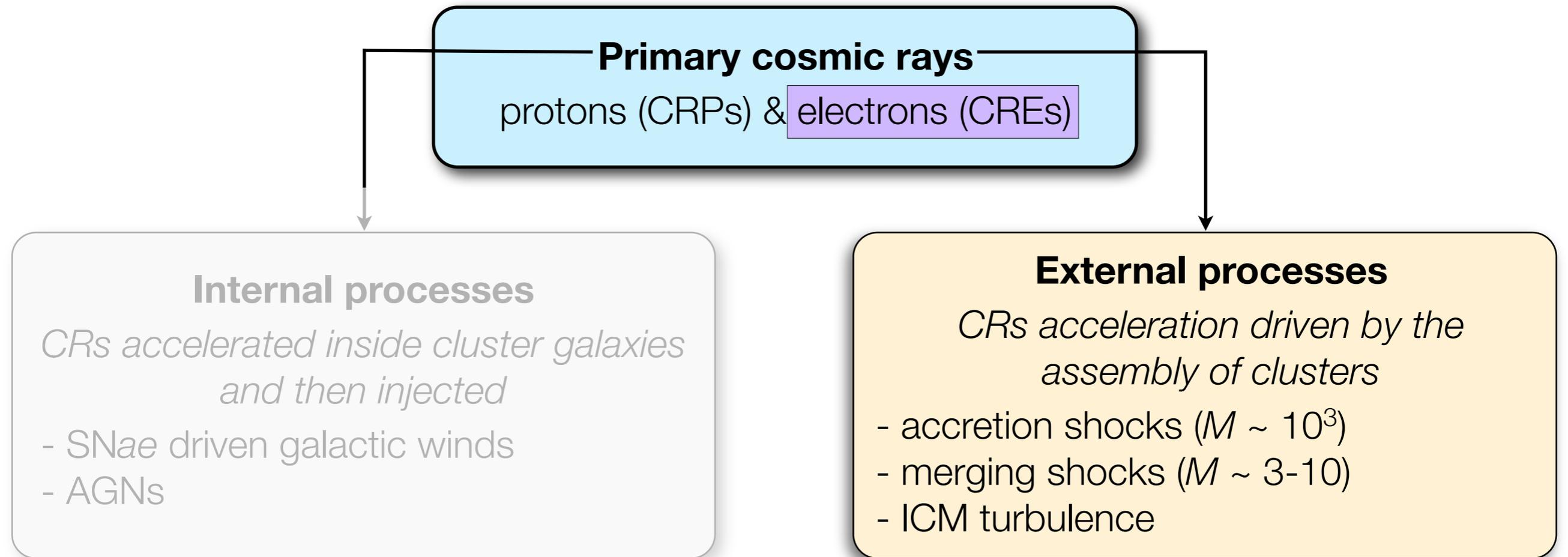
# Numerical Simulations of Large Scale Structure Formation

Kapferer et al., Innsbruck University

# ORIGIN OF INTRACLUSTER PRIMARY COSMIC RAYS



# ORIGIN OF INTRACLUSTER PRIMARY COSMIC RAYS



# ORIGIN OF INTRACLUSTER PRIMARY COSMIC RAYS

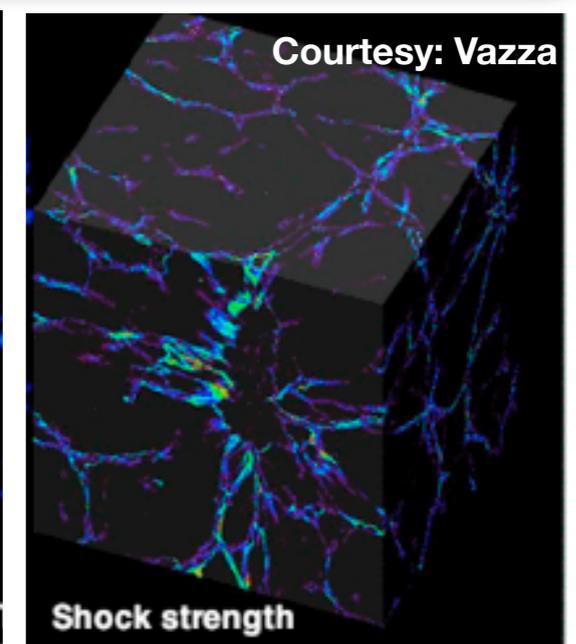
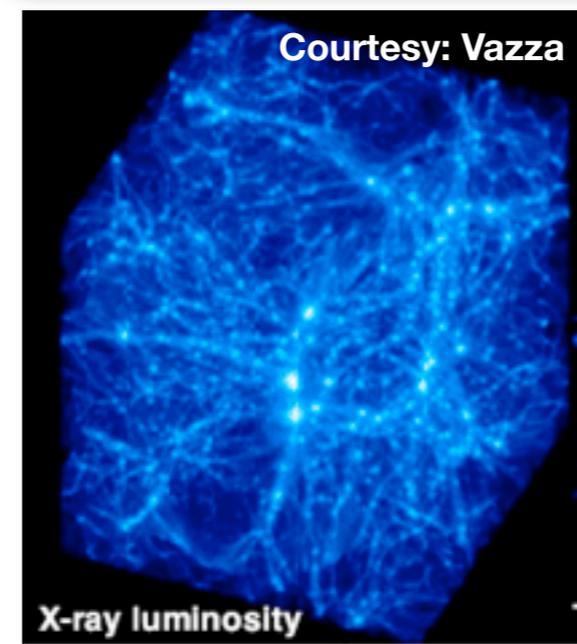
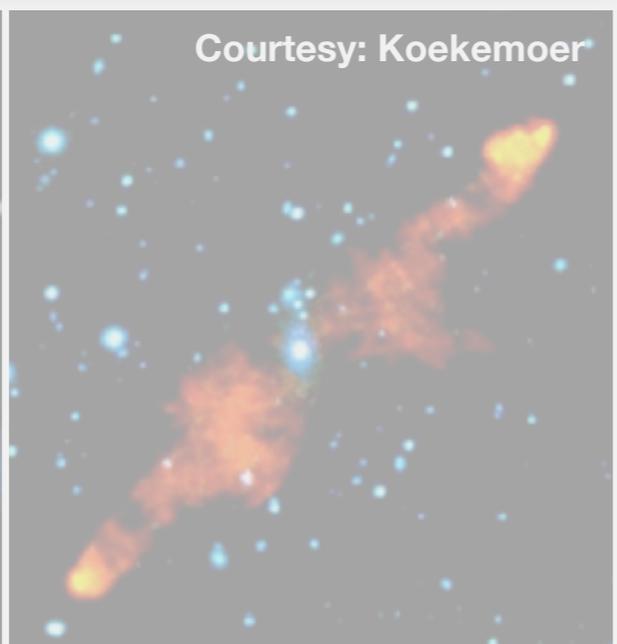
**Primary cosmic rays**  
protons (CRPs) & electrons (CREs)

**Internal processes**  
*CRs accelerated inside cluster galaxies and then injected*

- SNaE driven galactic winds
- AGNs

**External processes**  
*CRs acceleration driven by the assembly of clusters*

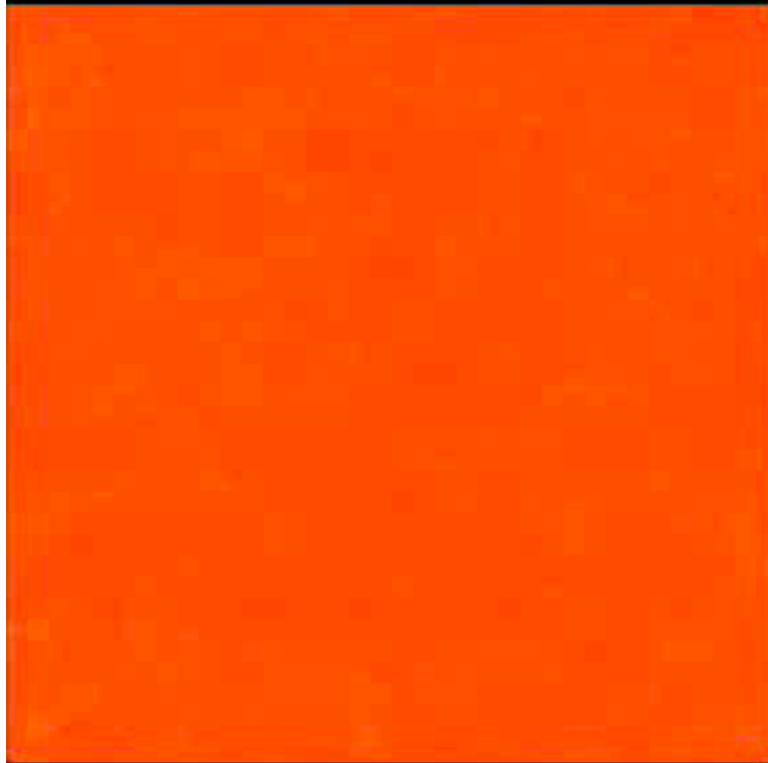
- accretion shocks ( $M \sim 10^3$ ) GeV
- merging shocks ( $M \sim 3-10$ ) TeV
- ICM turbulence GeV



Thermal energy of the ICM

Baryon density

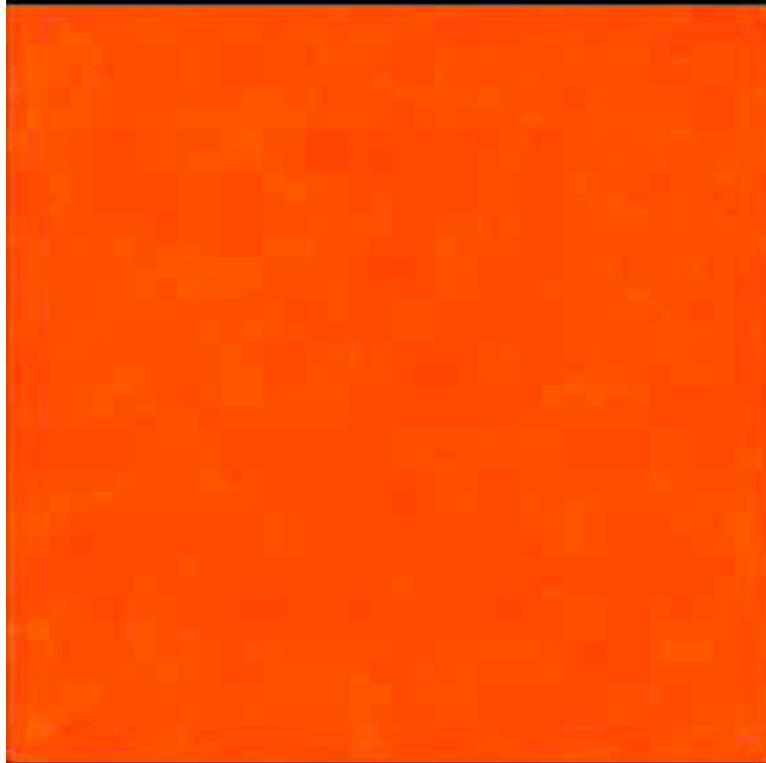
Mach number



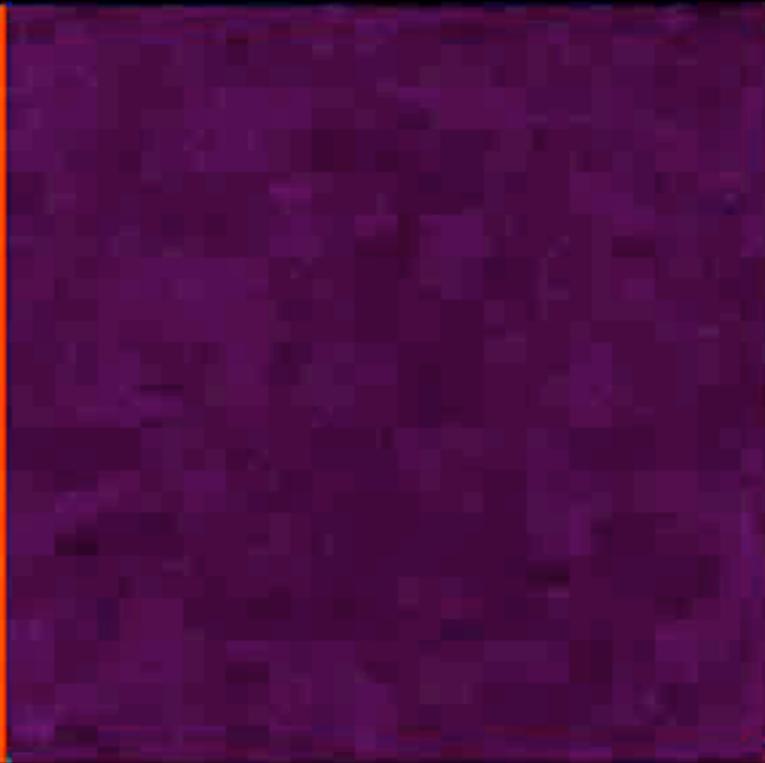
0.0 0.5 1.0 1.5 2.0 2.5 3.0 3.5 4.0 4.5 5.0

Credits: F. Vazza

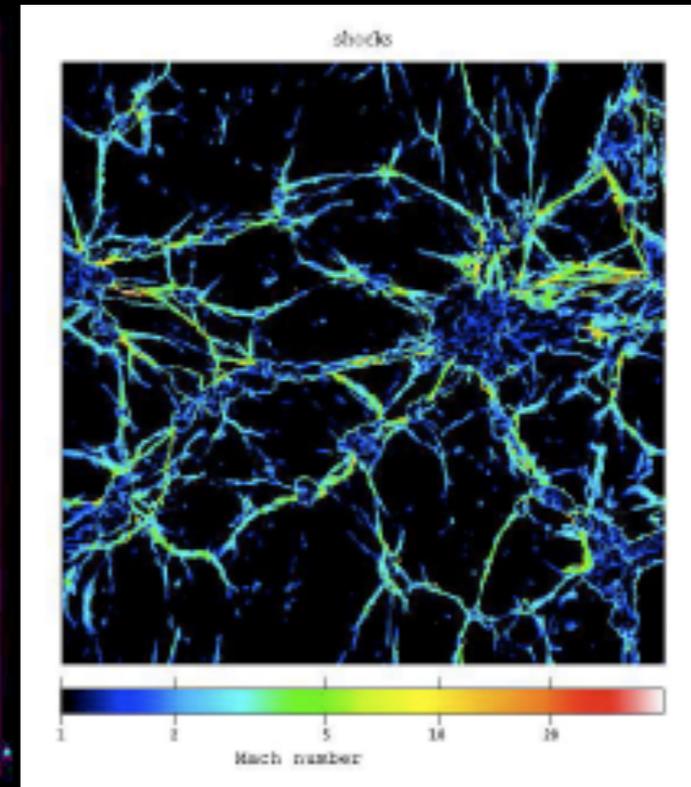
Thermal energy of the ICM



Baryon density



Mach number



Credits: F. Vazza

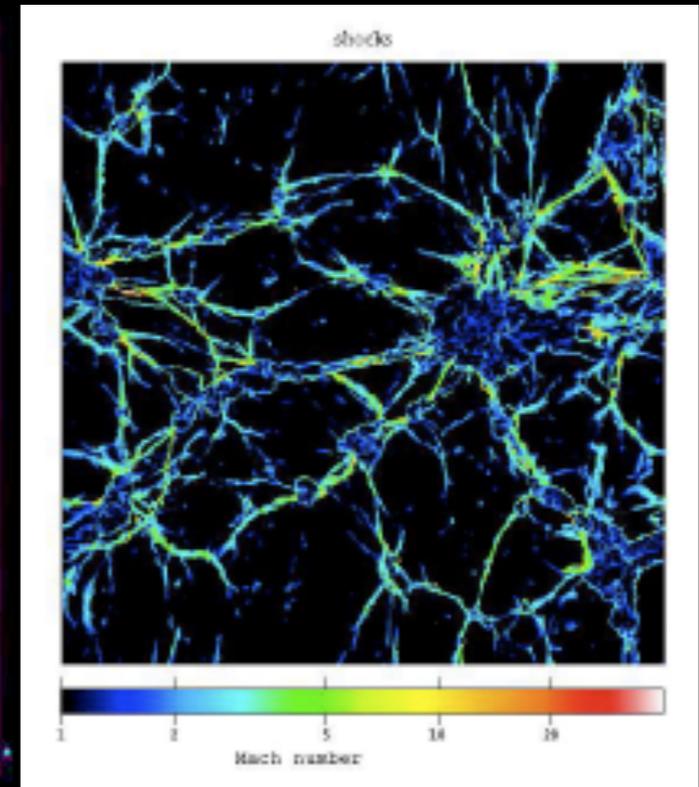
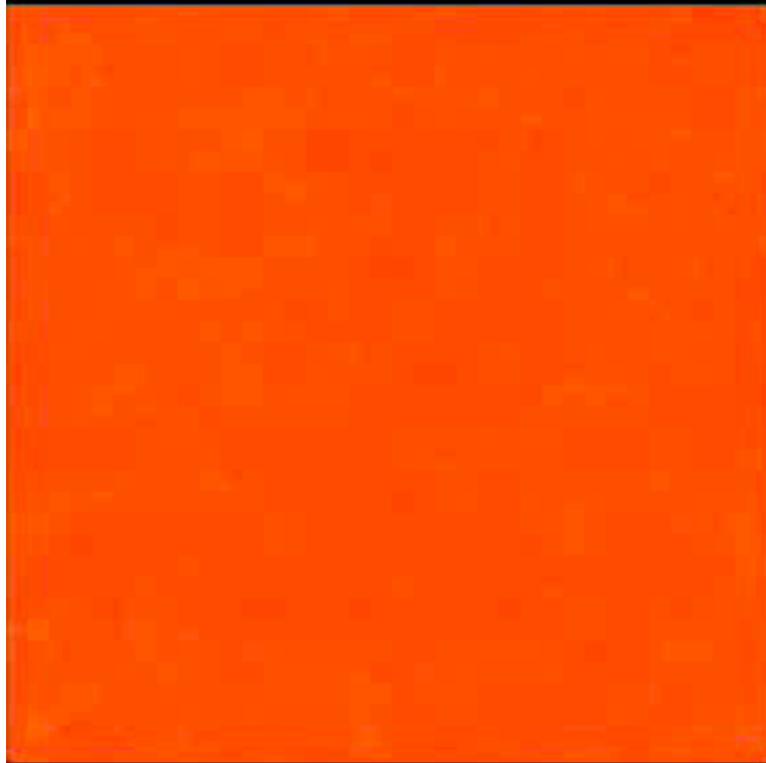
$$N(\gamma) d\gamma \sim N_0 \gamma^{-\mu} d\gamma$$

