Emission Mechanisms

Lesson 3

Chiara Ferrari

Reference Texts:

"Astrophysical Processes" by H. Bradt
Cambridge University Press (2008)

"Radiative Processes in Astrophysics" by G. B. Rybicki & A. P. Lightman
Wiley-VCH (1979, 2004)

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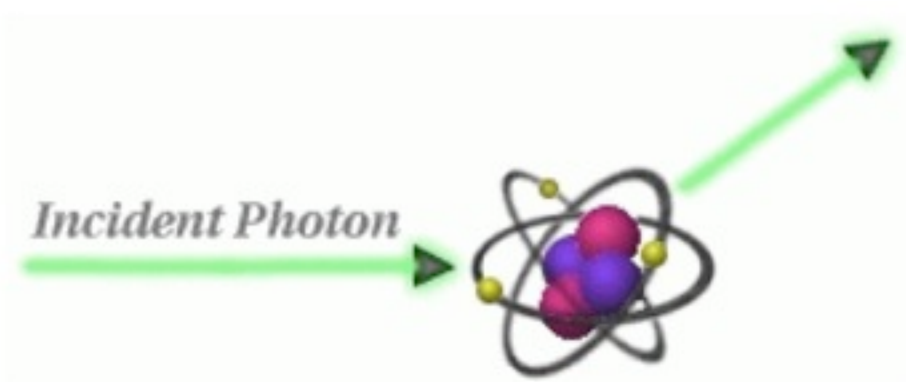
Interactions between electrons and photons

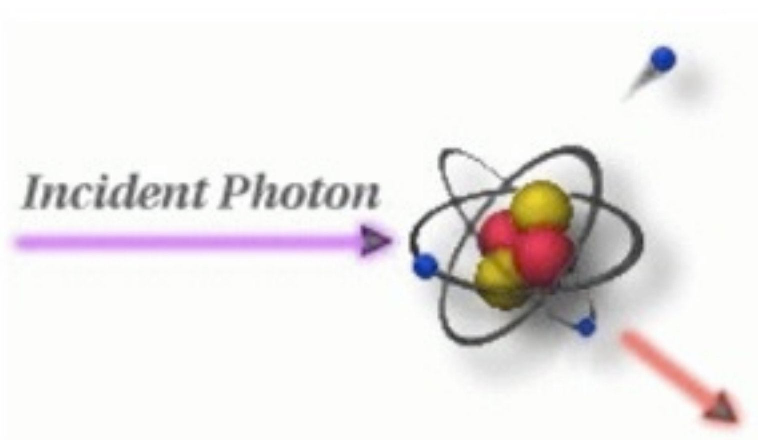
Thomson & Compton scattering

Inverse Compton effect

The interaction between particles and light is able to scatter photons from lower to higher energies (or vice versa) in interactions with electrons of higher (or lower) energies. The kind of interaction between photons and electrons depends from their relative energy: if the electrons is (nearly) not moving, we will speak about ***Thomson*** or ***Compton scattering*** as a function of the photon energy. If the electron is moving at high speed, we will be in the ***Inverse Compton*** case.



Thomson scattering



Compton scattering

Matter-photon interactions

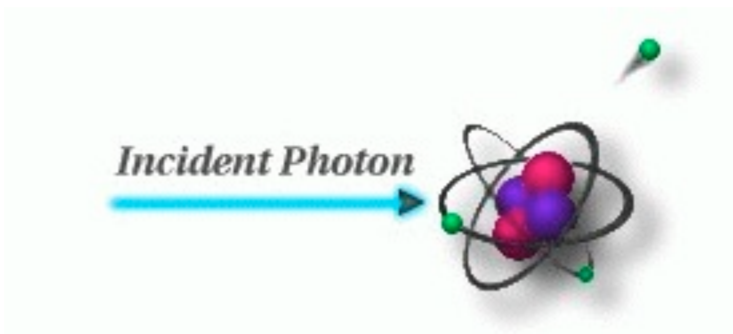
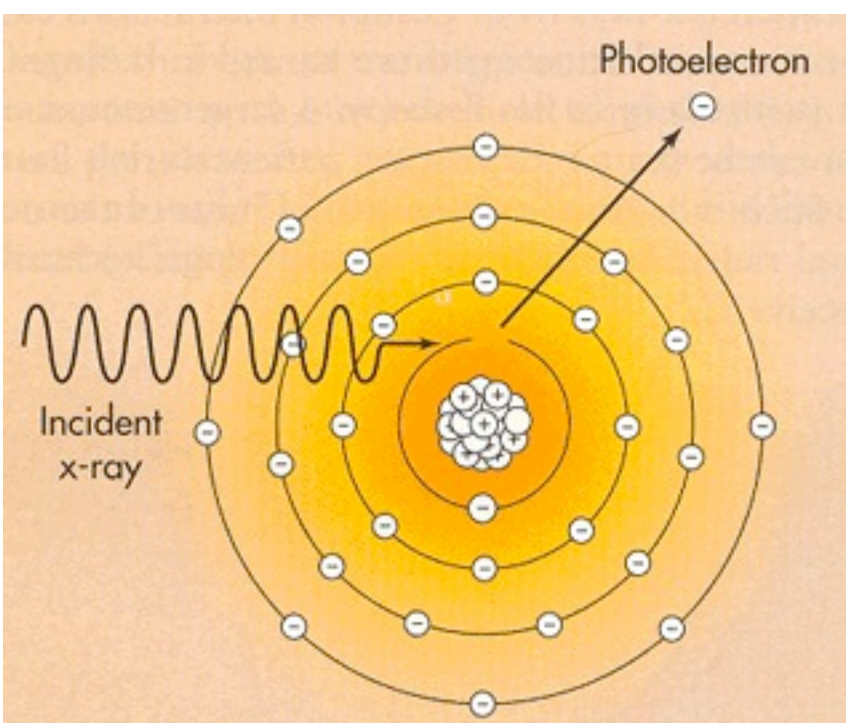
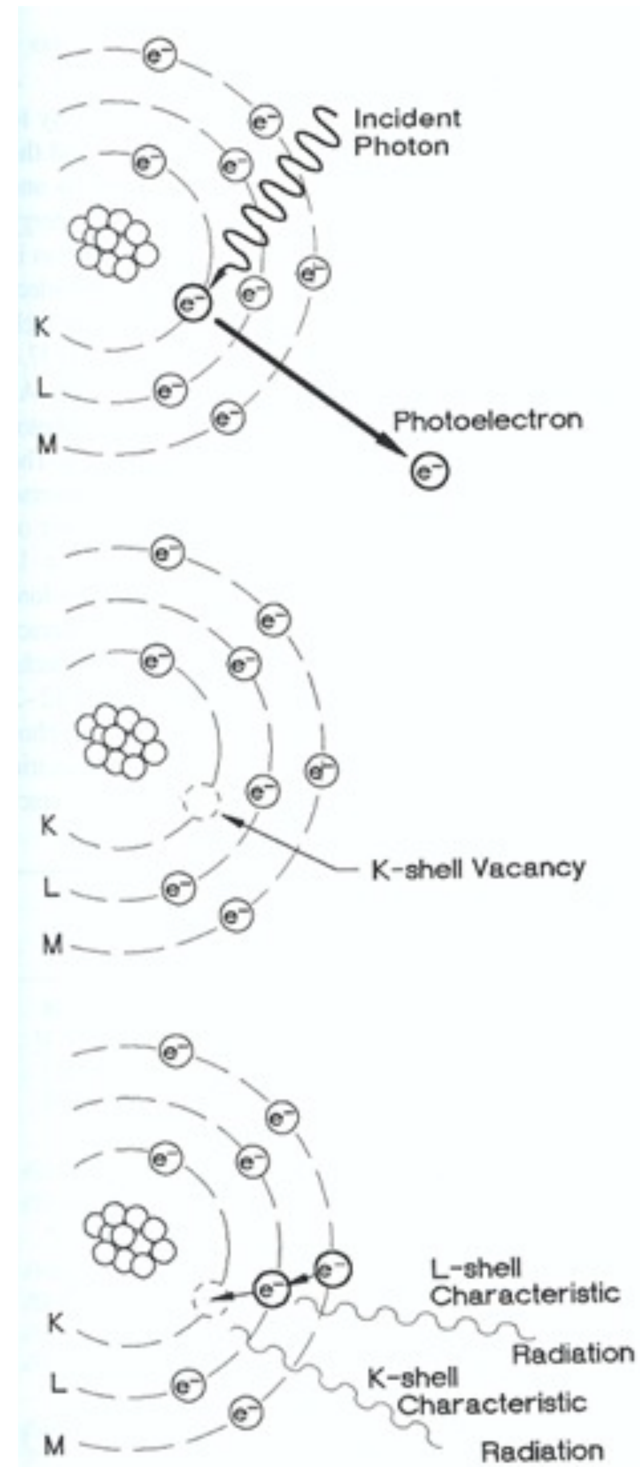
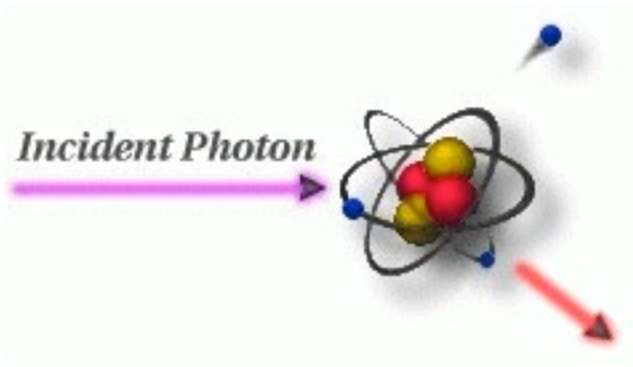