$$\mu = \frac{2(M^2 + 1)}{M^2 - 1}$$

Thermal energy of the ICM

Baryon density

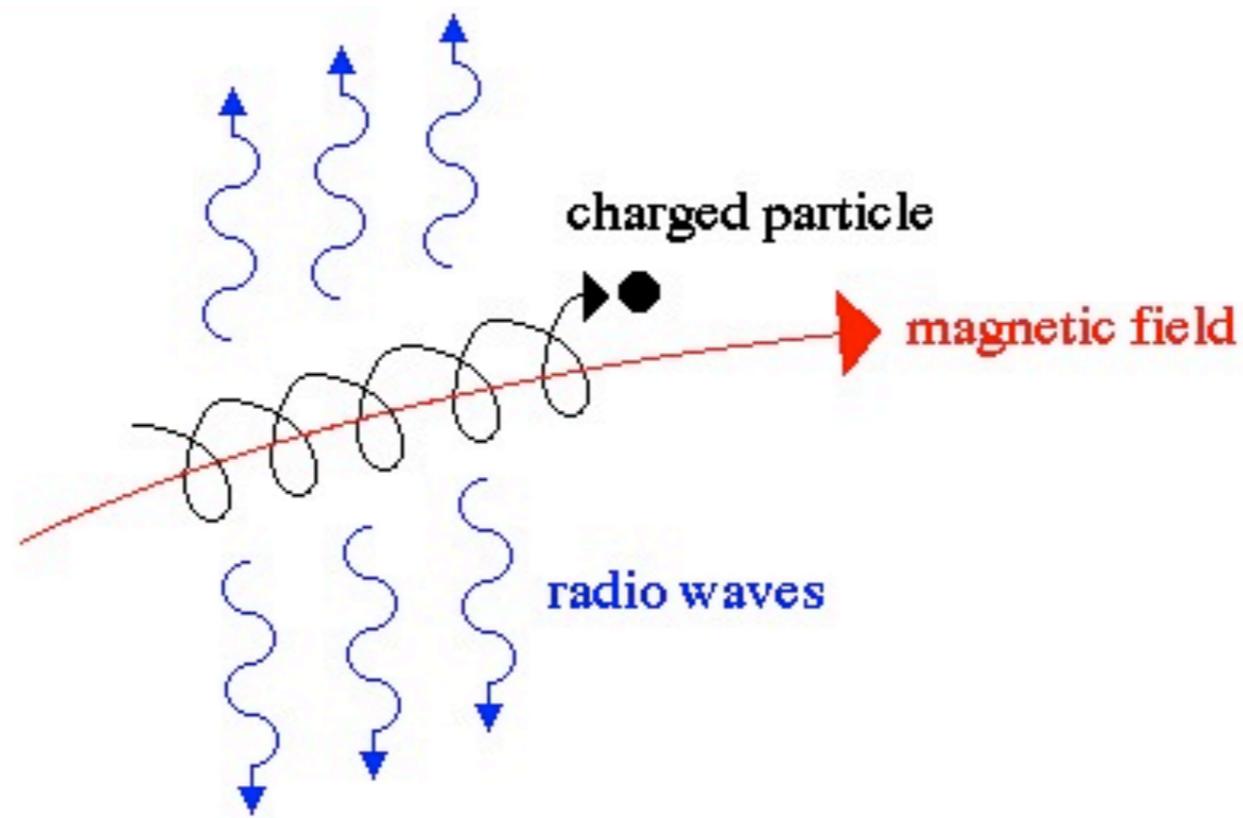
Mach number



Credits: F. Vazza

# NT ELECTRONS & MAGNETIC FIELDS IN THE UNIVERSE

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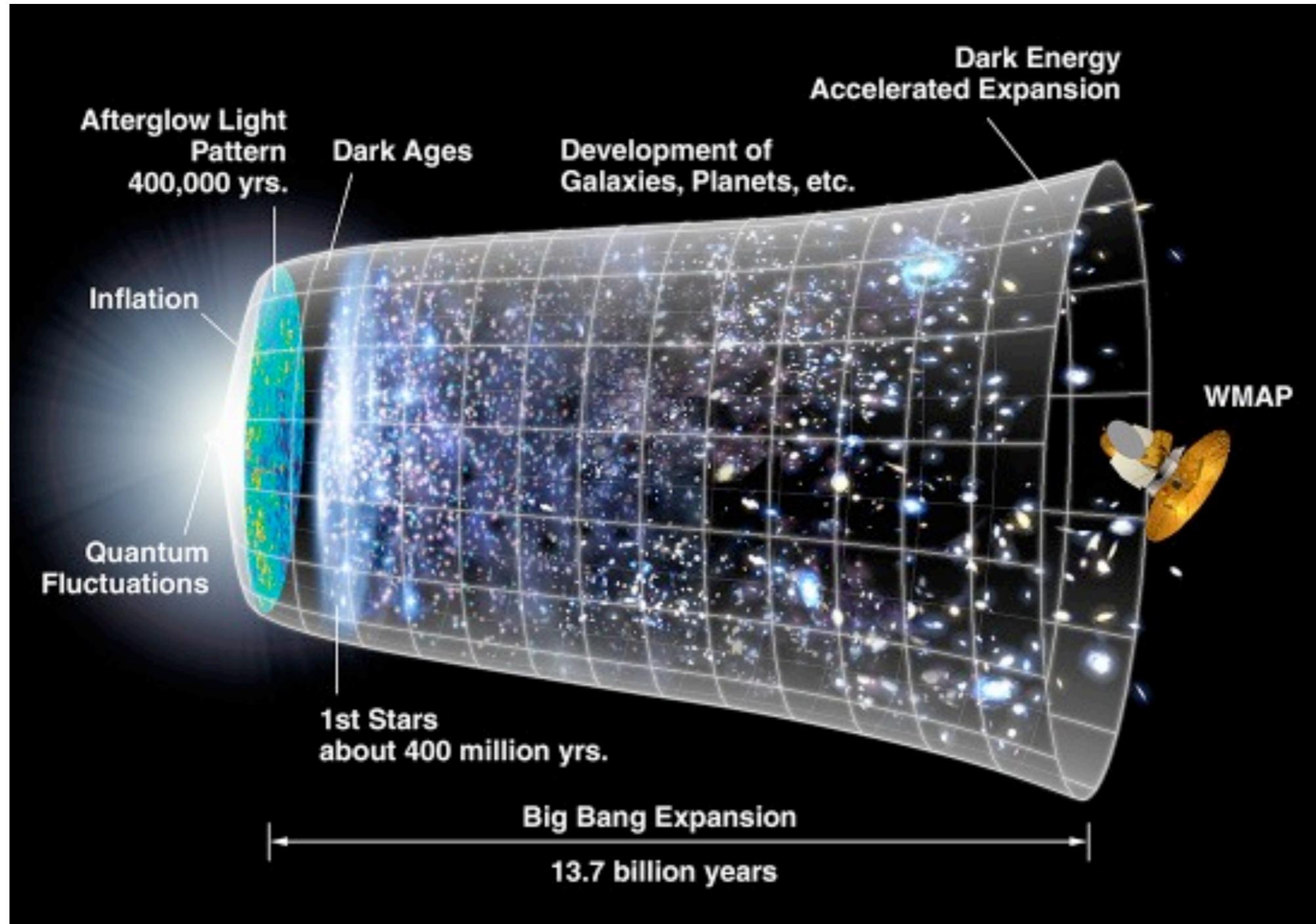


Synchrotron

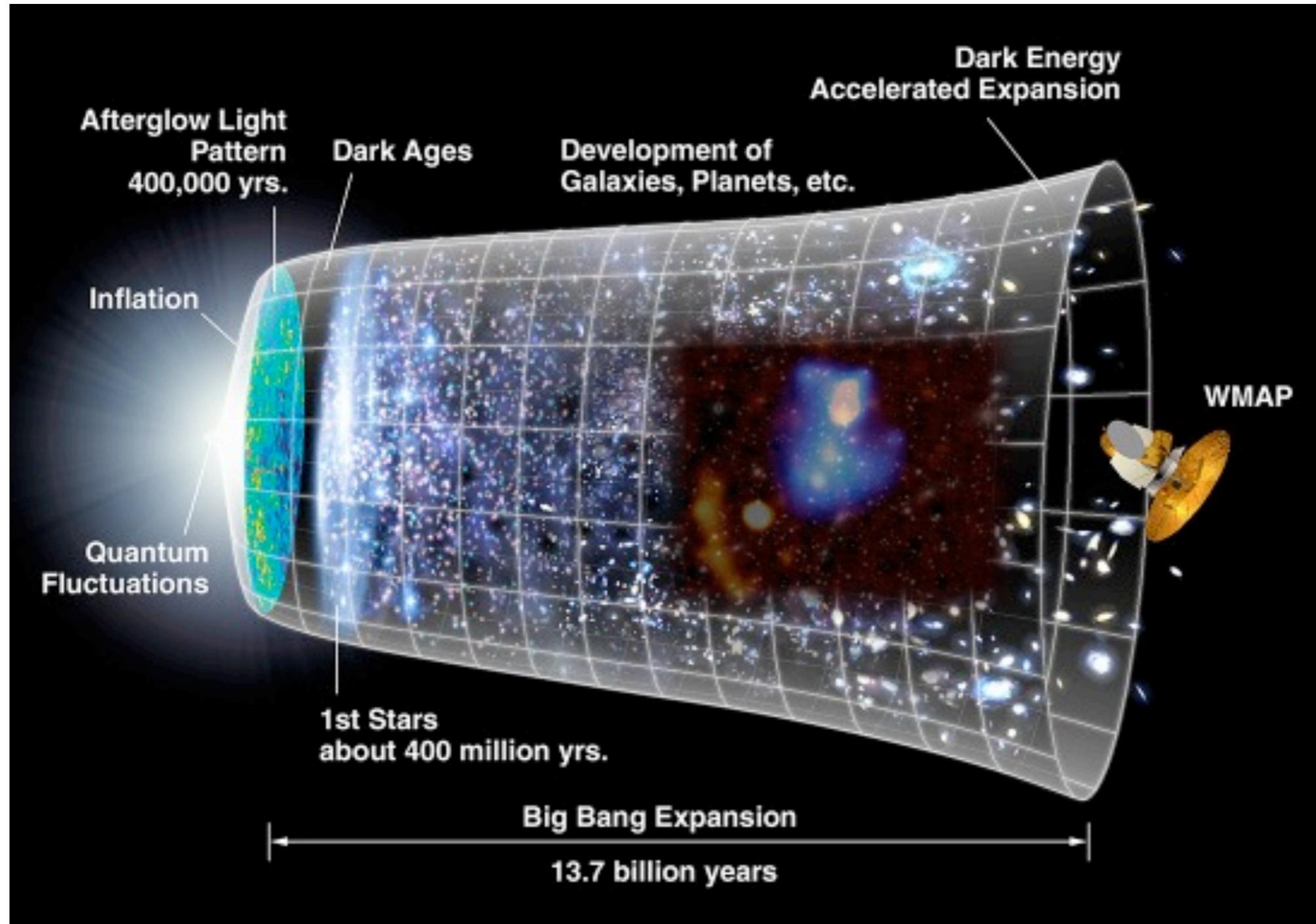
**GeV electrons** → Radio emission

**TeV electrons** → Hard X-ray emission

# PHOTONS & ELECTRONS IN THE UNIVERSE



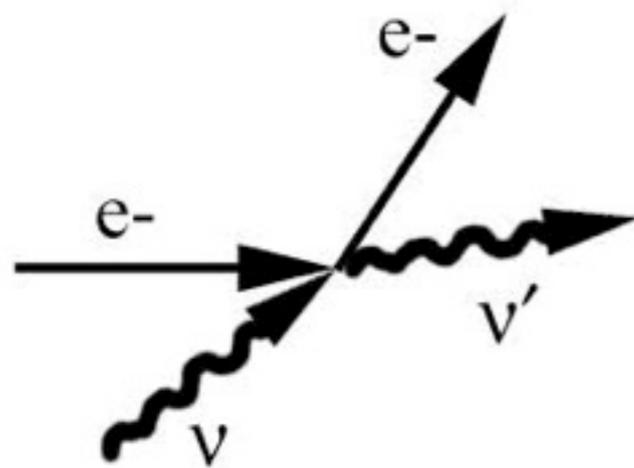
# PHOTONS & ELECTRONS IN THE UNIVERSE



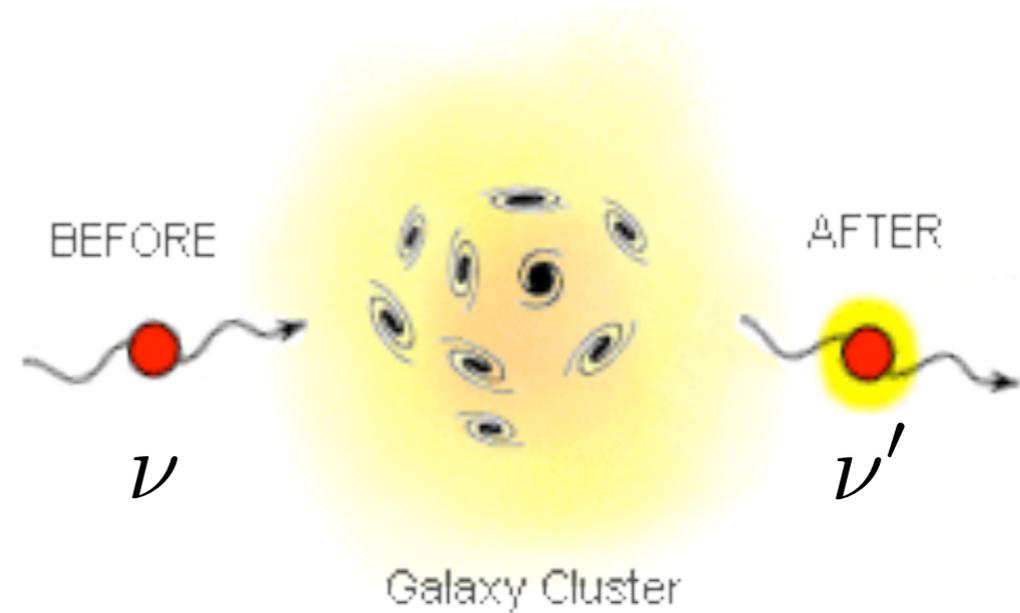
# PHOTONS & ELECTRONS IN THE UNIVERSE

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Inverse Compton scattering



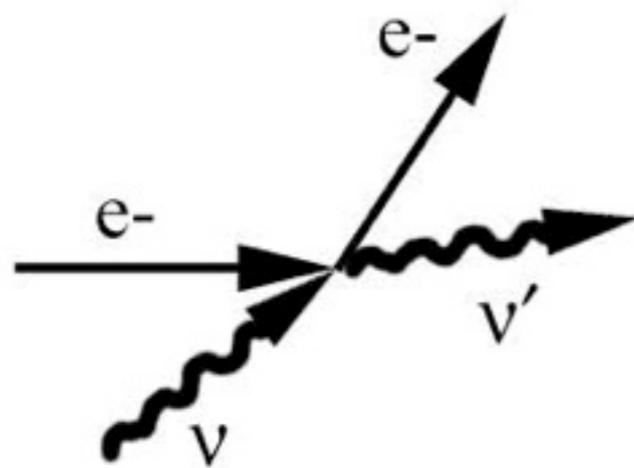
$\nu' > \nu$   
High energy  $e^-$  initially  
 $e^-$  loses energy



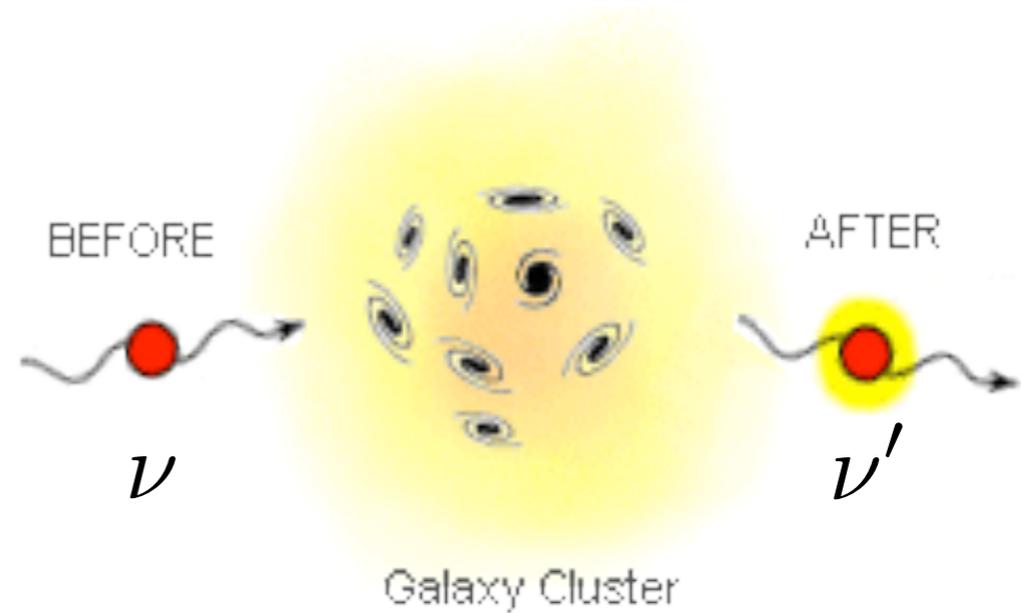
$$\frac{\langle \nu' \rangle}{\nu} = \frac{4}{3} \gamma_{el}^2$$

# PHOTONS & ELECTRONS IN THE UNIVERSE

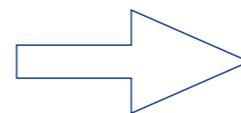
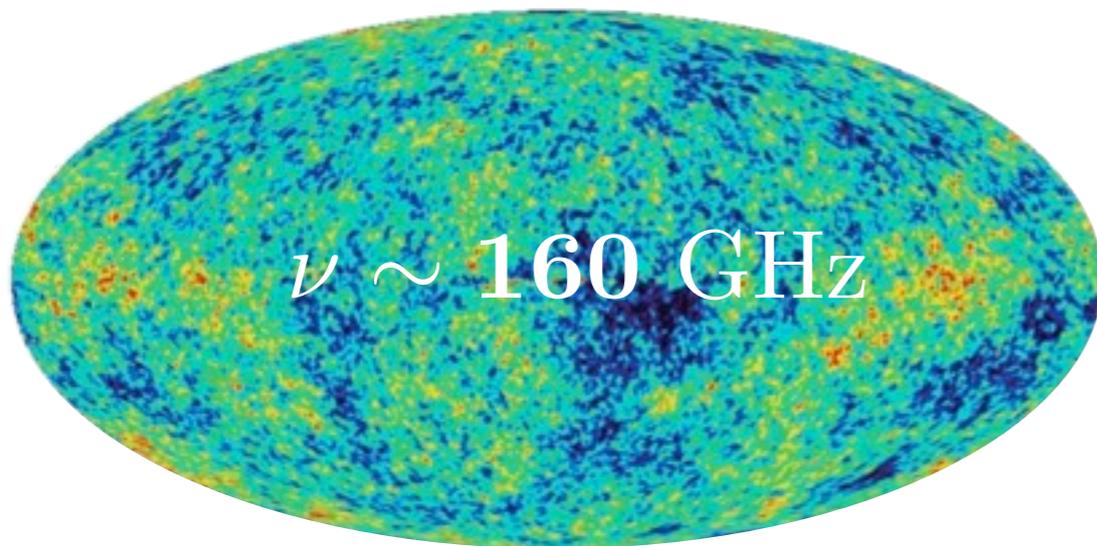
Inverse Compton scattering



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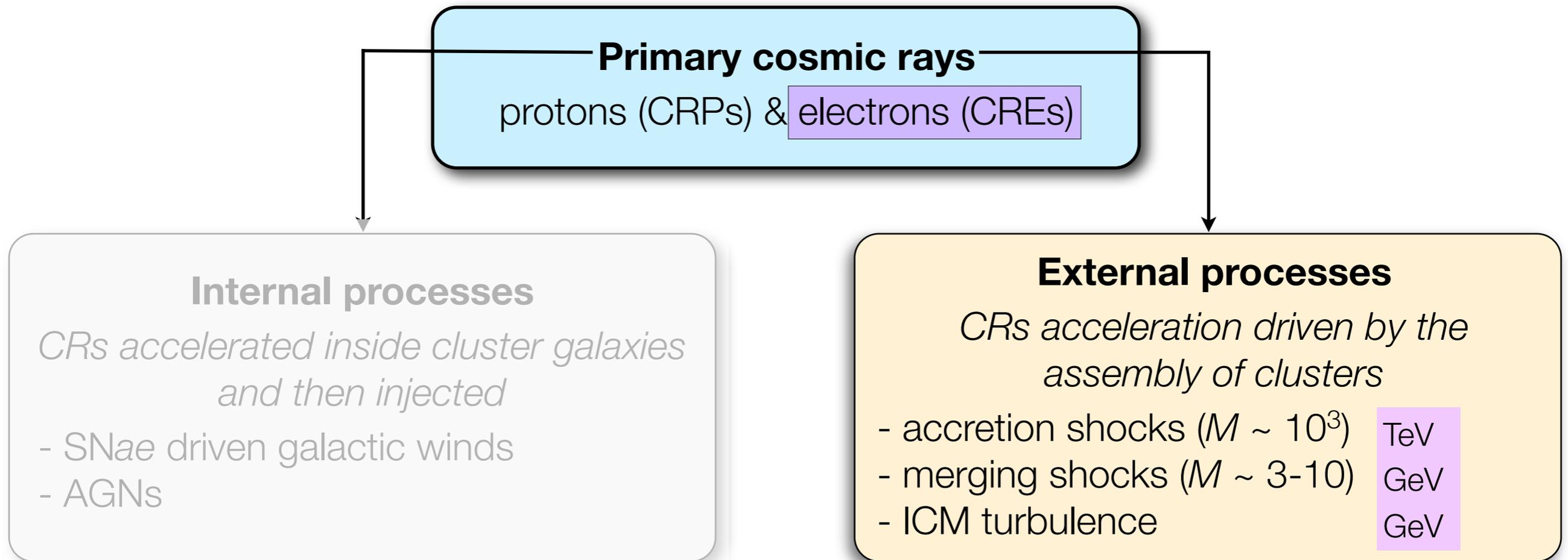


$$\frac{\langle \nu' \rangle}{\nu} = \frac{4}{3} \gamma_{el}^2$$



- keV electrons** → Sub-mm photons
- GeV electrons** → Hard X-ray photons
- TeV electrons** → Gamma-ray photons

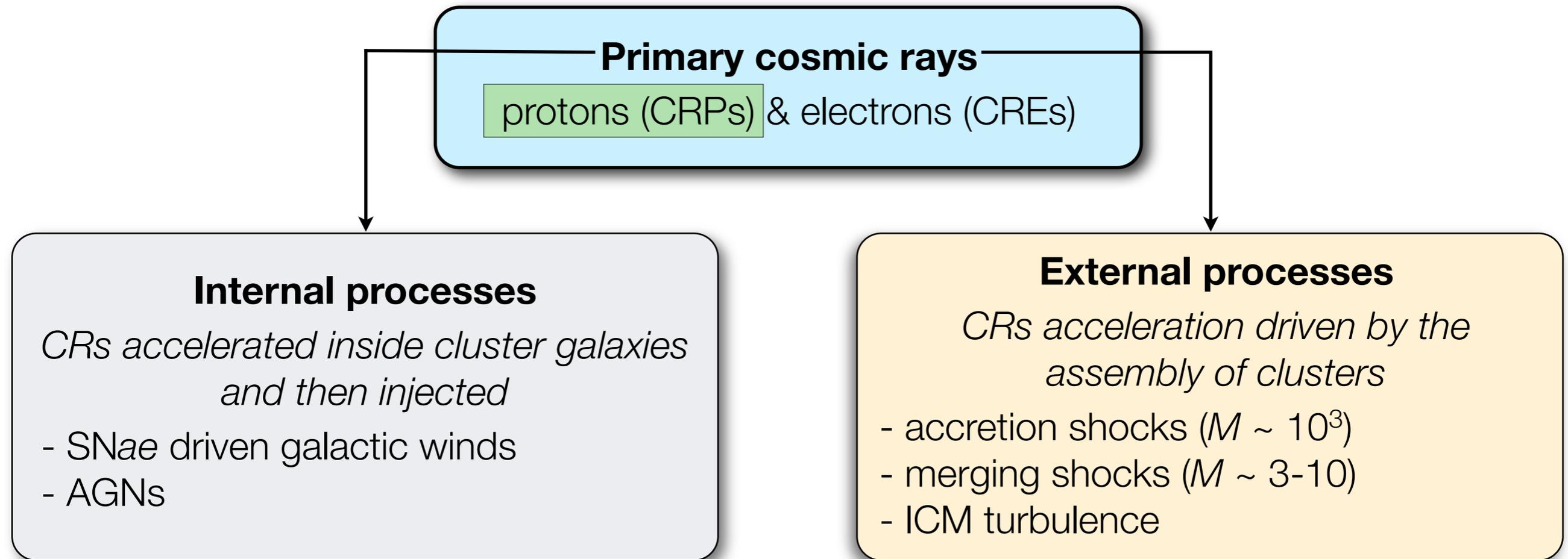
# ORIGIN OF INTRACLUSTER PRIMARY COSMIC RAYS



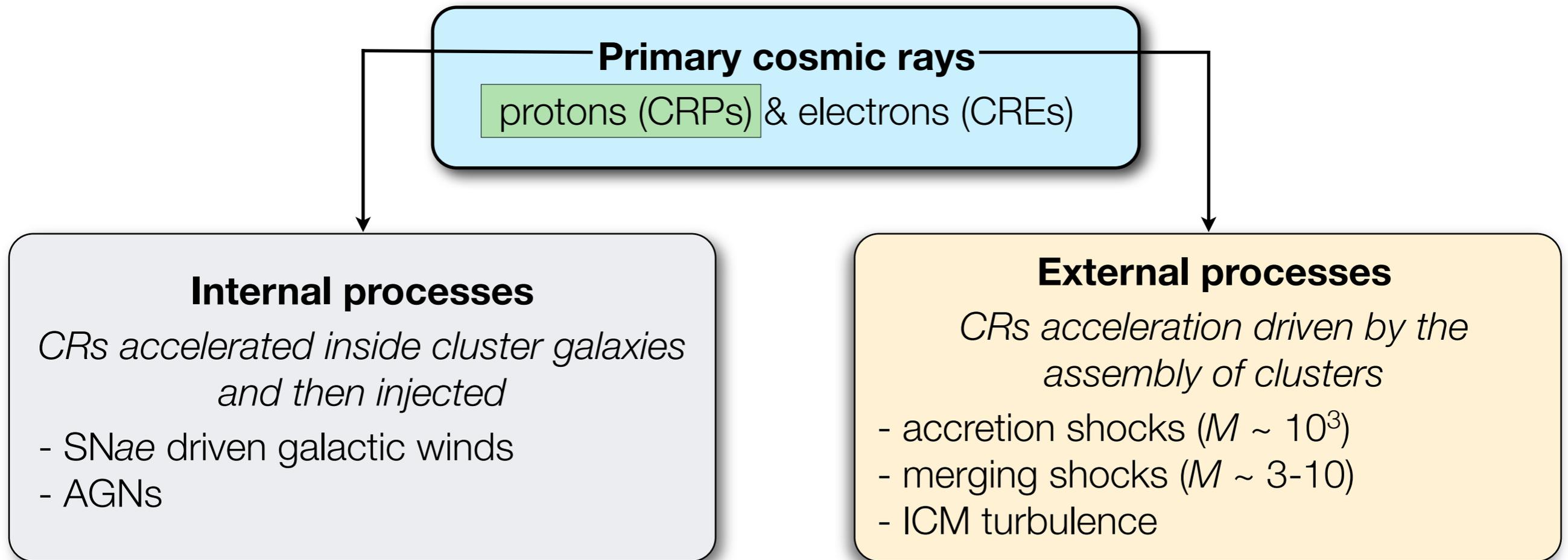
$$\tau \approx 2 \times 10^3 \gamma^{-1} \left[ (1+z)^4 + \left( \frac{B}{3.3 \mu\text{G}} \right)^2 \right]^{-1} \text{Gyr} \begin{cases} E_{e^-} \approx \text{GeV} \rightarrow \tau \lesssim \text{Gyr} \\ E_{e^-} \approx \text{TeV} \rightarrow \tau \lesssim \text{Myr} \end{cases}$$

# ORIGIN OF INTRACLUSTER PRIMARY COSMIC RAYS

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# ORIGIN OF INTRACLUSTER PRIMARY COSMIC RAYS



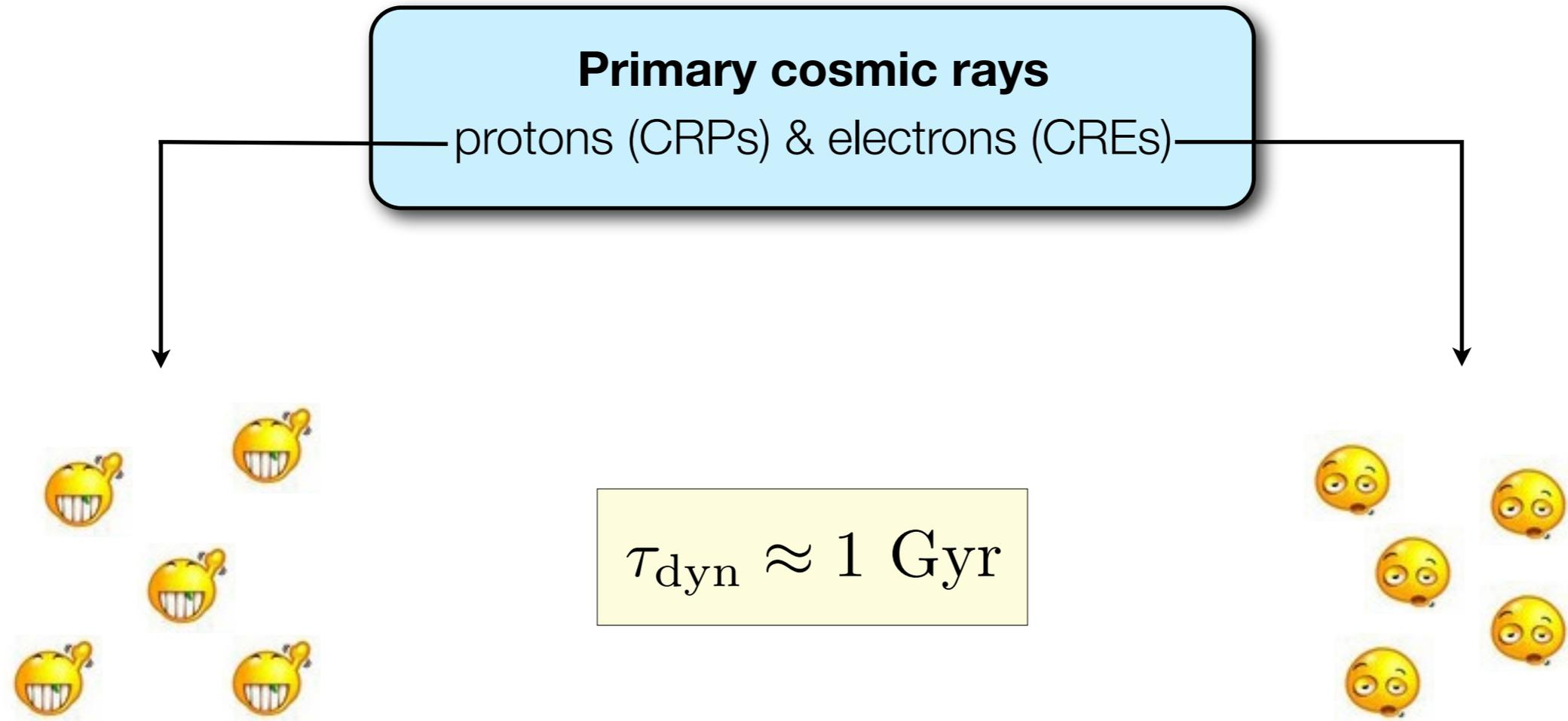
- Time scale for energy losses longer than Hubble time
- Time scale for diffusion out of clusters longer than Hubble time for energies  $< 10^6$  GeV
- $E_{\max} \sim 10^{20}$  eV + Acceleration over  $\sim 10^{10}$  yrs

→ effective accumulation of high energy particles in clusters

e.g. Sarazin 2002, Vannoni et al. 2009 and refs. therein

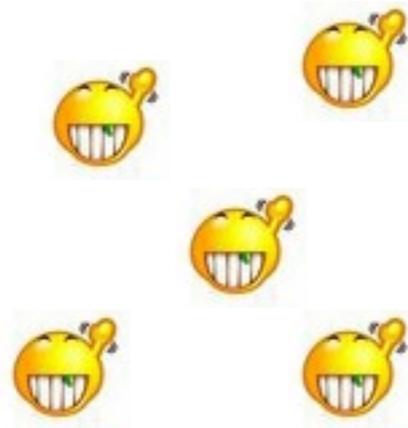
# CRs OVER THE DYNAMICAL TIME SCALE OF A CLUSTER

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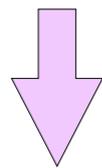
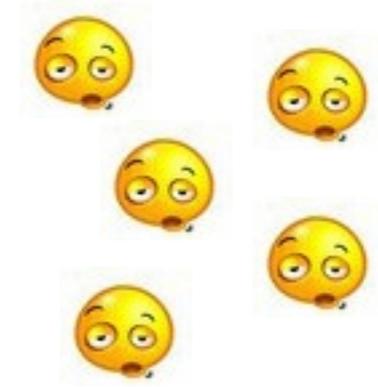


# CRs OVER THE DYNAMICAL TIME SCALE OF A CLUSTER

**Primary cosmic rays**  
protons (CRPs) & electrons (CREs)



$\tau_{\text{dyn}} \approx 1 \text{ Gyr}$



Production of secondary CREs

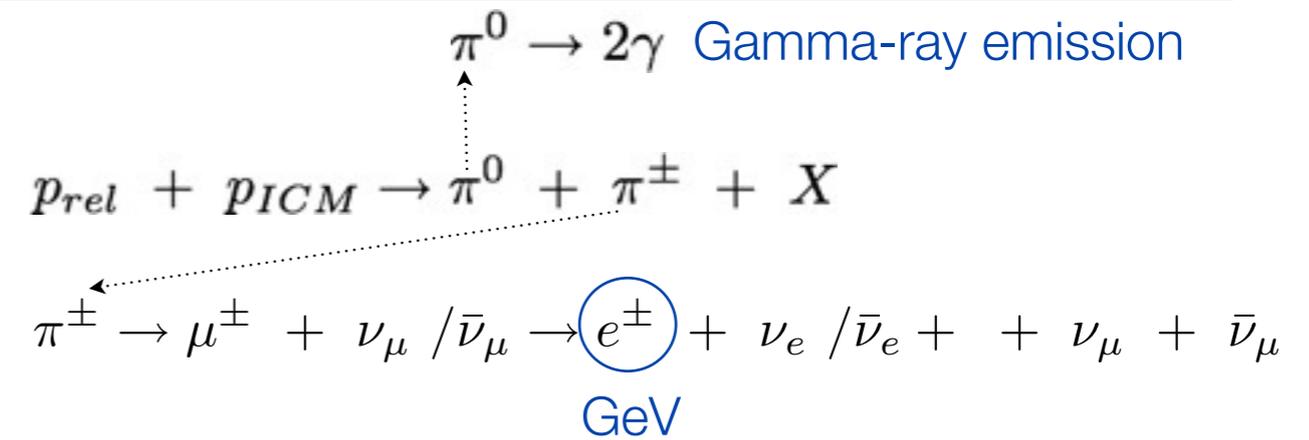
# SECONDARY RELATIVISTIC ELECTRONS

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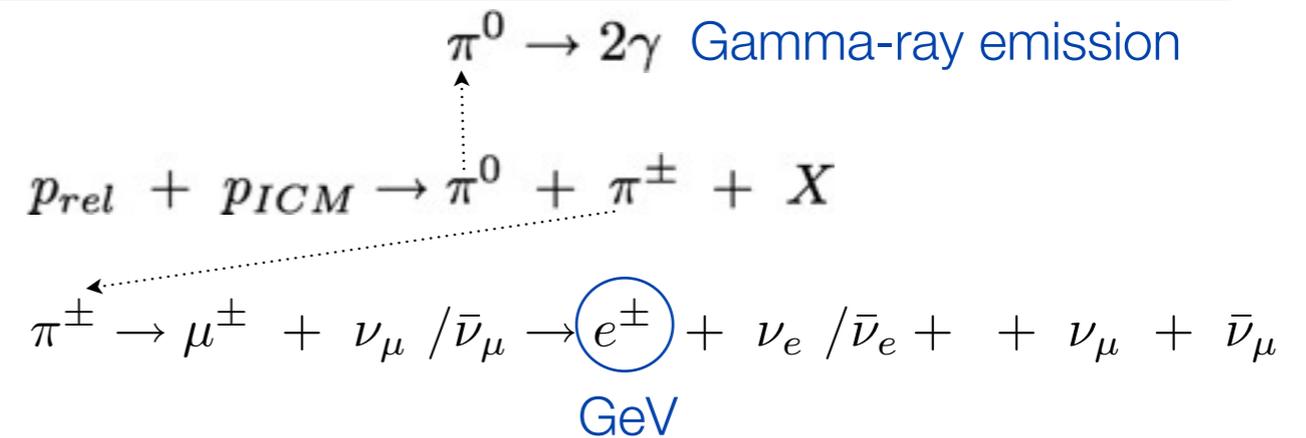
- Inelastic collision between relativistic protons and target protons provided by the hot intracluster medium - e.g. Dennison 1980



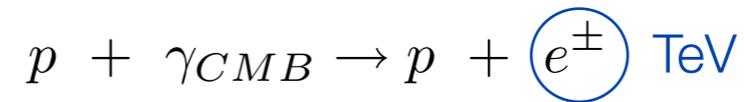
# SECONDARY RELATIVISTIC ELECTRONS

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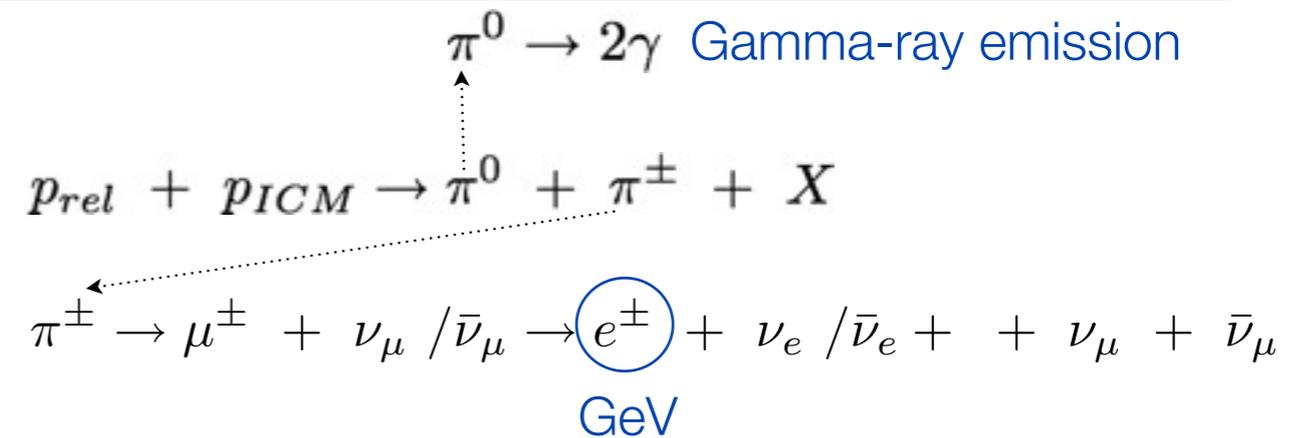


- Interaction between ultra-relativistic protons and CMB photons (Bethe-Heitler process) - e.g. Inoue et al. 2005

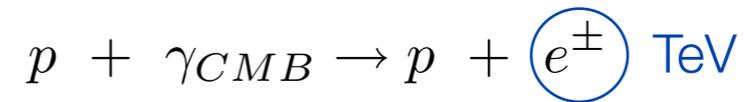


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- Inelastic collision between relativistic protons and target protons provided by the hot intracluster medium - e.g. Dennison 1980



- Interaction between ultra-relativistic protons and CMB photons (Bethe-Heitler process) - e.g. Inoue et al. 2005



- Interaction of very high-energy photons (gamma-rays) with the CMB radiation field - e.g. Timokhin et al. 2004



# COSMIC RAYS, PHOTONS, MAGNETIC FIELDS: MULTI-WAVELENGTH EMISSION

---

“Product” CRs	Radio	Hard X-rays	Gamma-rays	Secondary relativistic electrons (GeV)	Secondary ultra-relativistic electrons (TeV)
Relativistic electrons (GeV)	+ Magnetic field (synchrotron)	+ CMB photons (IC)			
Relativistic protons (GeV)			+ ICM ions (hadronic processes)	+ ICM ions (hadronic processes)	
Ultra-relativistic electrons (TeV)		+ Magnetic field (synchrotron)	+ CMB photons (IC)		
Ultra-relativistic protons (TeV)					+ CMB photons (Bethe-Heitler)

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Ultra-relativistic protons (TeV)					+ CMB photons (Bethe-Heitler)