Photo-electric effect

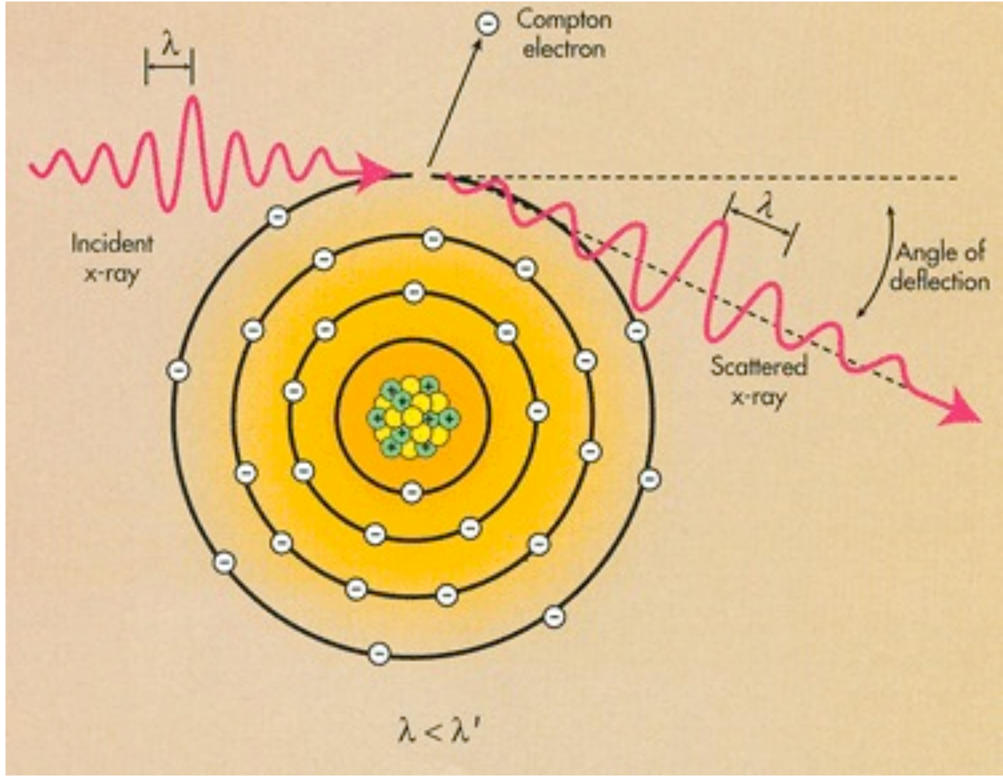


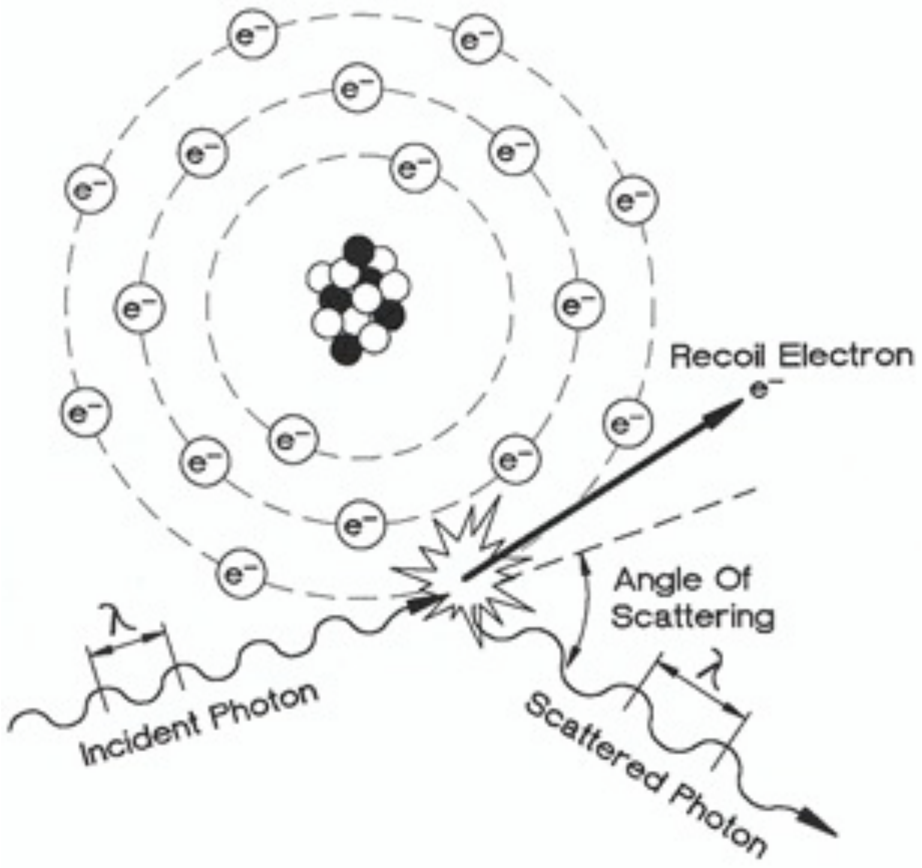


Matter-photon interactions



Compton scattering





Matter-photon interactions