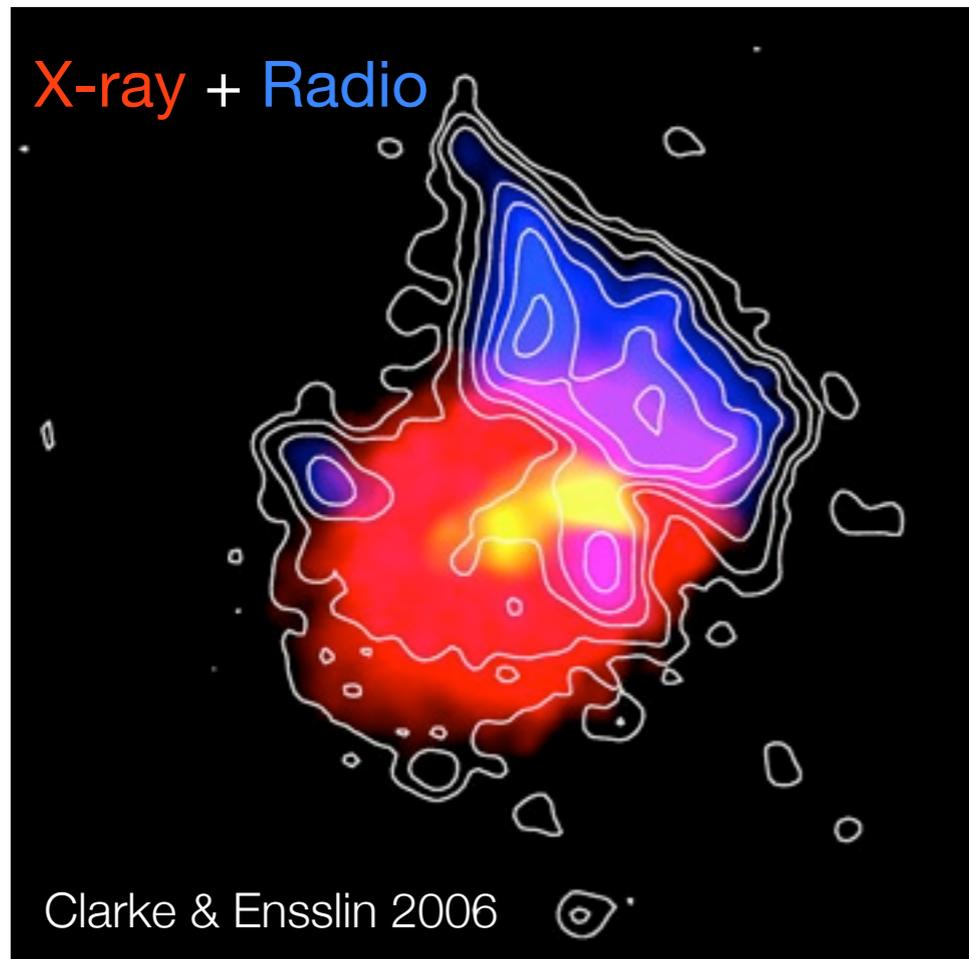
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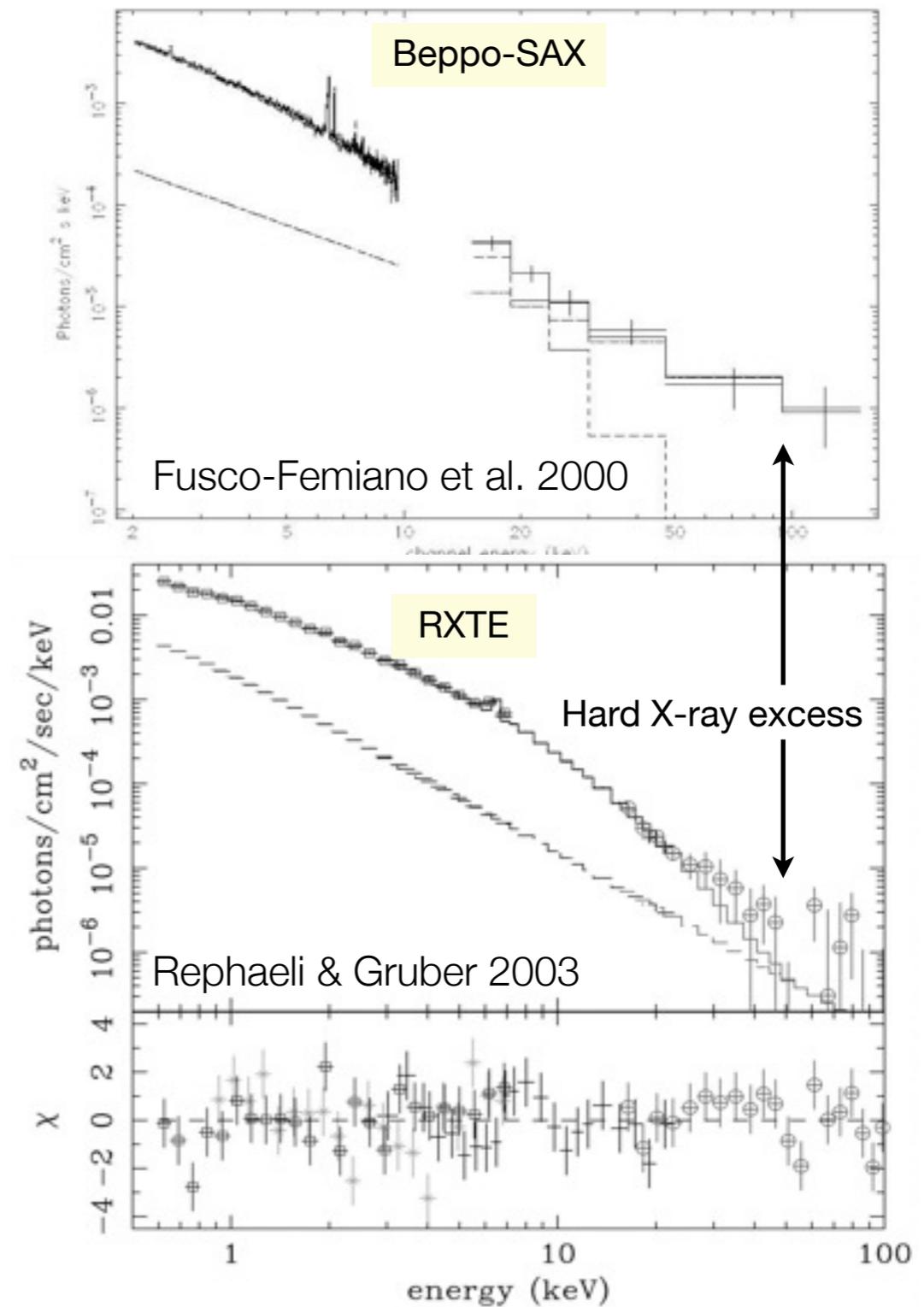
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Ultra-relativistic protons (TeV)					+ CMB photons (Bethe-Heitler)

# OBSERVATIONAL CONSTRAINTS - EXAMPLE 1



Abell 2256



# COSMIC RAYS, PHOTONS, MAGNETIC FIELDS: MULTI-WAVELENGTH EMISSION

“Product” CRs	Radio	Hard X-rays	Gamma-rays	Secondary relativistic electrons (GeV)	Secondary ultra-relativistic electrons (TeV)
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Ultra-relativistic protons (TeV)					+ CMB photons (Bethe-Heitler)

# ORIGIN OF RELATIVISTIC ELECTRONS (GEV)

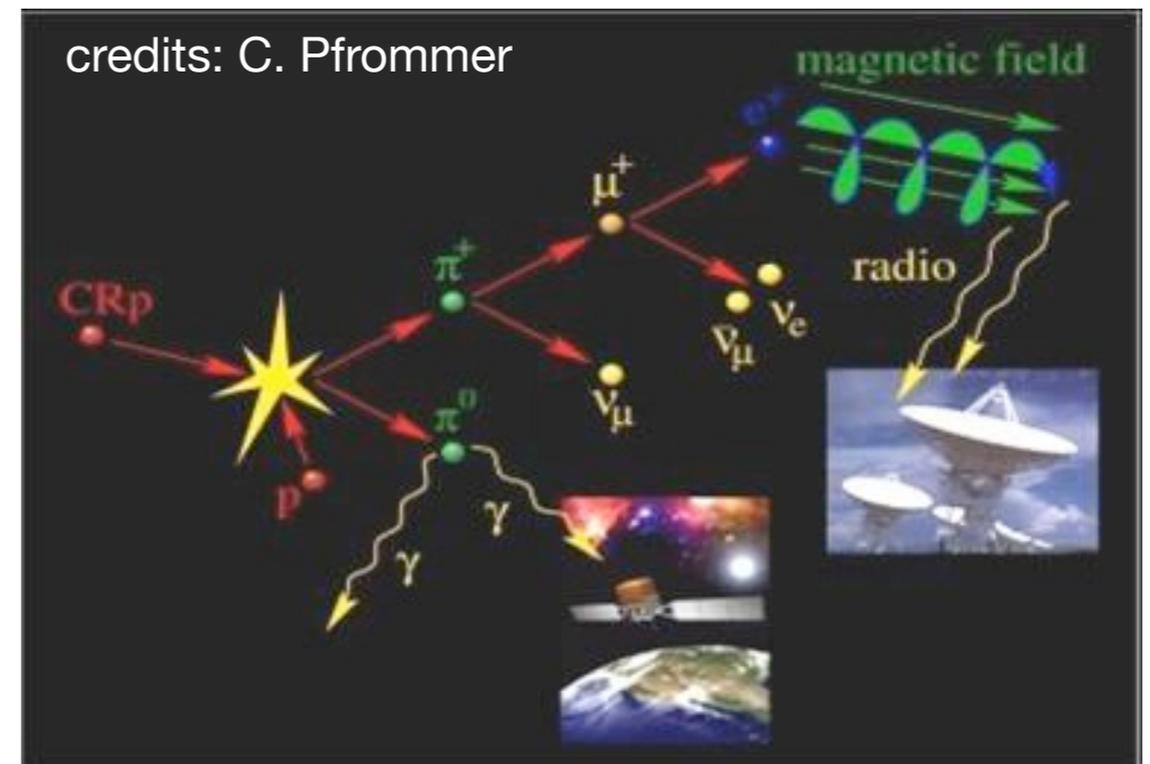
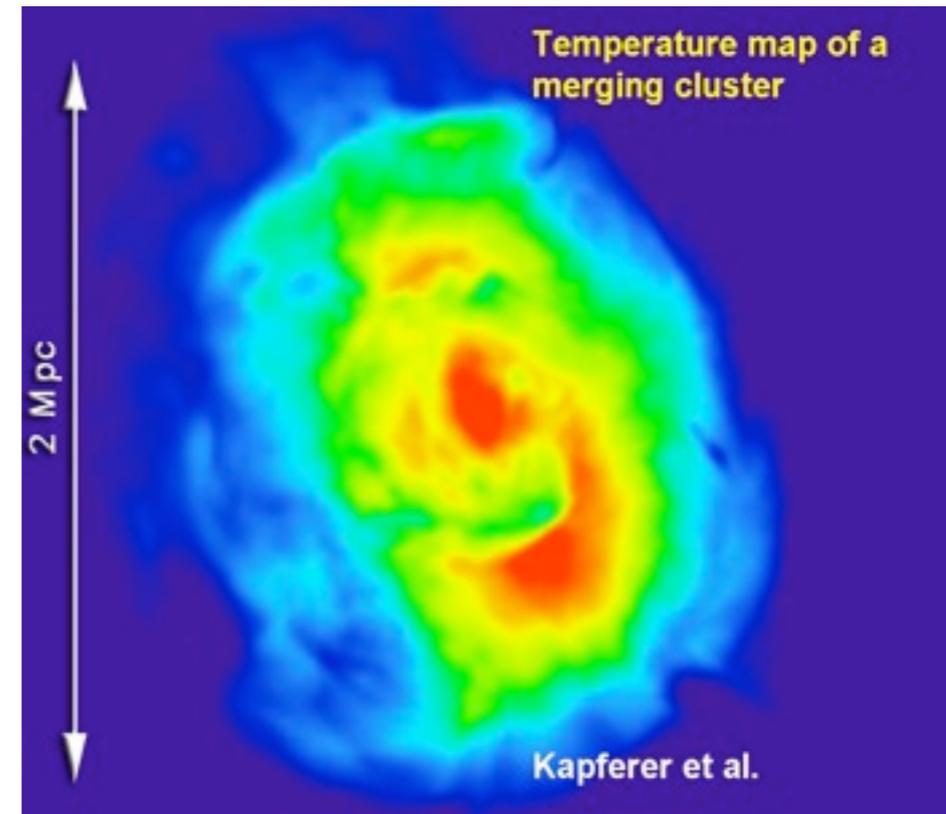
## Primary models

*(external processes related to cluster mergers)*

or

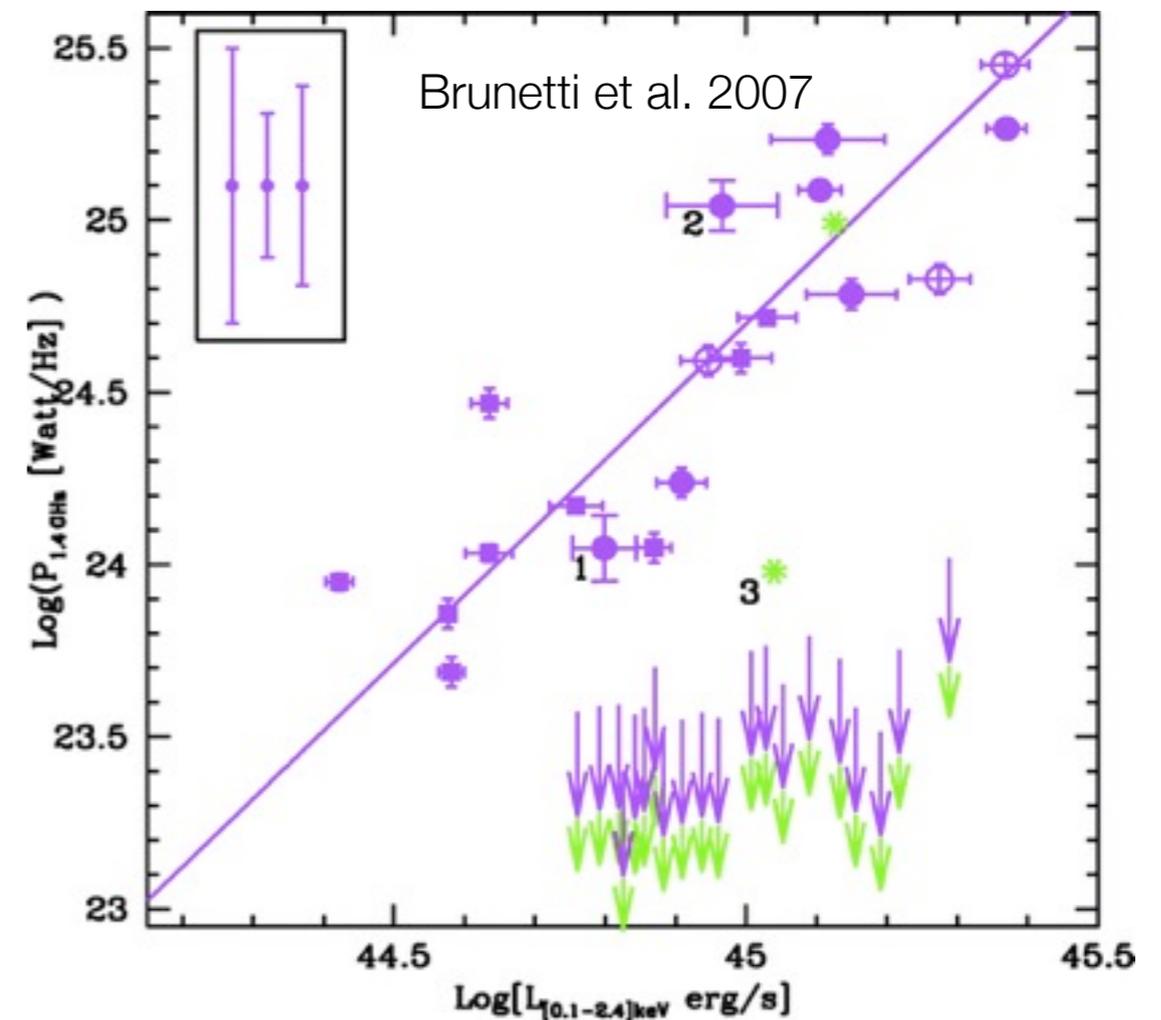
## Secondary models

*(external processes related to hadronic interactions)*



# OBSERVATIONAL EVIDENCE IN FAVOR OF PRIMARY MODELS

- Halos & Relics  $\leftrightarrow$  Cluster mergers
- $P_{1.4\text{GHz}} \leftrightarrow L_X (T_X)$
- Radio surface brightness  $\leftrightarrow$  X-ray surface brightness
- Radio spectral index  $\leftrightarrow \nu$
- Radio spectral index  $\leftrightarrow$  Distance from the X-ray centroid
- Radio spectral index  $\leftrightarrow T_X$

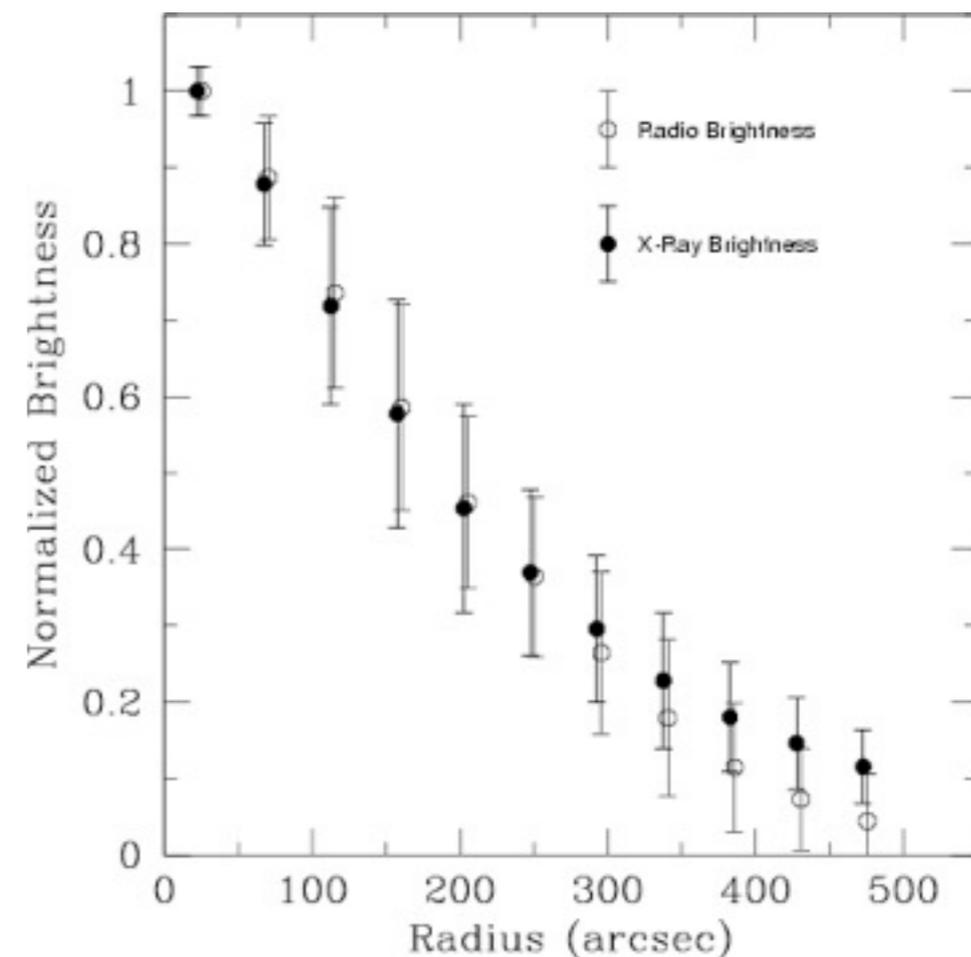


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\* Primary models:  $F_{\text{radio}} \propto F_X$

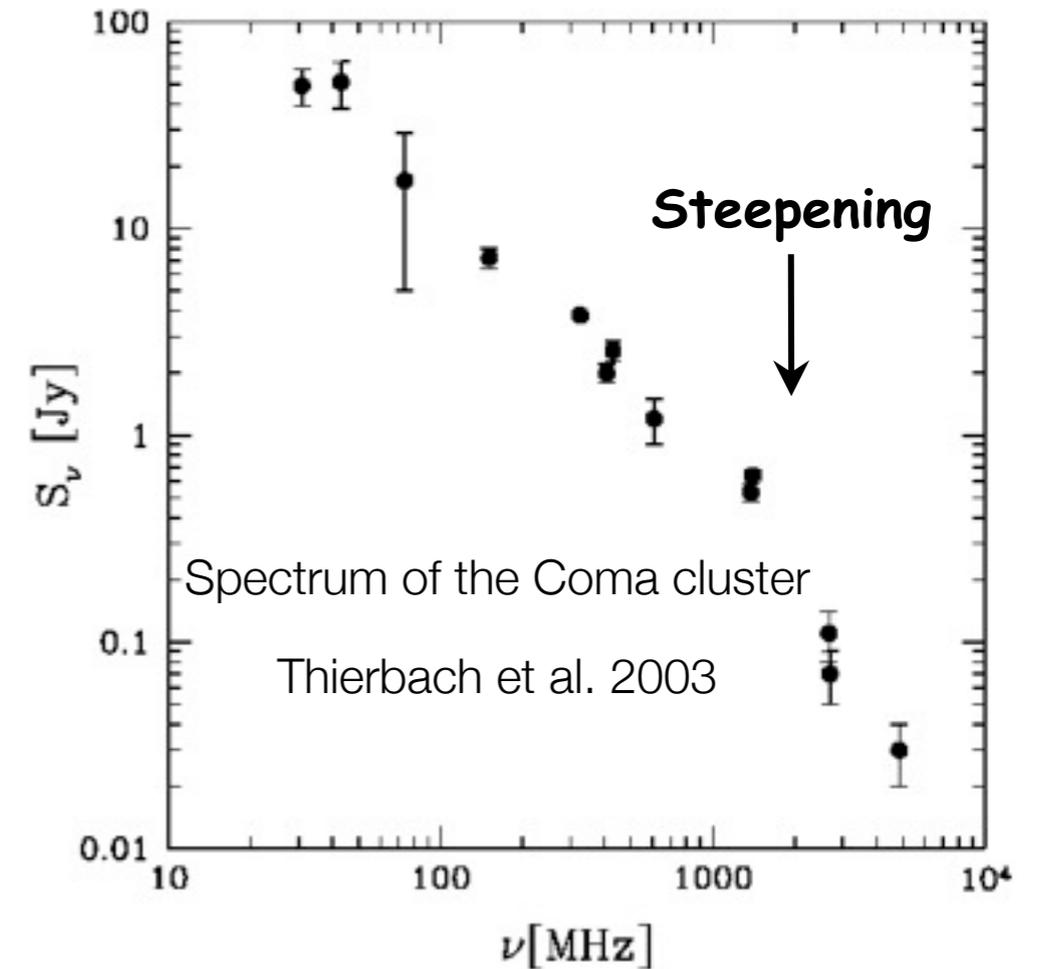
\* Secondary models:  $F_{\text{radio}} \propto F_X^a$   
( $a > 1$  for typical B values)



# OBSERVATIONAL EVIDENCE IN FAVOR OF PRIMARY MODELS

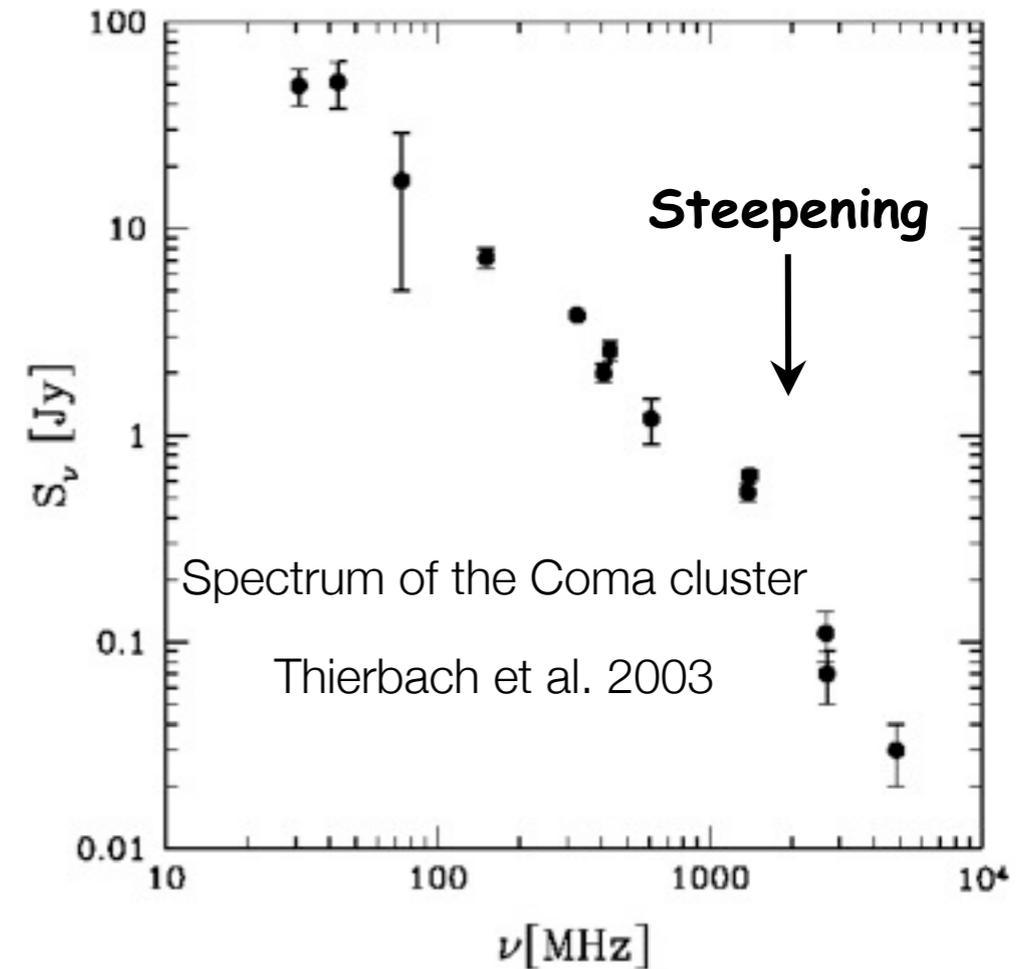
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- Radio spectral index  $\leftrightarrow \nu$      $S(\nu) \propto \nu^{-\alpha}$
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# OBSERVATIONAL EVIDENCE IN FAVOR OF PRIMARY MODELS

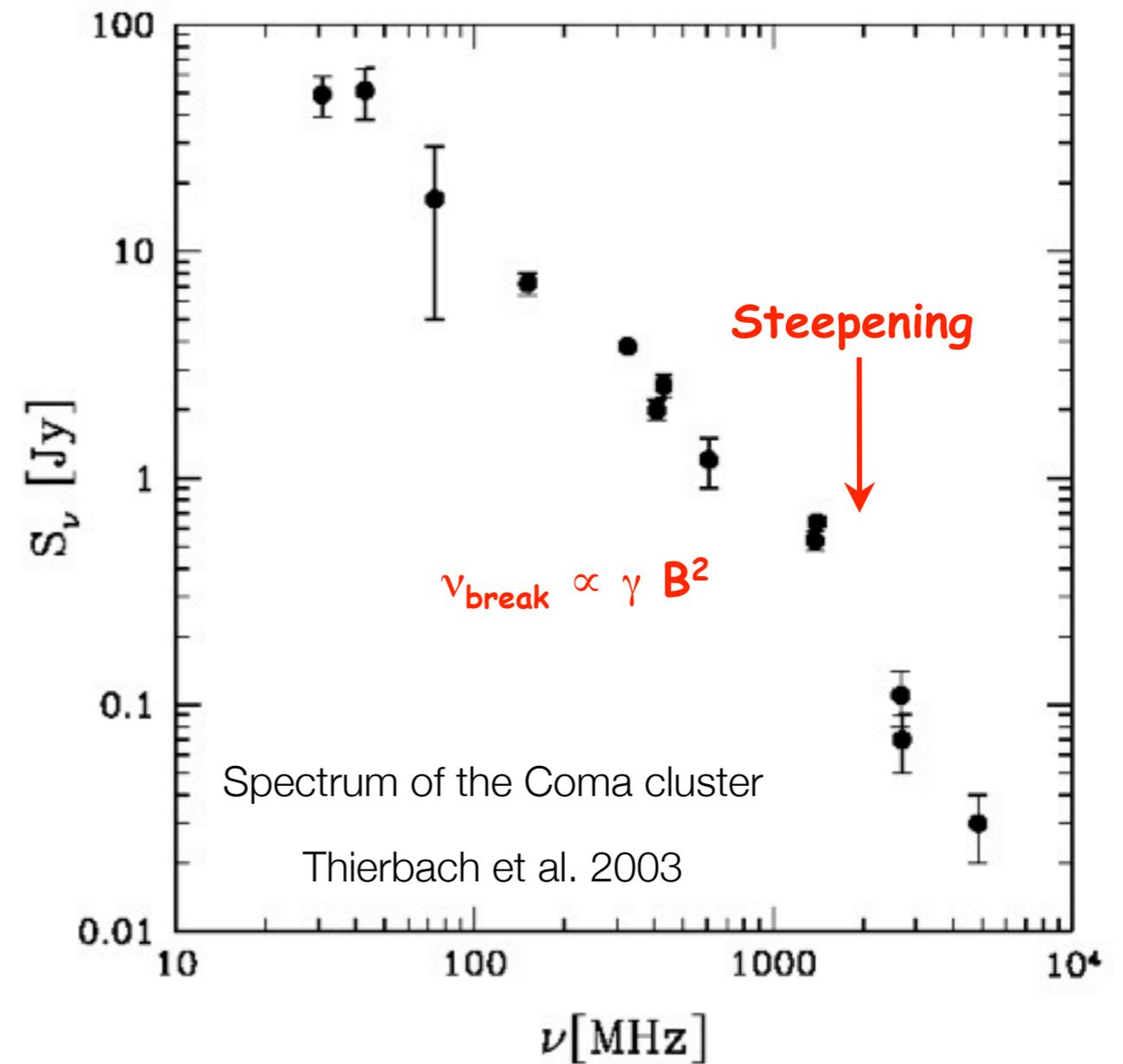
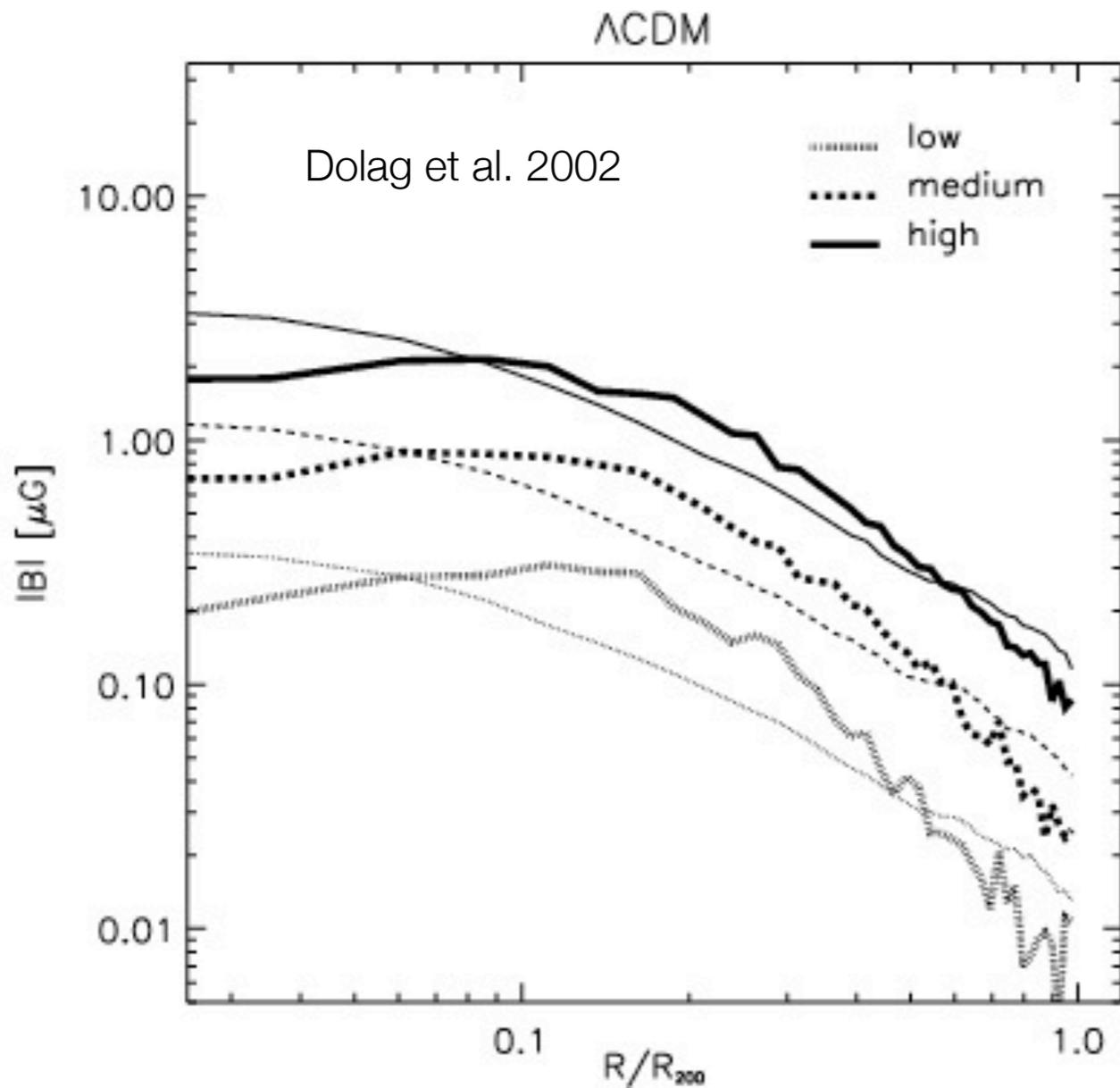
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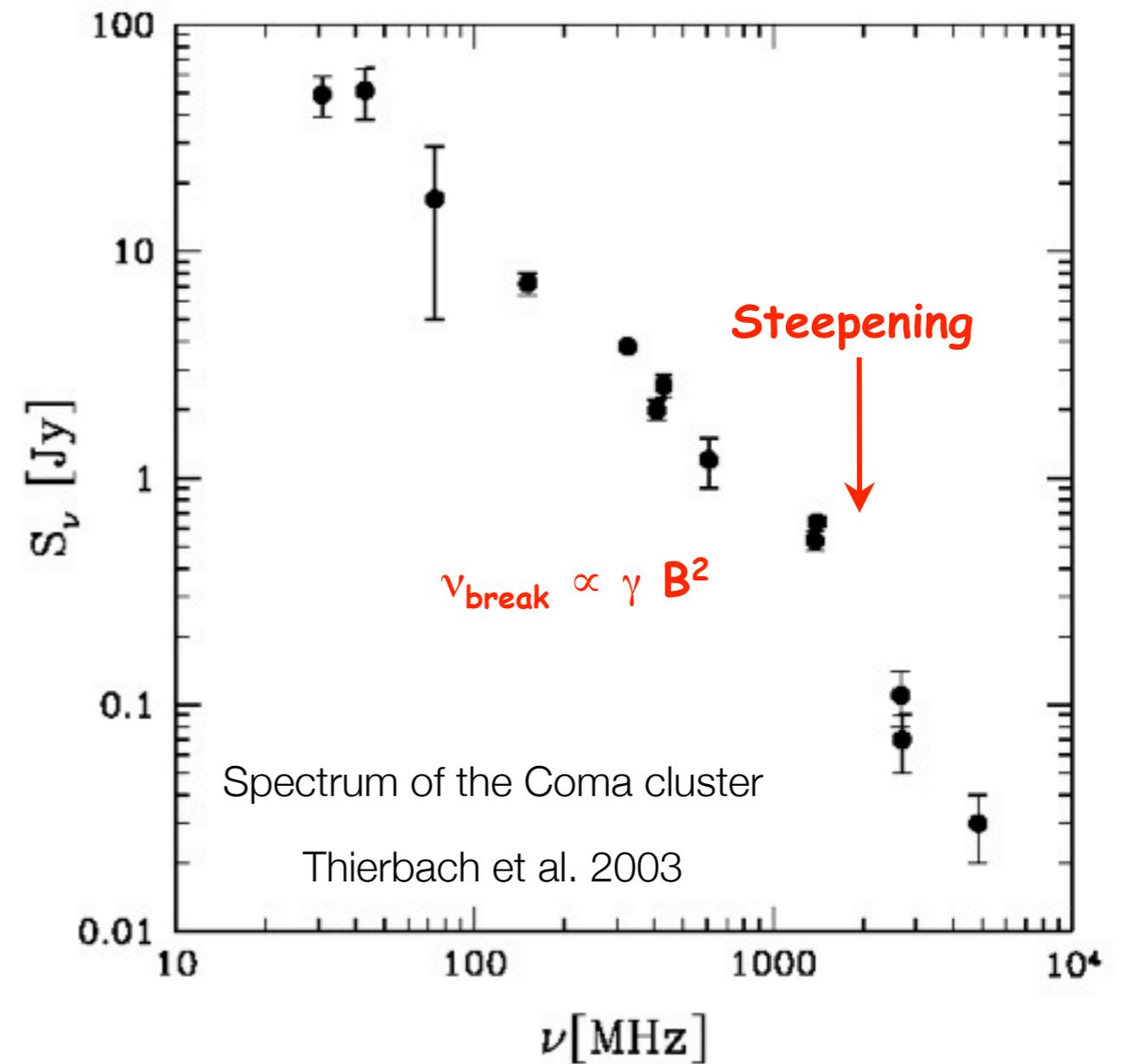
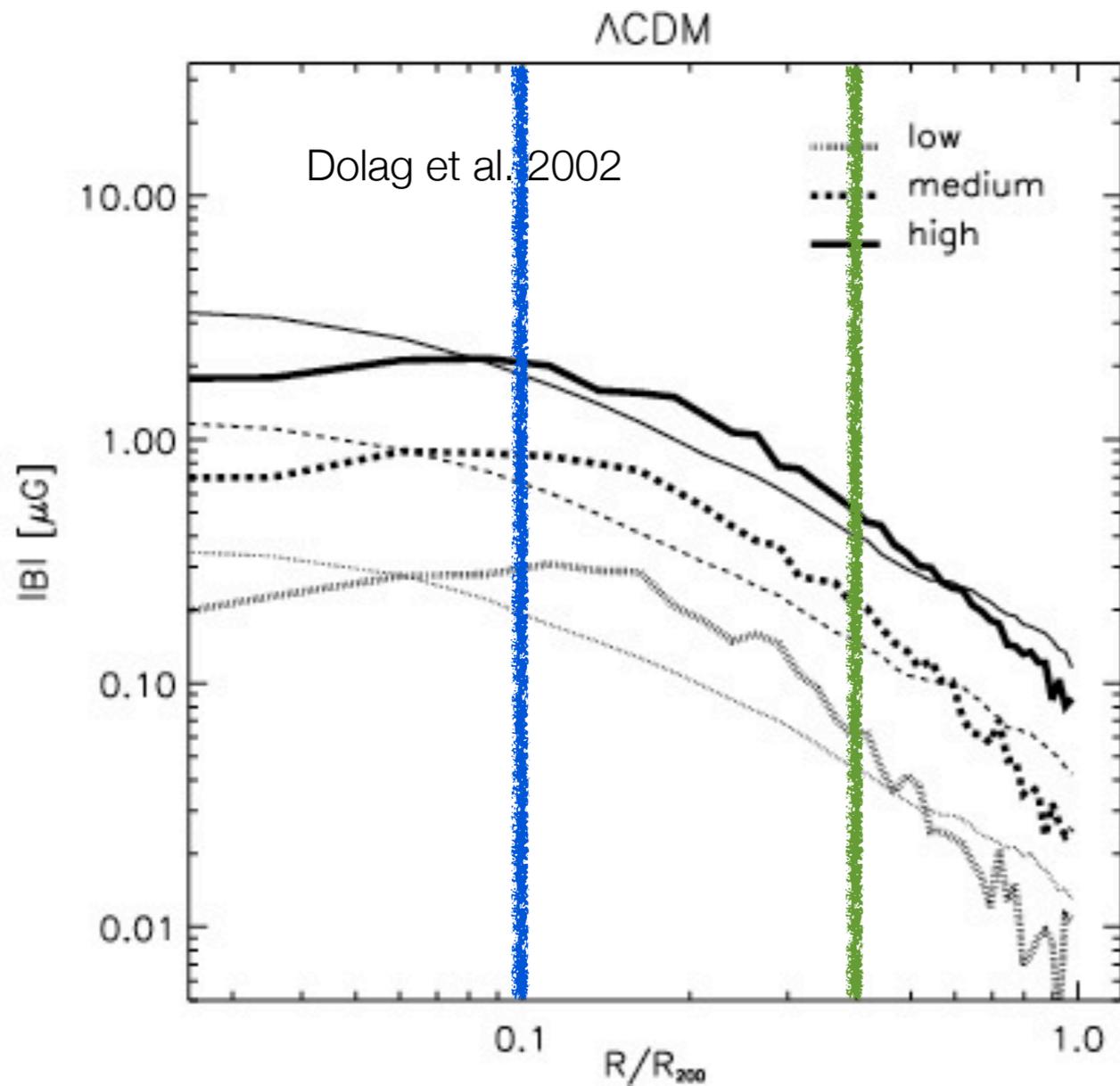
$$\nu_{\text{break}} \propto \gamma B^2$$