photo-electric effect or Compton scattering?

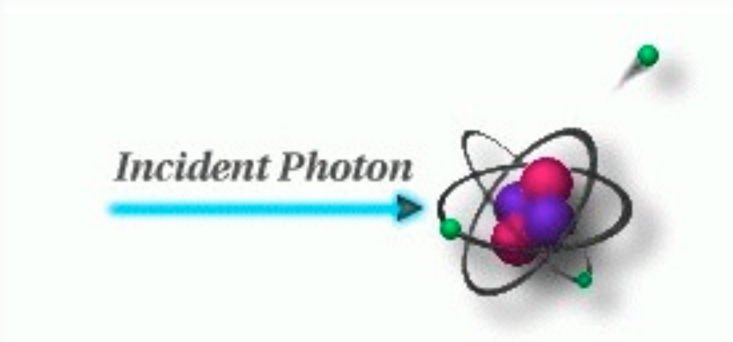
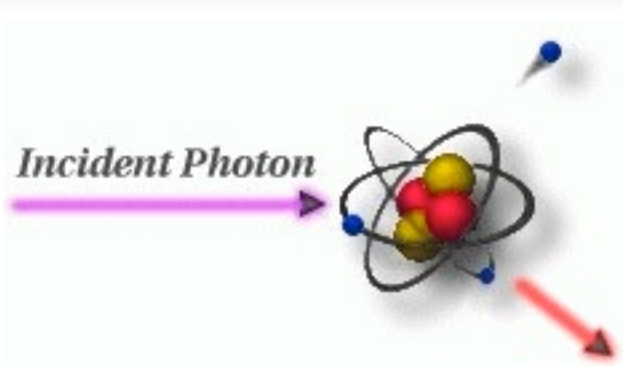
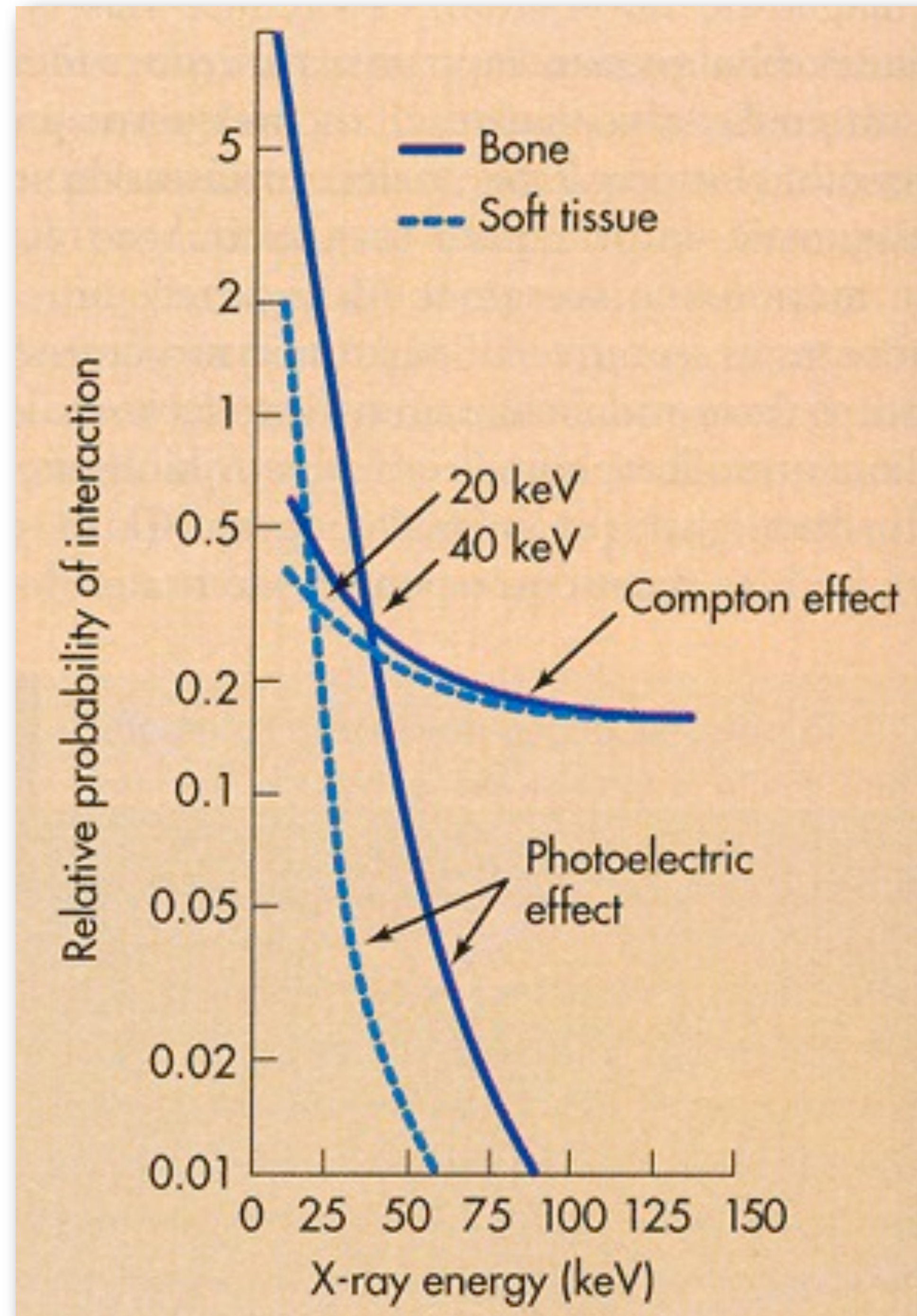