Brunetti et al. 2001

# OBSERVATIONAL EVIDENCE IN FAVOR OF PRIMARY MODELS

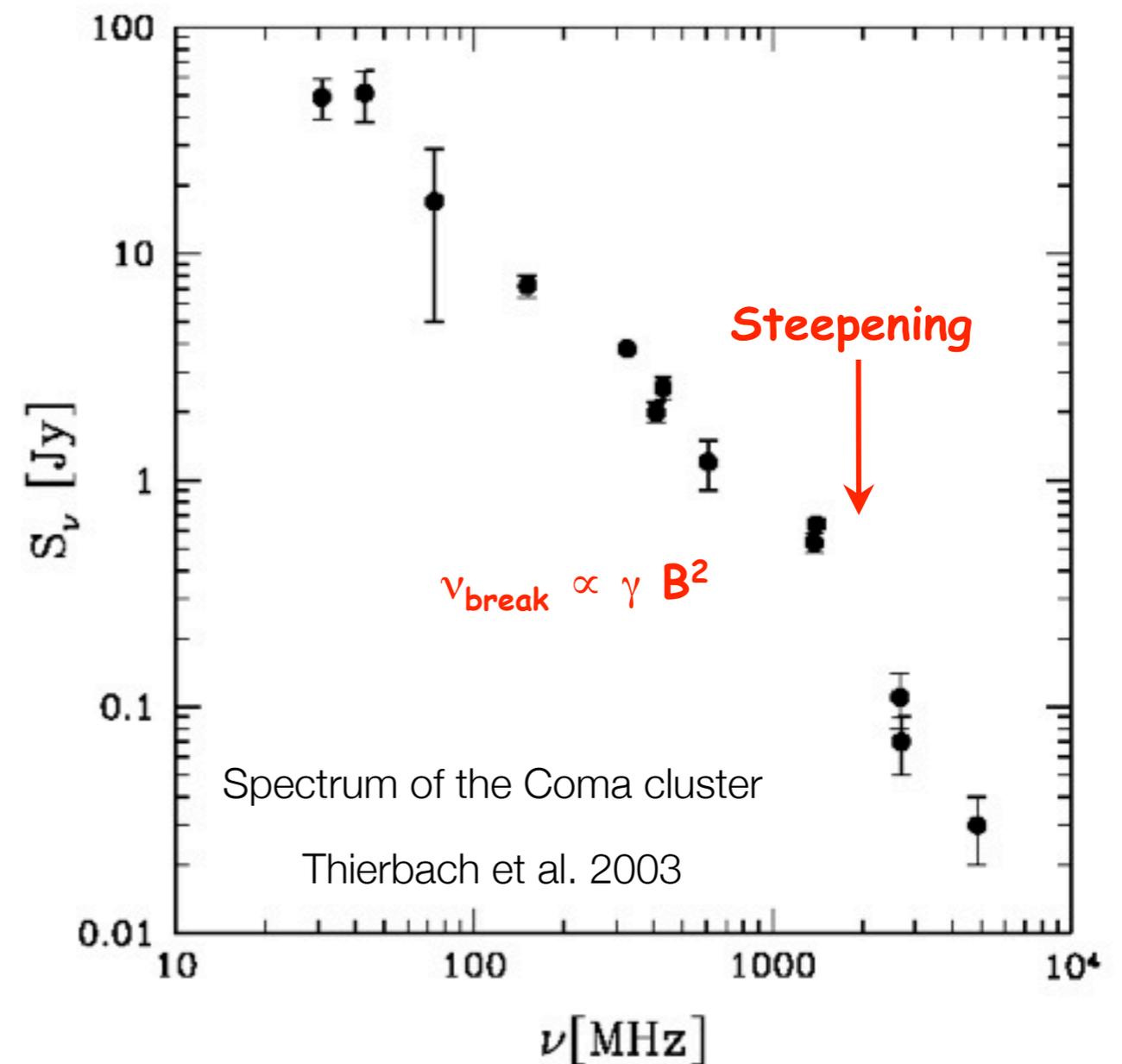
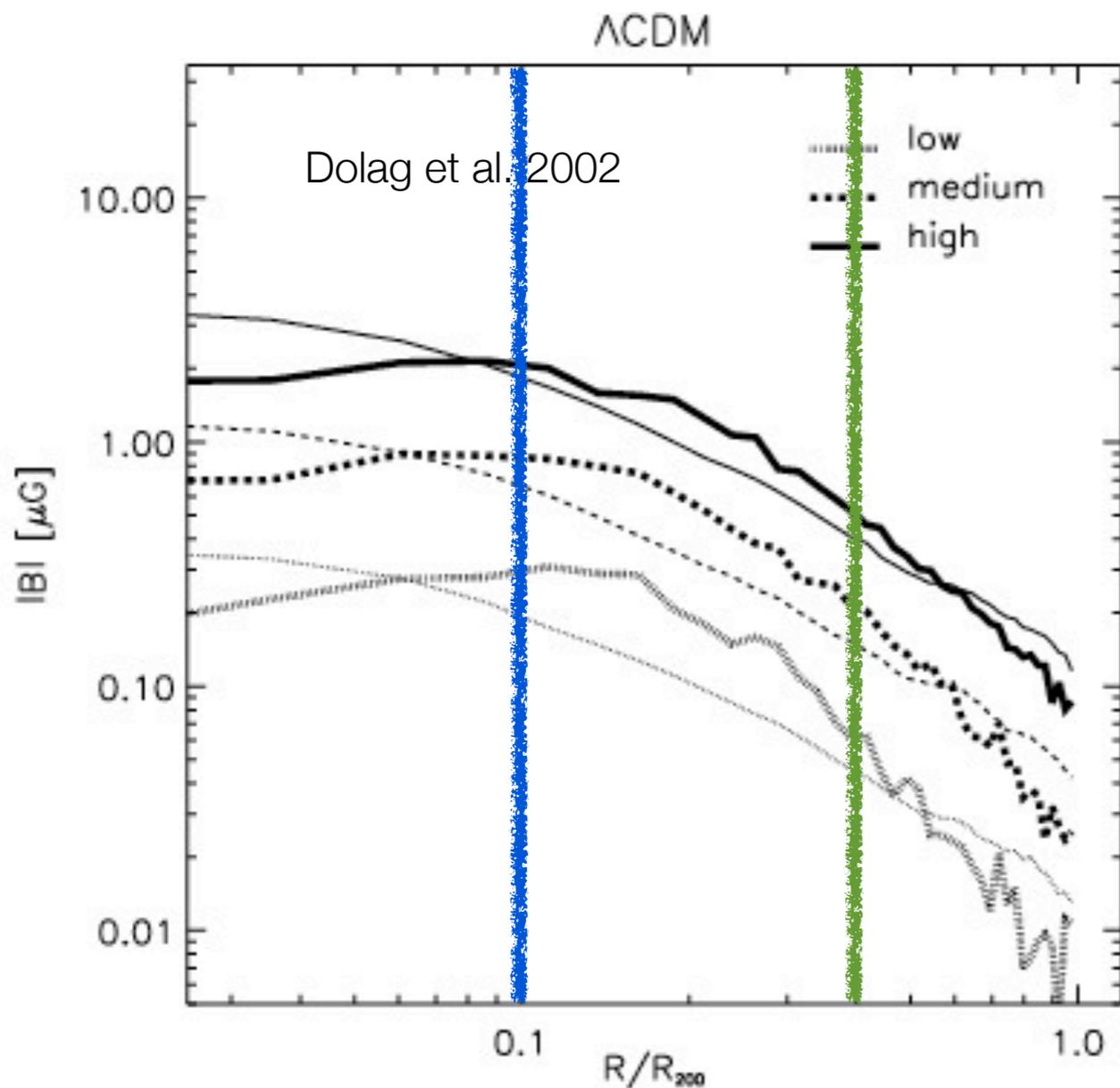


# OBSERVATIONAL EVIDENCE IN FAVOR OF PRIMARY MODELS



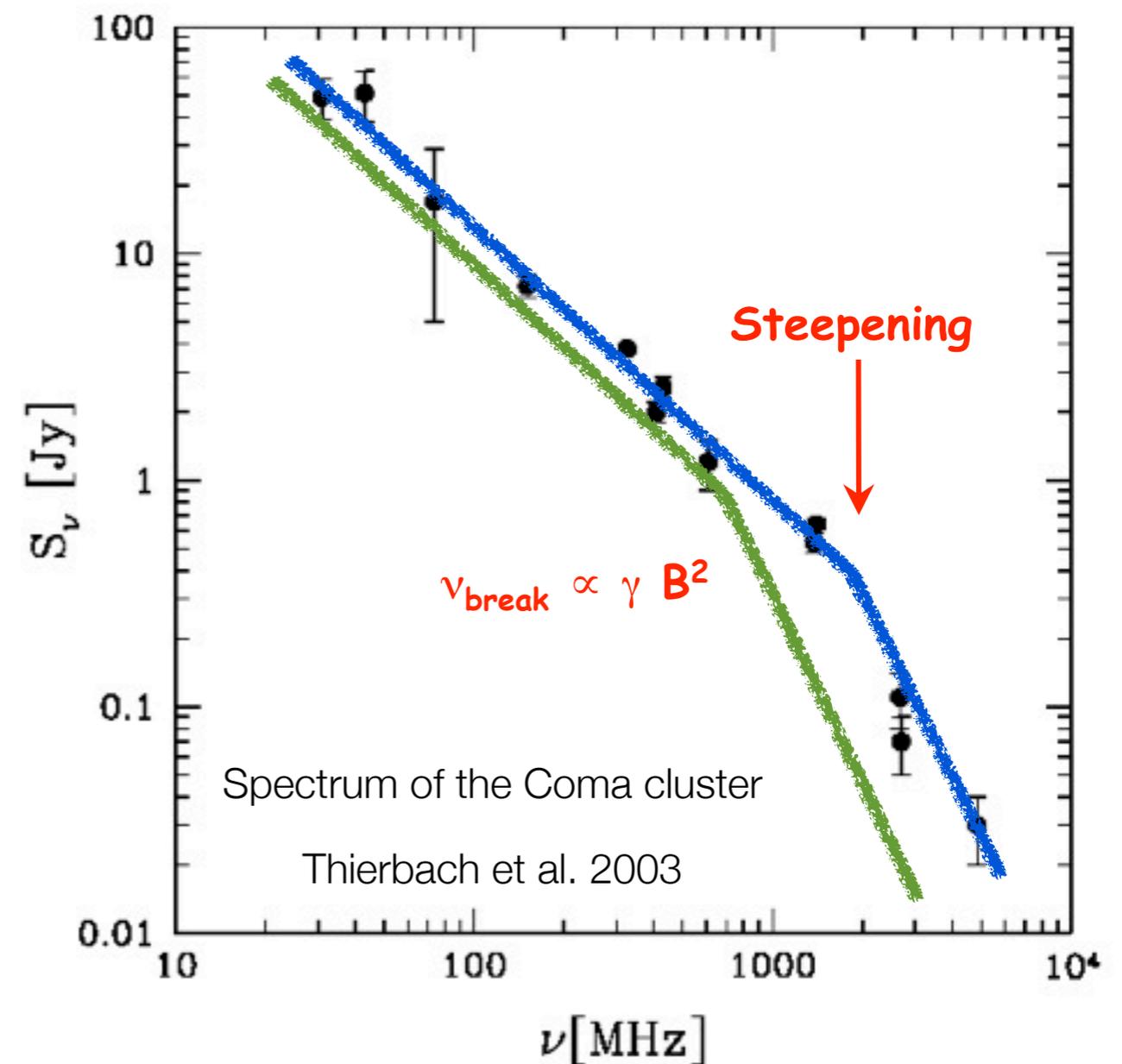
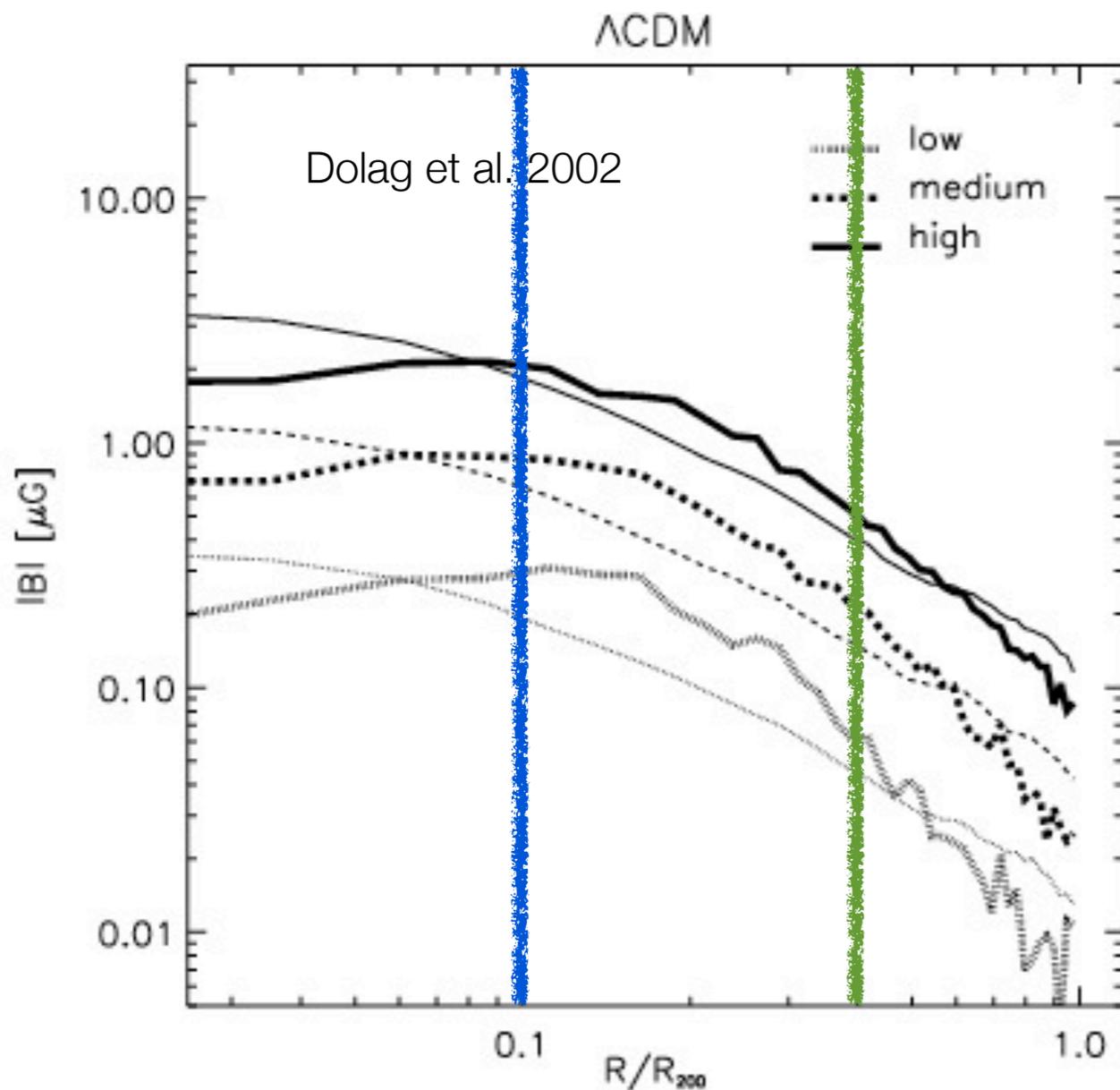
# OBSERVATIONAL EVIDENCE IN FAVOR OF PRIMARY MODELS

$$R_1 < R_2 \Rightarrow |B_1| > |B_2| \Rightarrow \nu_{\text{break1}} > \nu_{\text{break2}}$$



# OBSERVATIONAL EVIDENCE IN FAVOR OF PRIMARY MODELS

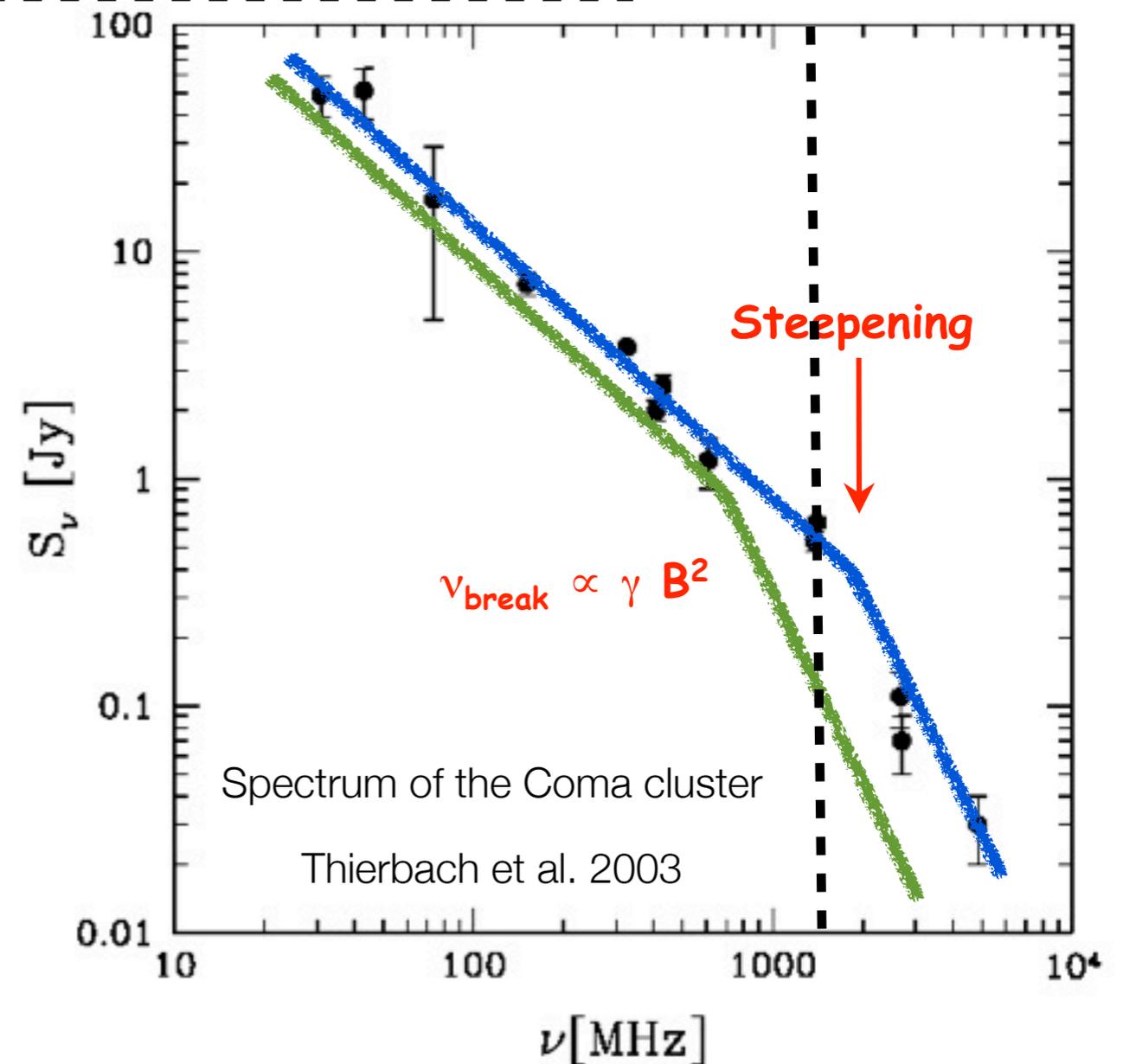
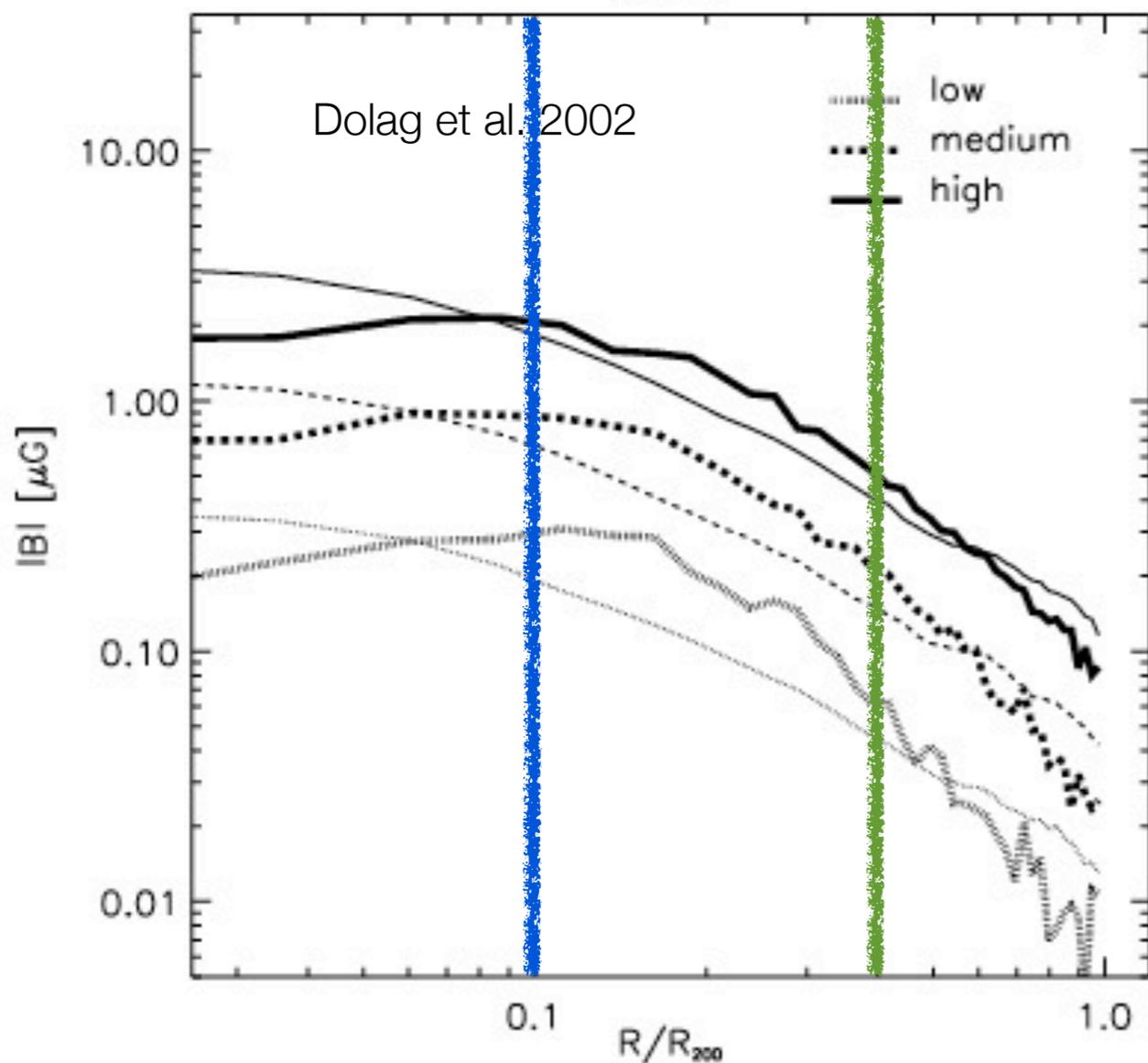
$$R_1 < R_2 \Rightarrow |B_1| > |B_2| \Rightarrow \nu_{\text{break}1} > \nu_{\text{break}2}$$