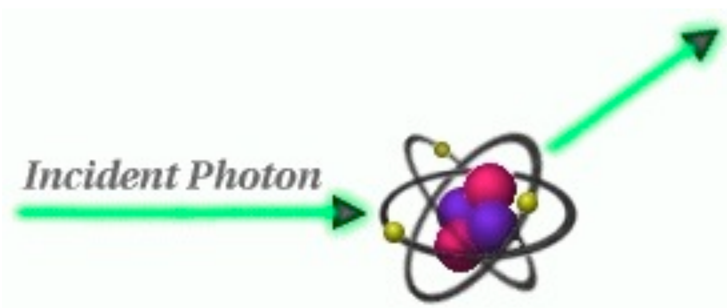
Photo-electric effect



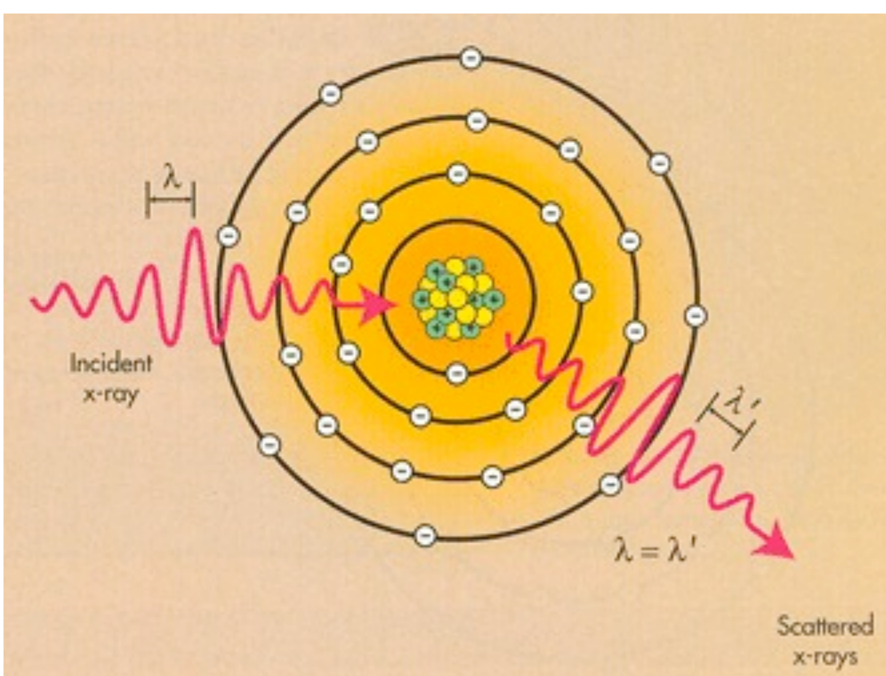
Compton scattering

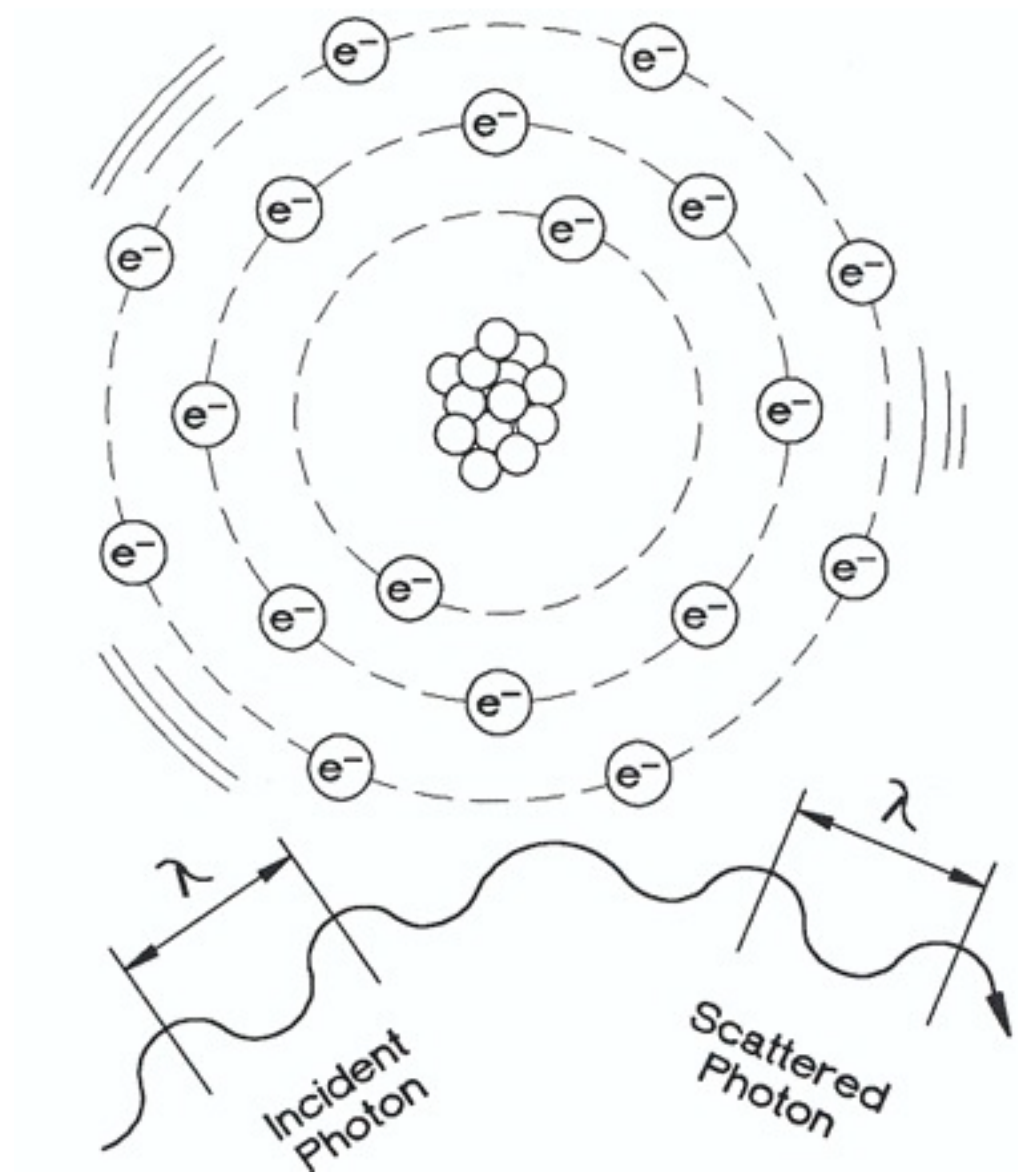


Matter-photon interactions



Thomson scattering

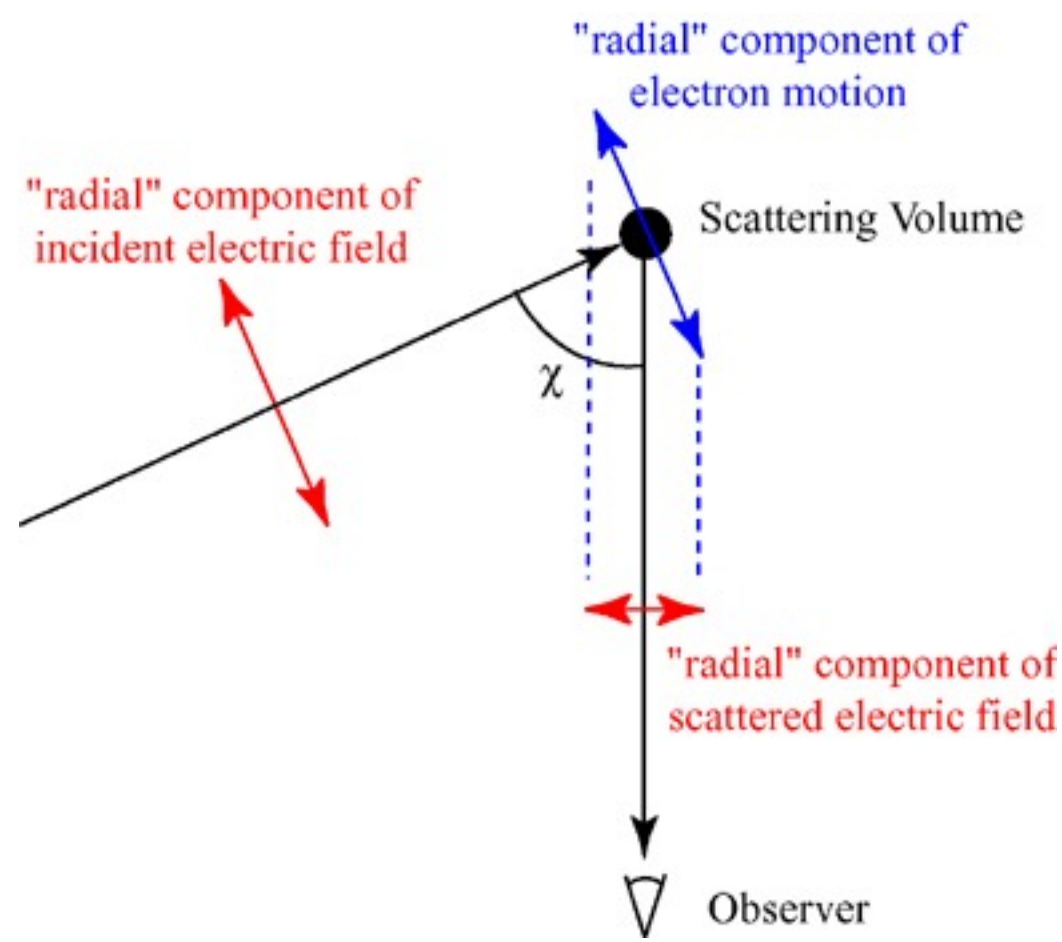




Thomson scattering

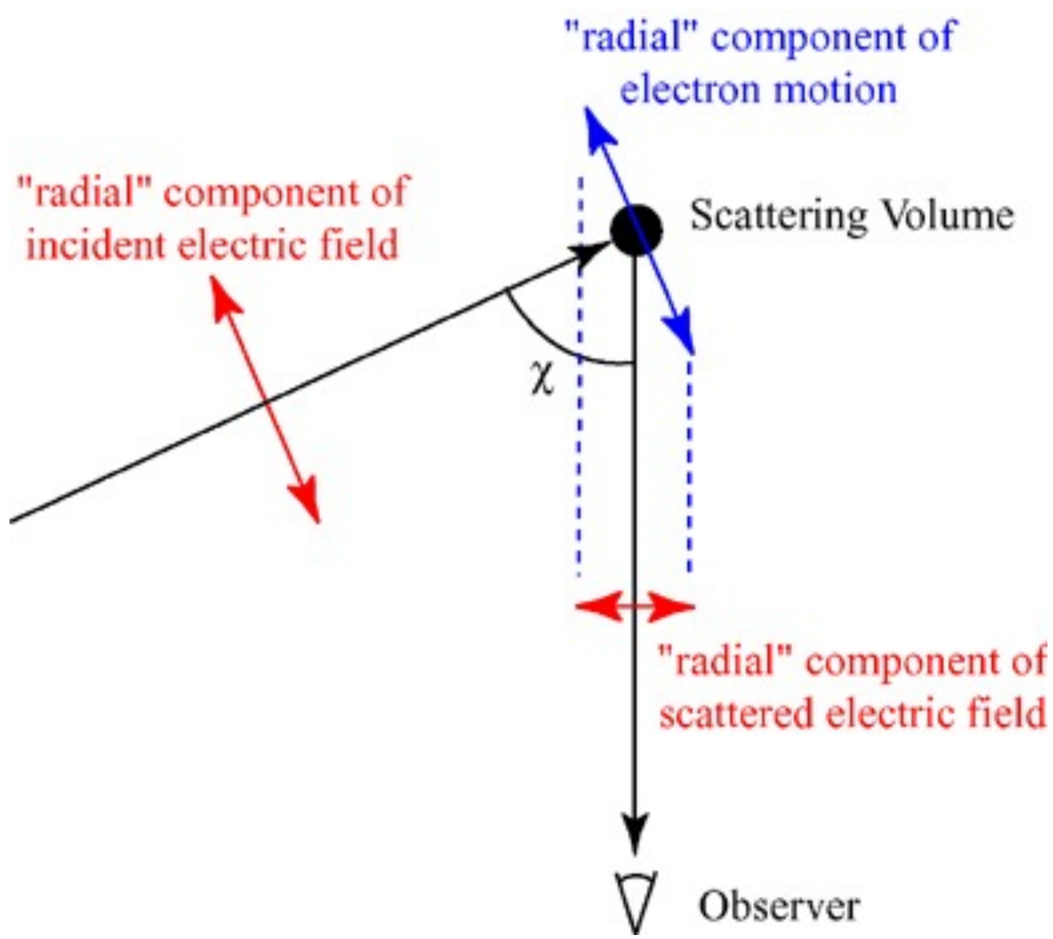
It is the interaction of an electron at rest with a low energy photon:

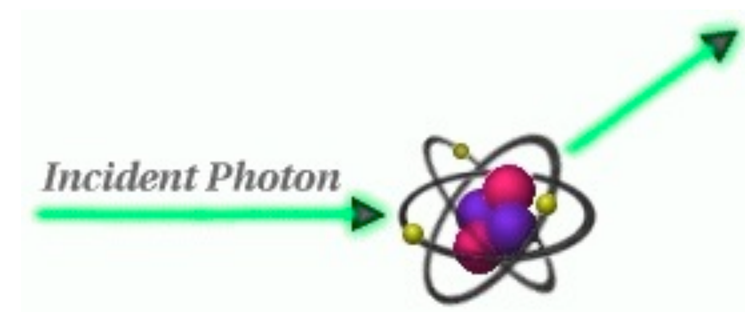
$$h\nu << m_e c^2$$



The incident photons can be considered as a continuous e.m. wave. When an e.m wave is incident on a charged particle, the electric and magnetic components of the wave exert a ***Lorentz force*** on the particle, setting it into motion. Since the wave is periodic in time, so is the motion of the particle, that will oscillate. The particle is accelerated and consequently emits radiation: energy is absorbed from the incident wave by the particle and re-emitted as e.m. radiation. Such a process is clearly equivalent to the scattering of the e.m. wave by the particle.

Thomson scattering





$$\vec{F}=\frac{d\,\vec{p}}{d\,t}=q\left(\vec{E}+\vec{v}\times\frac{\vec{H}}{c}\right)$$

Non relativistic particles

The main cause of acceleration of the particle will be due to the electric field component, along whose direction the particle will oscillate → parallel dipole radiation polarized along the direction of motion of the particle

Thomson scattering

The electron oscillates with an acceleration due to the electric field of the e.m. wave:

$$E = E_0 \cos(2\pi\nu t) \qquad\qquad a(t) = \frac{eE_0}{m_e} \cos(2\pi\nu t)$$

The electron radiates photons at the same frequency than the incoming radiation:
the energy of the incoming and radiated photons is the same.
The mean radiated power is the power of a dipole subject to an acceleration of amplitude:

$$a = \frac{eE}{m_e}$$

Thomson scattering

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The mean radiated power is the power of a dipole subject to an acceleration of amplitude:

$$a = \frac{eE}{m_e}$$

$$\mathcal{P}(t) = \frac{1}{6\pi\epsilon_0} \frac{q^2 a(t)^2}{c^3}$$

Larmor's formula [W]

$$\mathcal{F}_P = \epsilon_0 \; c \; E^2$$

Magnitude of Poynting vector [W/m^2]

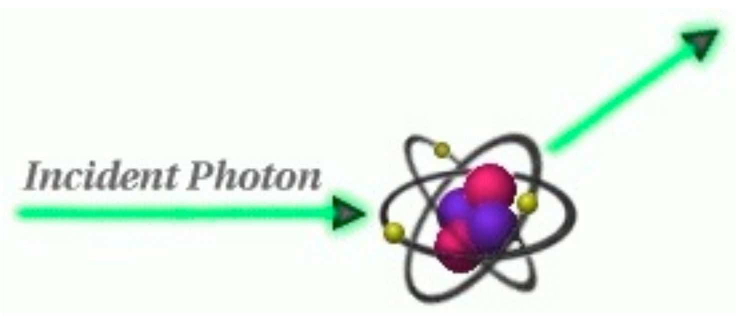
$$\sigma_T = \frac{\mathcal{P}}{\mathcal{F}_P} = \frac{8\pi}{3} r_e^2 = 6.6525 \times 10^{-29} \text{ m}^2$$

Thomson cross section

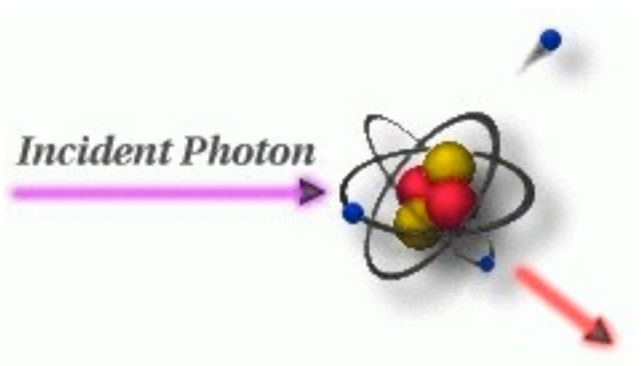
$$r_e = \frac{1}{4\pi\epsilon_0} \frac{e^2}{m_e c^2} = 2.8179 \times 10^{-15} \text{ m}$$

Classical radius of the electron

Scattering from electrons at rest
Quantum effects



Thomson scattering

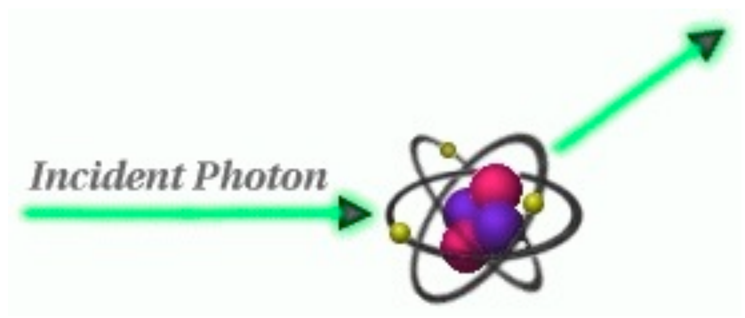


Compton scattering

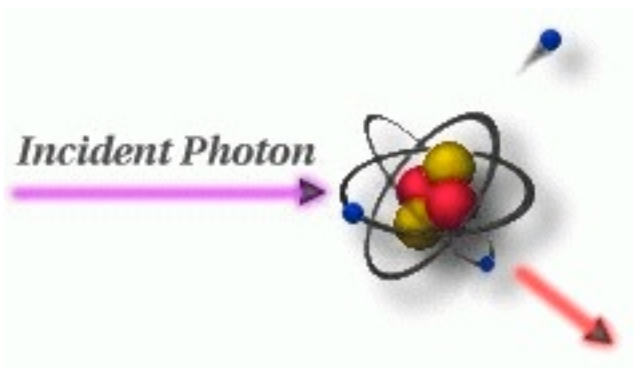
Quantum effects appear in two ways:

- kinematics of the scattering process
- alteration of the cross sections

Scattering from electrons at rest
Quantum effects



Thomson scattering



Compton scattering

Quantum effects appear in two ways:

- **kinematics of the scattering process**
- alteration of the cross sections

Compton scattering

The kinematic effects occur because a photon possesses a momentum (hc/ν) as well as an energy (hν)

$$h\nu + mc^2 = h\nu_s + \gamma mc^2 \quad \text{(Energy conservation)}$$

$$\frac{h\nu}{c} = \frac{h\nu_s}{c}\cos\theta + \gamma\ \beta\ m\ c\ \cos\phi \quad \text{(Longitudinal momentum)}$$

$$0 = \frac{h\nu_s}{c}\sin\theta - \gamma\ \beta\ m\ c\ \sin\phi \quad \text{(Transverse momentum)}$$