# OBSERVATIONAL EVIDENCE IN FAVOR OF PRIMARY MODELS

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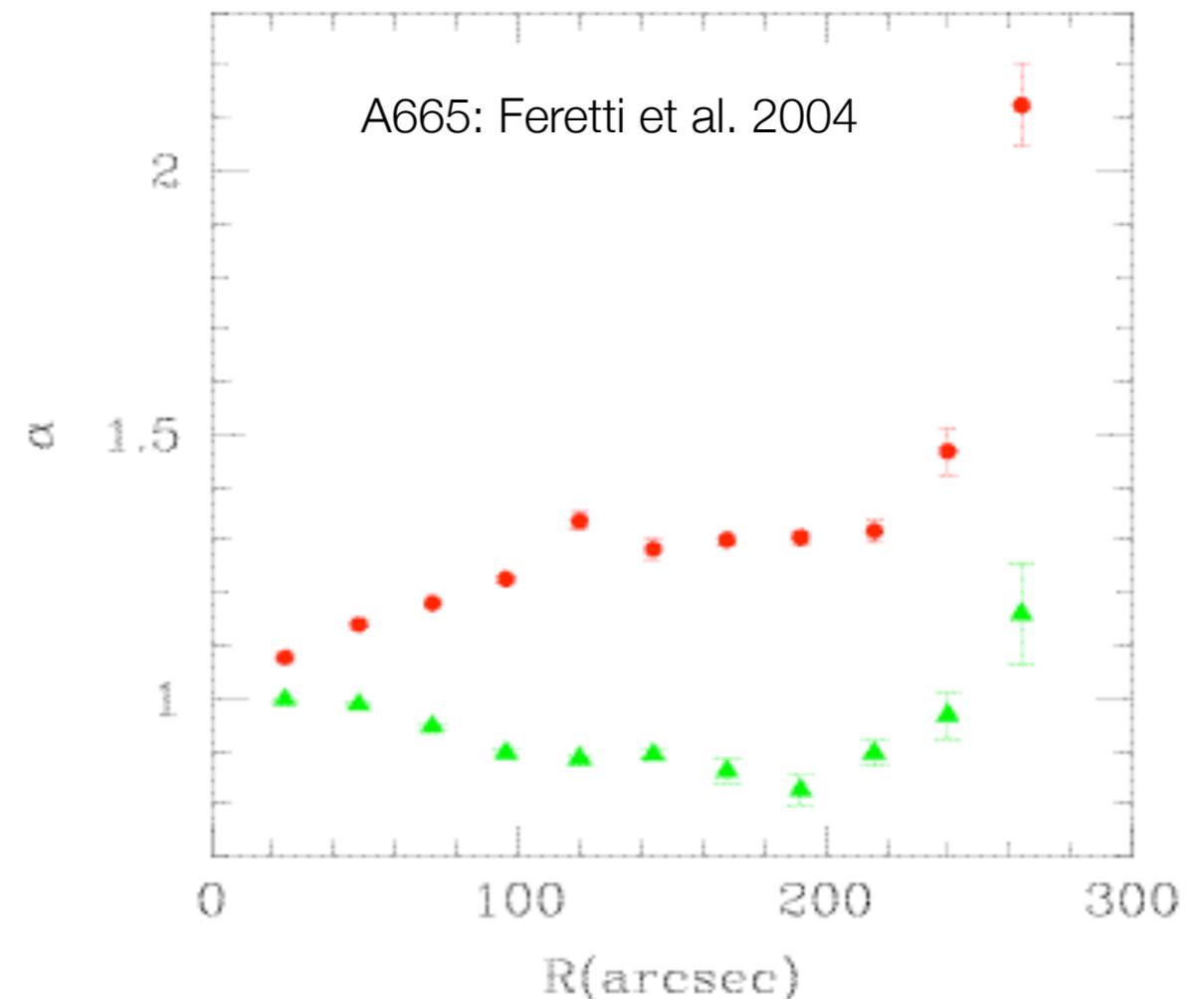
⇒ The spectrum is steeper at  $R_2$  than at  $R_1$



# OBSERVATIONAL EVIDENCE IN FAVOR OF PRIMARY MODELS

---

- Halos & Relics  $\leftrightarrow$  Cluster mergers
- $P_{1.4\text{GHz}} \leftrightarrow L_X (T_X)$
- Radio surface brightness  $\leftrightarrow$  X-ray surface brightness
- Radio spectral index  $\leftrightarrow \nu$
- Radio spectral index  $\leftrightarrow$  Distance from the X-ray centroid
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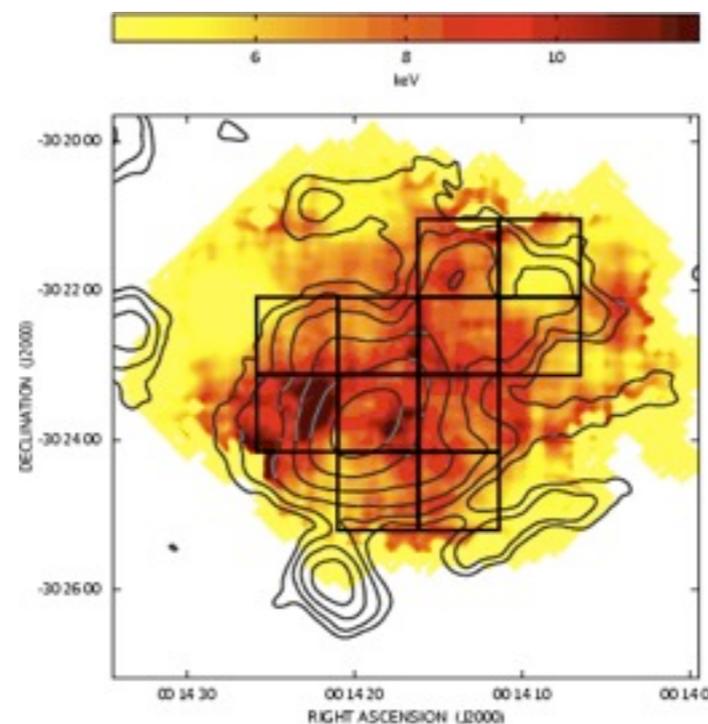
- Radio surface brightness  $\leftrightarrow$  X-ray

surface brightness

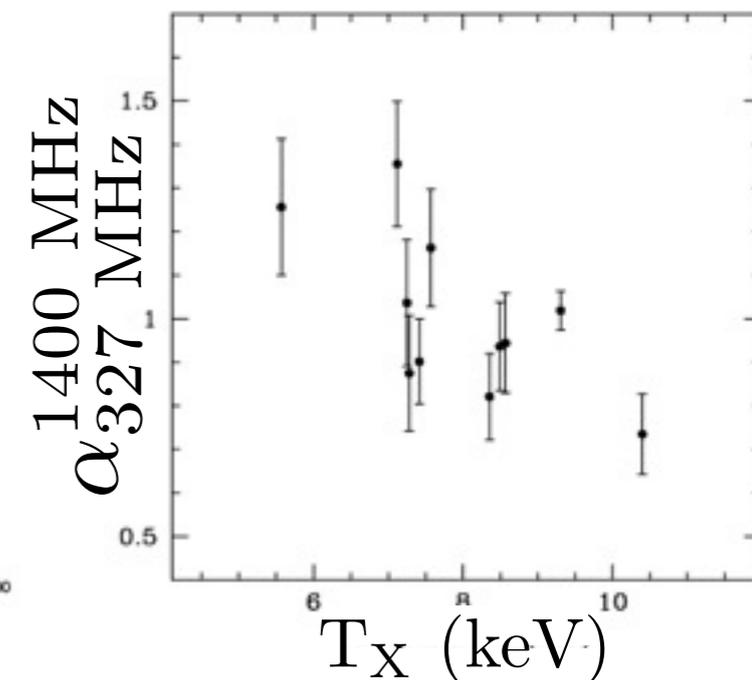
- Radio spectral index  $\leftrightarrow \nu$

- Radio spectral index  $\leftrightarrow$  Distance from the X-ray centroid

- Radio spectral index  $\leftrightarrow T_X$



A2744: Orrù et al. 2007



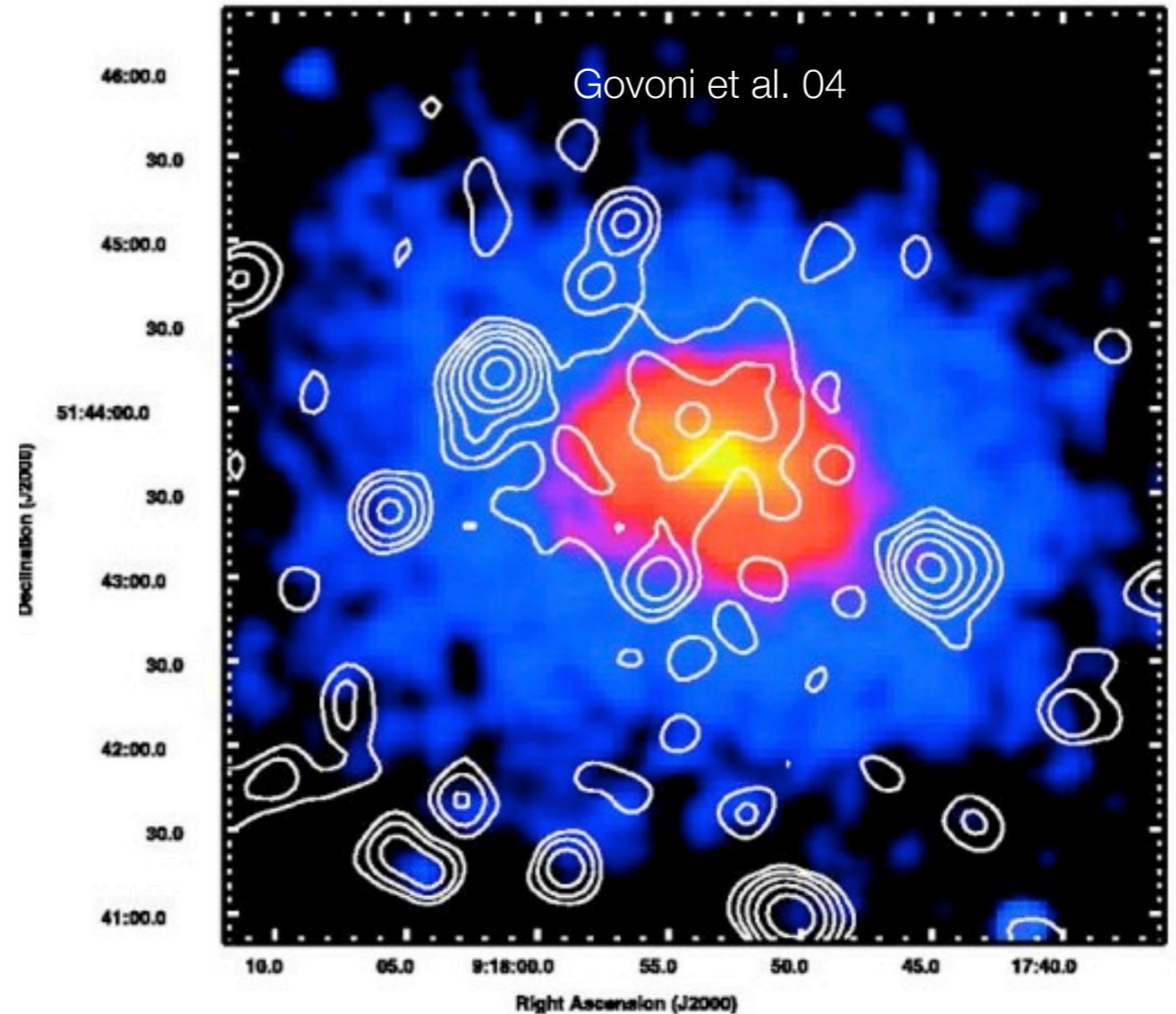
# HALOS, RELICS AND MINI-HALOS

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# HALOS, RELICS AND MINI-HALOS

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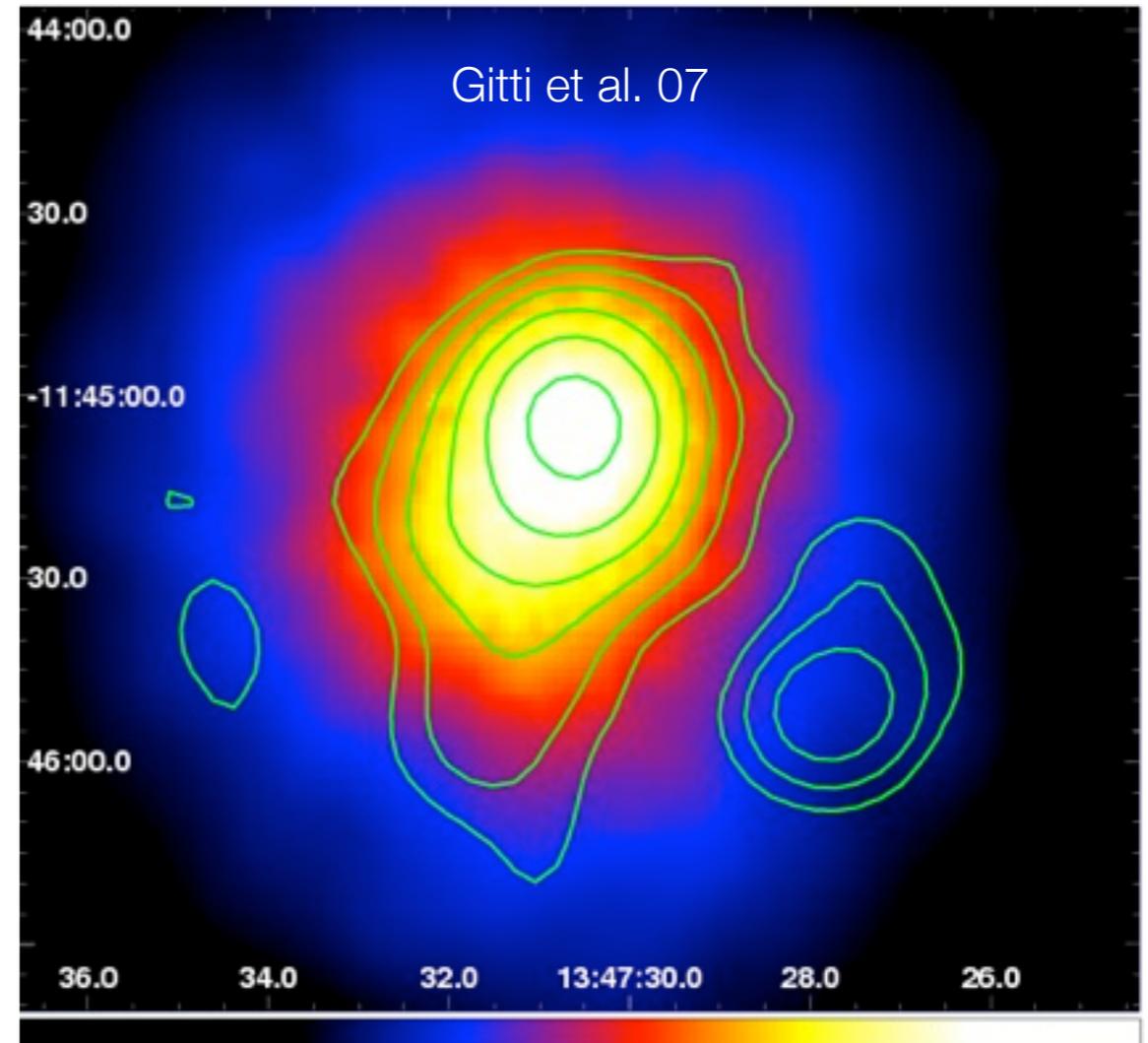
- Halos  $\Leftrightarrow$  ICM turbulence due to cluster merging



# HALOS, RELICS AND MINI-HALOS

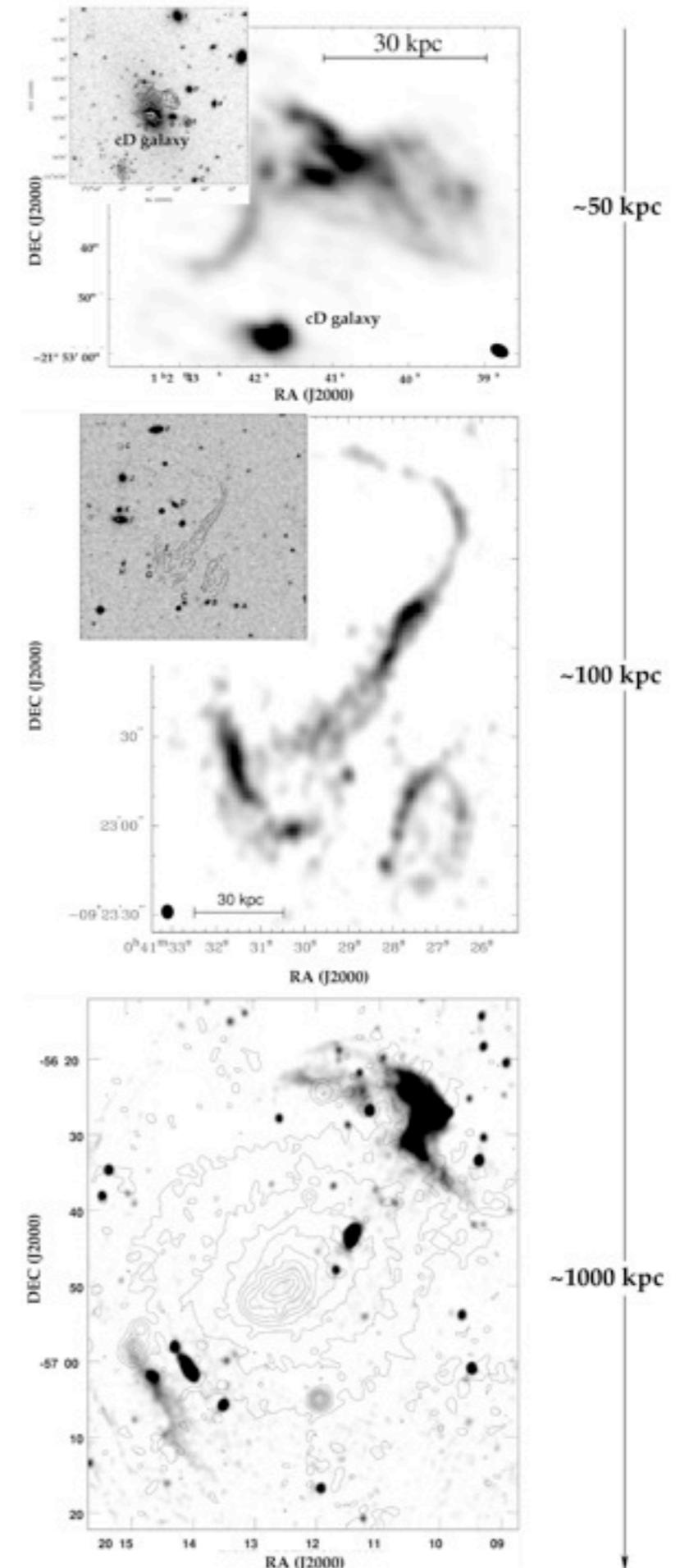
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- Halos  $\Leftrightarrow$  ICM turbulence due to cluster merging
- Mini-halos  $\Leftrightarrow$  Cooling core turbulence & AGN emission



# HALOS, RELICS AND MINI-HALOS

- Halos  $\Leftrightarrow$  ICM turbulence due to cluster merging
- Mini-halos  $\Leftrightarrow$  Cooling core turbulence & AGN emission
- Relics:
  - \* ICM shocks due to cluster merging
  - \* Old relativistic plasma