Normal Compton Effect

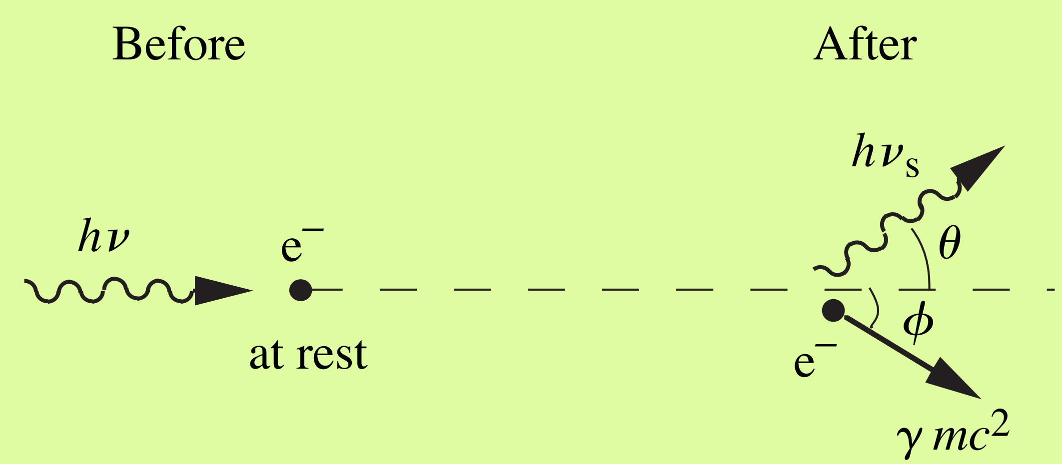


Fig.9.1: Astrophysics Processes (CUP), © H. Bradt 2008

Collision of a photon with a stationary free electron:
both the electron and the photon are treated as particles

Compton scattering

Collision of a photon with a stationary free electron:
both the electron and the photon are treated as particles

(a) $h\nu + mc^2 = h\nu_s + \gamma mc^2$ (Energy conservation)

(b) $\dfrac{h\nu}{c} = \dfrac{h\nu_s}{c}\cos\theta + \gamma\ \beta\ m\ c\ \cos\phi$ (Longitudinal momentum)

(c) $0 = \dfrac{h\nu_s}{c}\sin\theta - \gamma\ \beta\ m\ c\ \sin\phi$ (Transverse momentum)

Normal Compton Effect

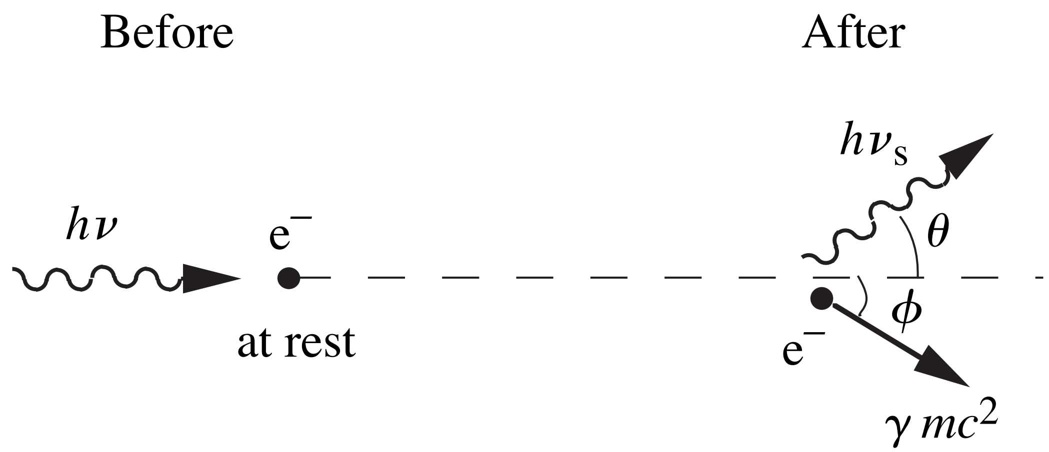


Fig.9.1: Astrophysics Processes (CUP), © H. Bradt 2008

- Solve (b) for cosΦ and (c) for sinΦ ; remember! $\cos^2\Phi + \sin^2\Phi = 1$ → (bc)’
- Square (a) with the γmc^2 term isolated on the right side → (a’)
- (a’) - (bc)’

Compton scattering

Collision of a photon with a stationary free electron:
both the electron and the photon are treated as particles

(a) $h\nu + mc^2 = h\nu_s + \gamma mc^2 \quad \text{(Energy conservation)}$

(b) $\frac{h\nu}{c} = \frac{h\nu_s}{c}\cos\theta + \gamma\ \beta\ m\ c\ \cos\phi \quad \text{(Longitudinal momentum)}$

(c) $0 = \frac{h\nu_s}{c}\sin\theta - \gamma\ \beta\ m\ c\ \sin\phi \quad \text{(Transverse momentum)}$

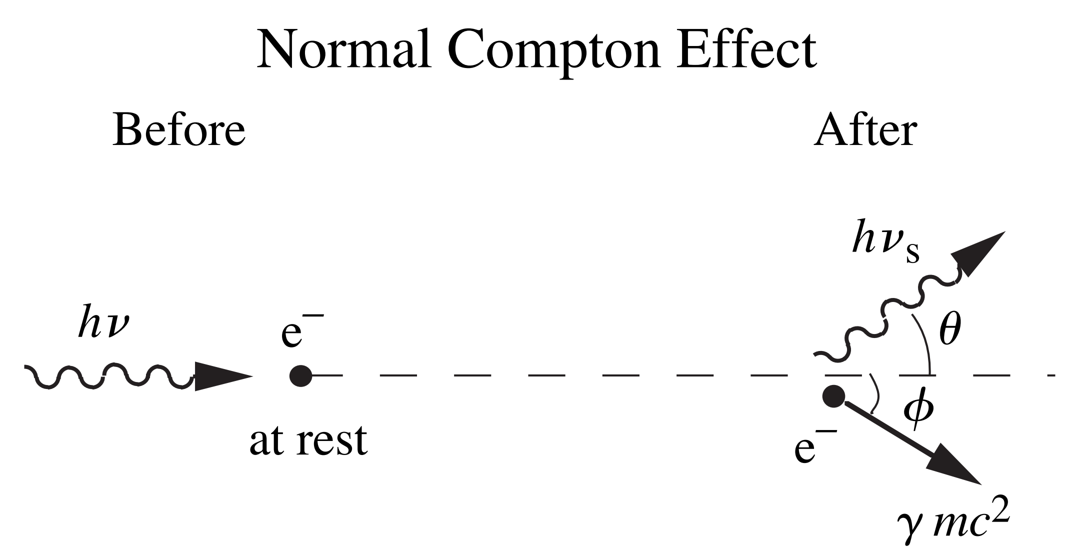


Fig.9.1: Astrophysics Processes (CUP), © H. Bradt 2008

$$\longrightarrow \quad \frac{1}{h\nu_s} - \frac{1}{h\nu} = \frac{1-\cos\theta}{mc^2}$$

$$h\nu_s = \frac{h\nu}{1 + \frac{h\nu}{mc^2}(1-\cos\theta)}$$

Compton scattering

Compton scattering

Collision of a photon with a stationary free electron:
both the electron and the photon are treated as particles