# HYBRID MODELS: PRIMARY + SECONDARY

---

Time elapsed since last MHD  
turbulence injection

0.75 Gyr

Brunetti et al. 2008

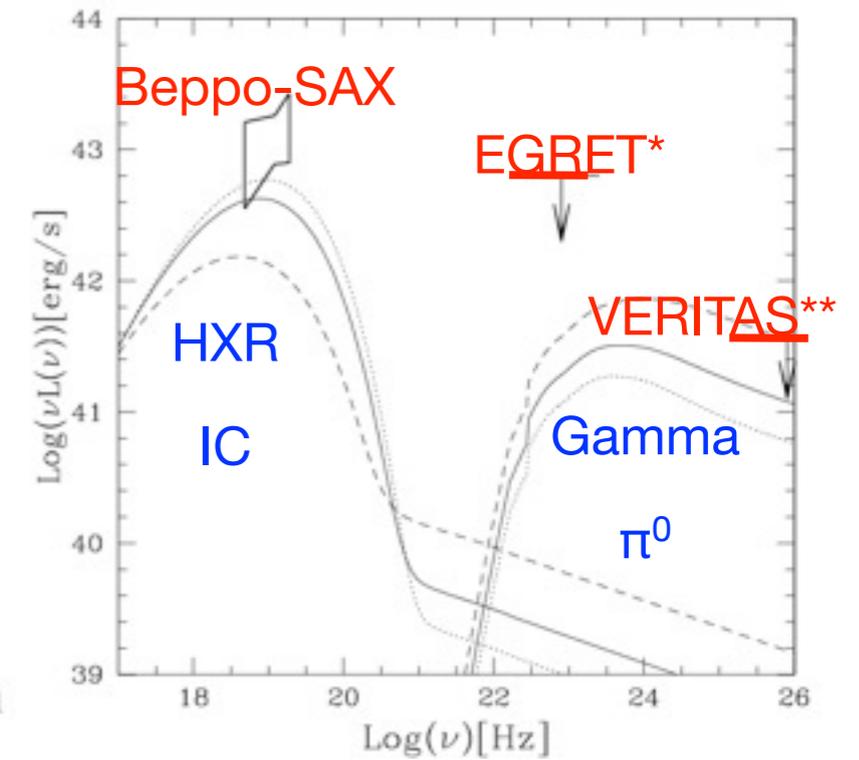
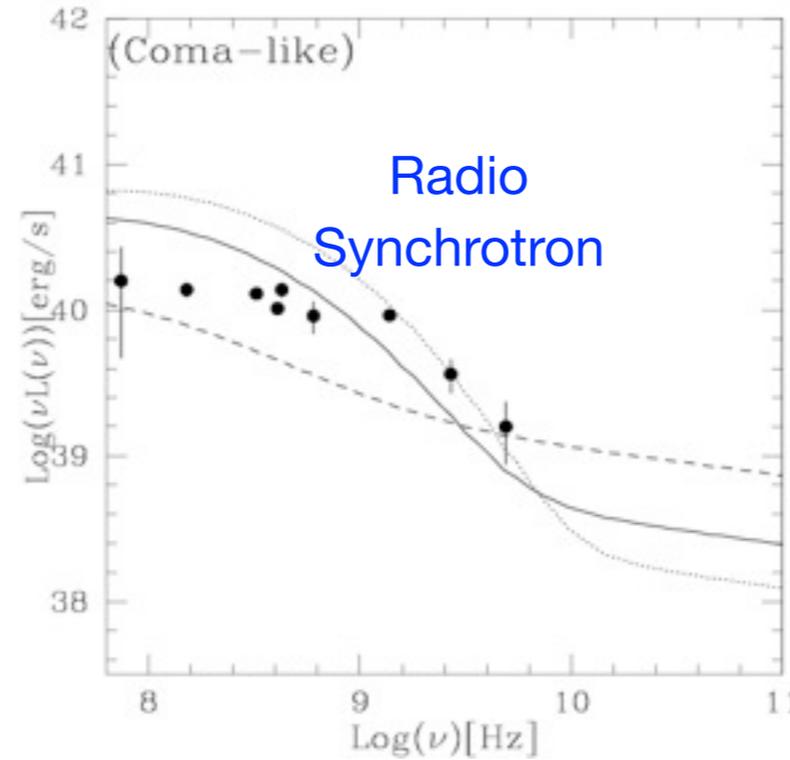
1.75 Gyr

# HYBRID MODELS: PRIMARY + SECONDARY

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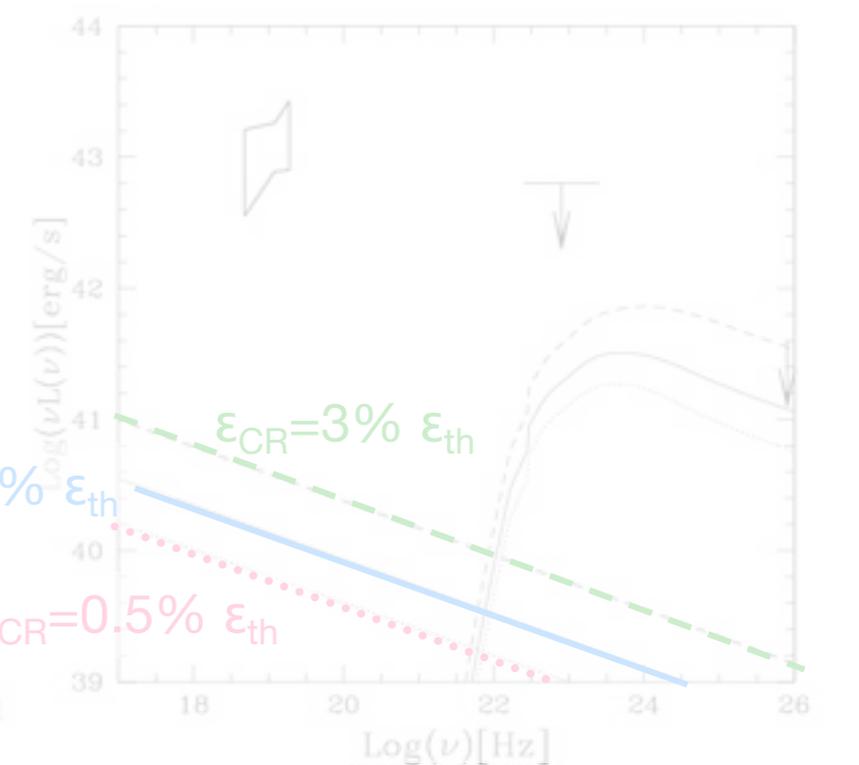
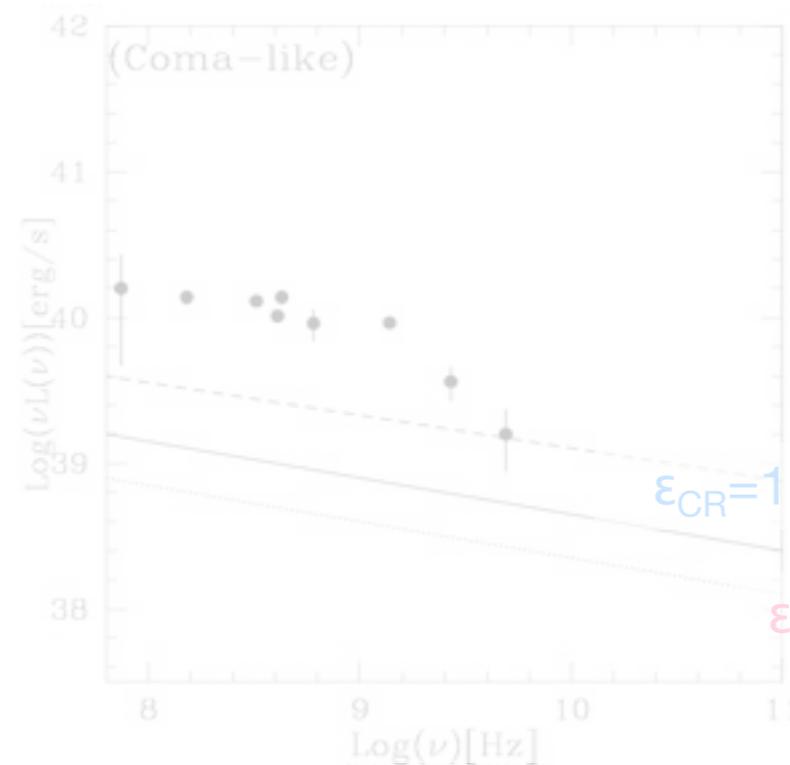
Brunetti et al. 2008



1.75 Gyr

\*Reimer et al. 2003

\*\*Perkins et al. 2008

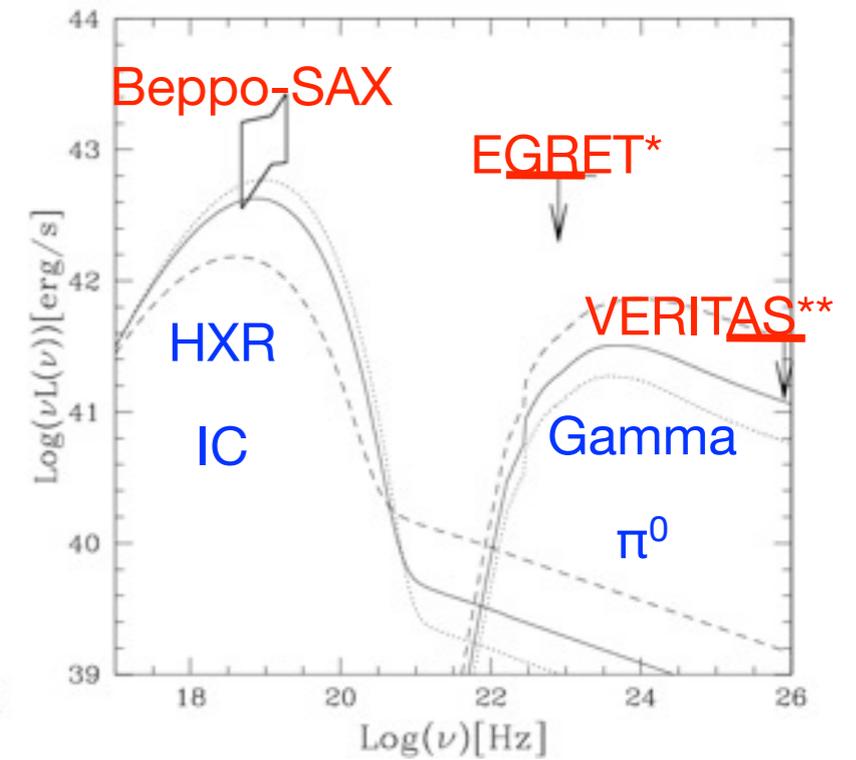
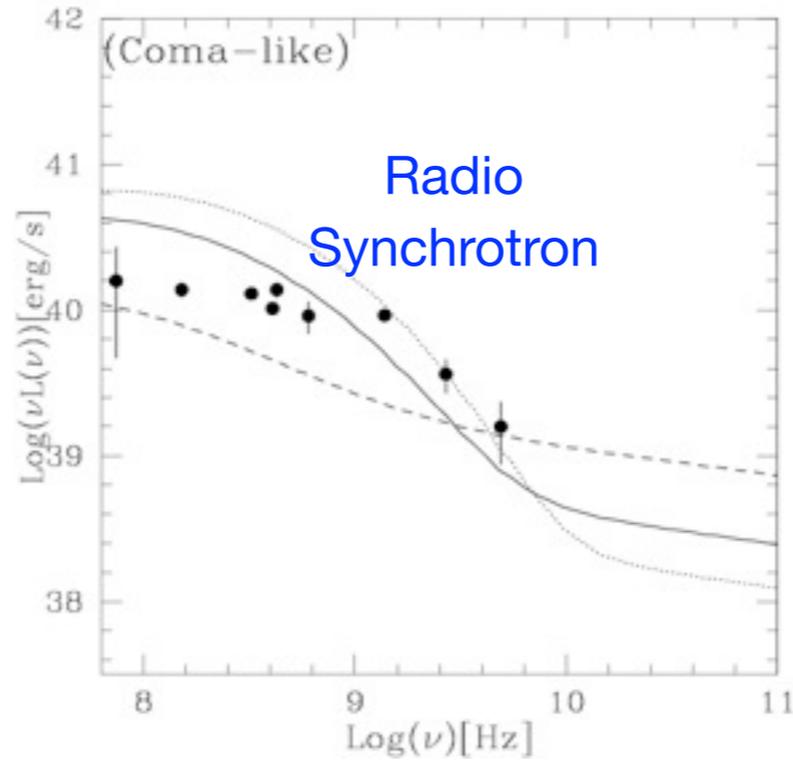


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Time elapsed since last MHD turbulence injection

0.75 Gyr

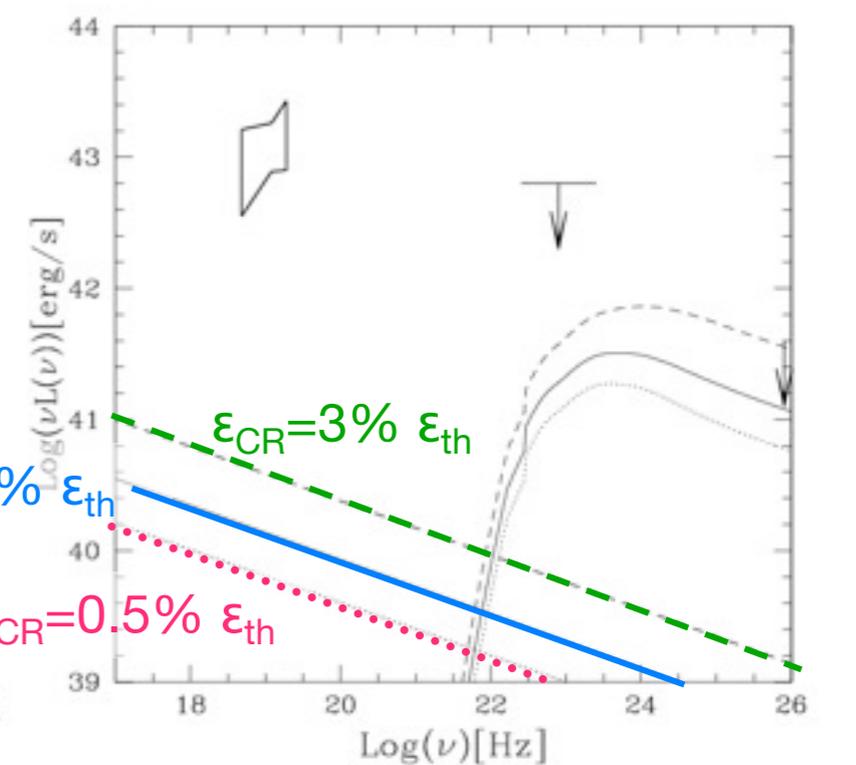
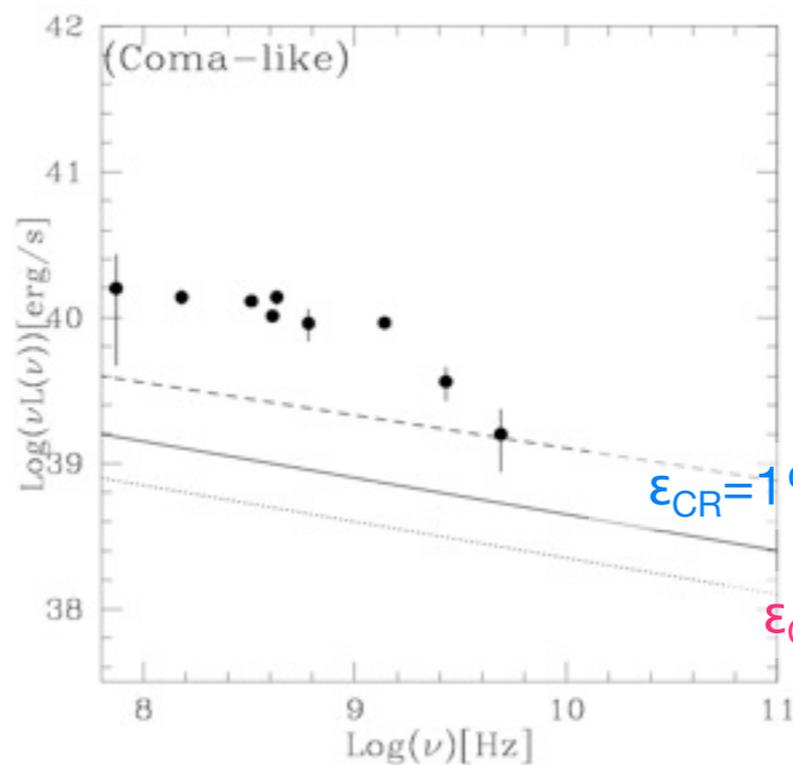
Brunetti et al. 2008



1.75 Gyr

\*Reimer et al. 2003

\*\*Perkins et al. 2008



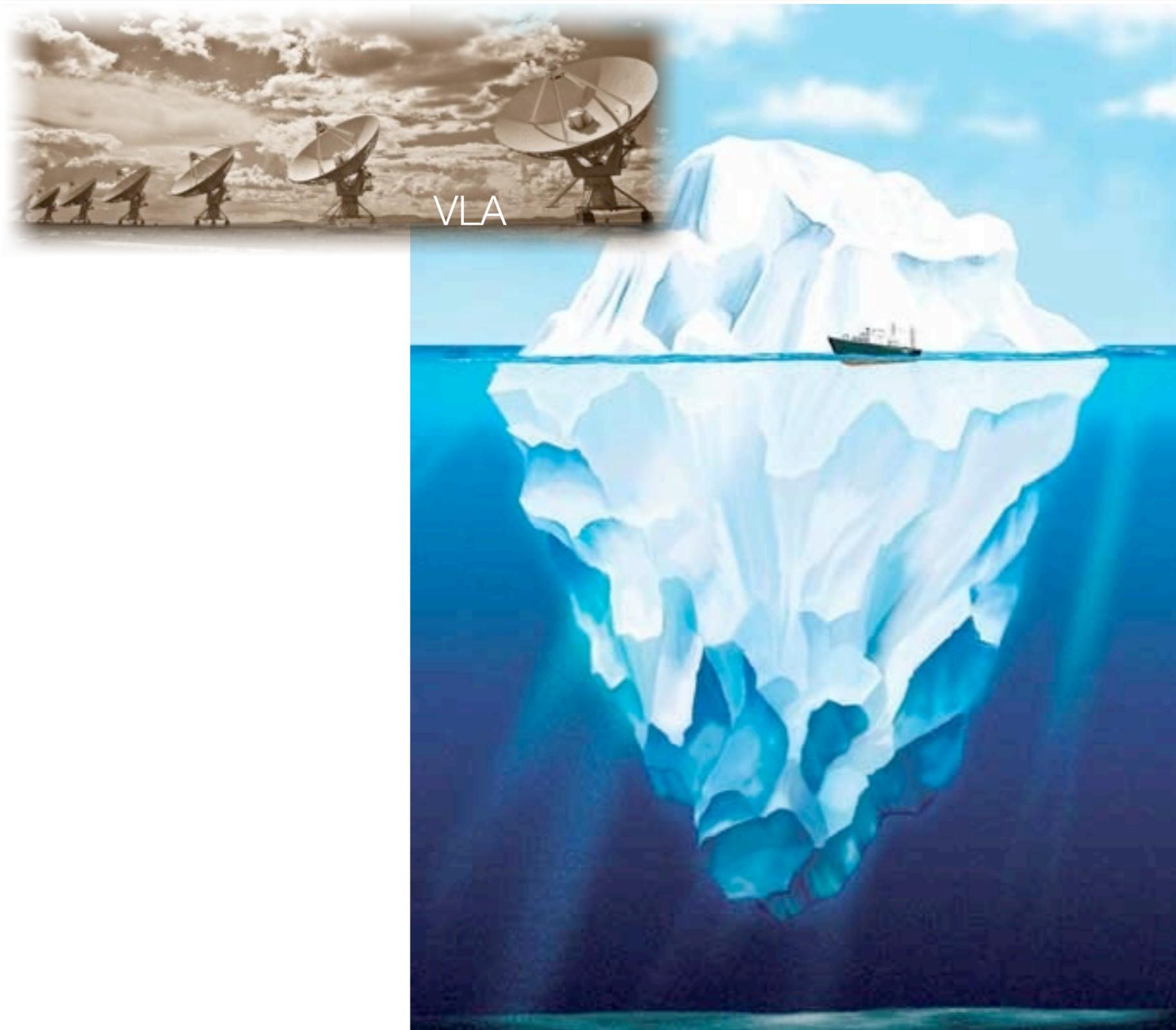
# THE GOLDEN-AGE OF NON-THERMAL CLUSTER STUDIES

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# THE GOLDEN-AGE OF NON-THERMAL CLUSTER STUDIES



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