$$h\nu_s = \frac{h\nu}{1 + \frac{h\nu}{mc^2}(1 - \cos\theta)}$$

$$\lambda_s - \lambda = \frac{h}{mc}(1 - \cos\theta)$$

Normal Compton Effect

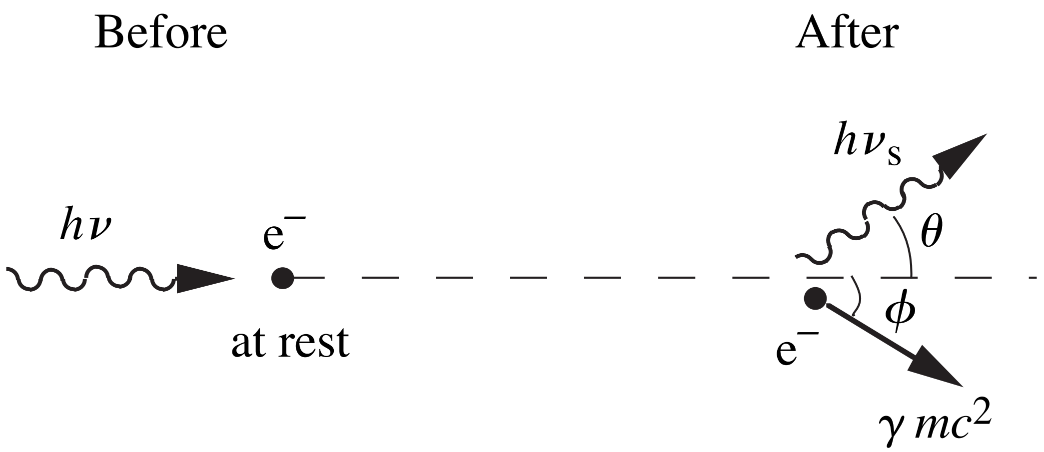


Fig.9.1: Astrophysics Processes (CUP), © H. Bradt 2008

$$\lambda_c = \frac{h}{mc} = 2.43 \times 10^{-12}\ \text{m} \rightarrow \frac{c}{\lambda_c} = 1.23 \times 10^{20}\ \text{Hz} \rightarrow 0.511\ \text{MeV}$$

Compton wavelength

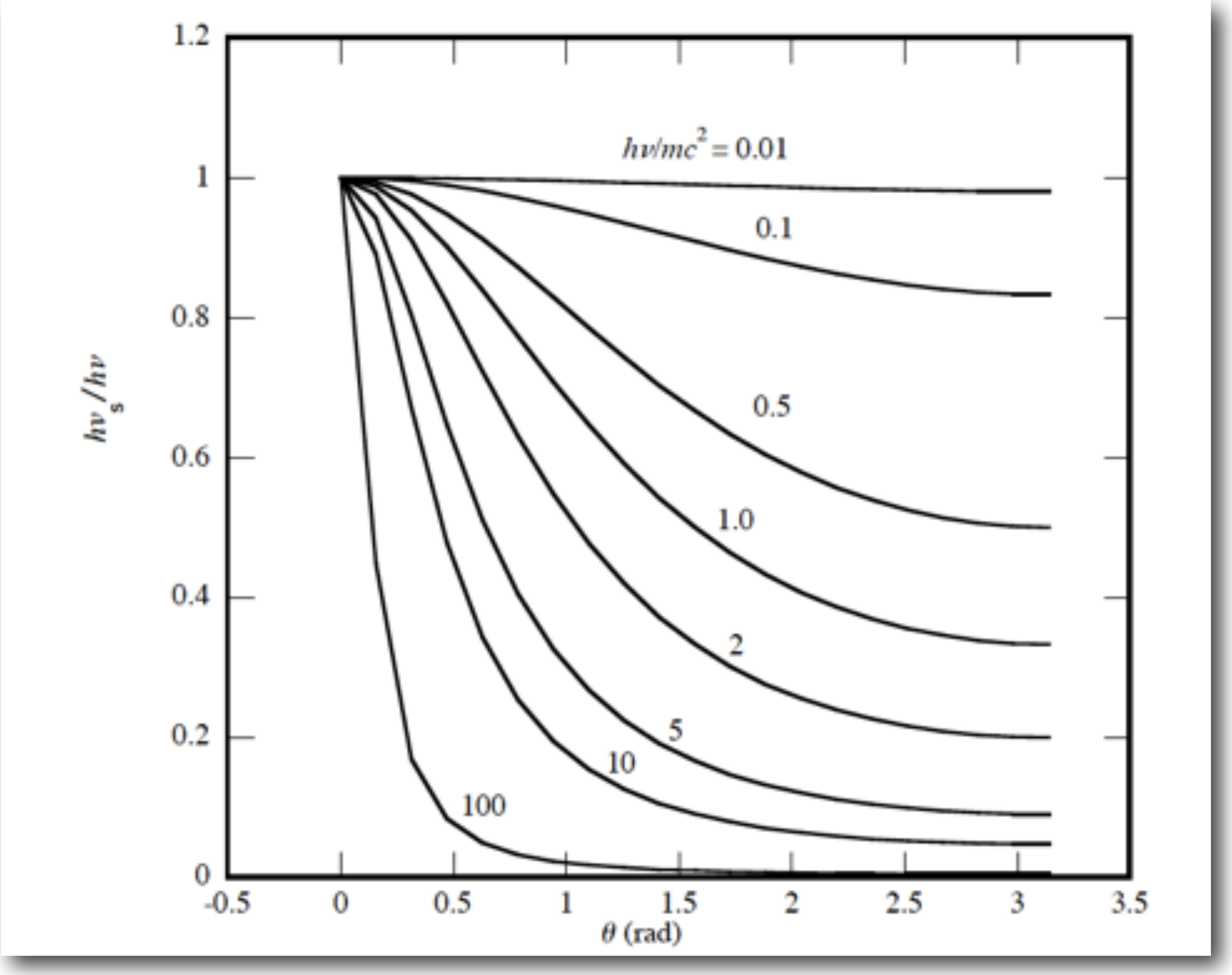
Result relativistically correct

Compton scattering

Collision of a photon with a stationary free electron:
both the electron and the photon are treated as particles

$$h\nu_s = \frac{h\nu}{1 + \frac{h\nu}{mc^2}(1 - \cos\theta)}$$

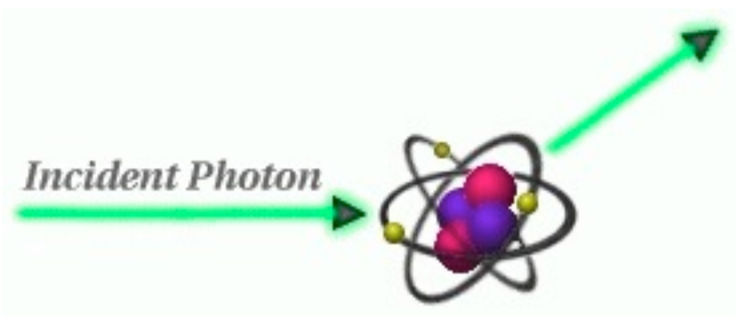
The scattered photon energy is shifted significantly as the incident photon energy becomes comparable to the rest energy of the electron



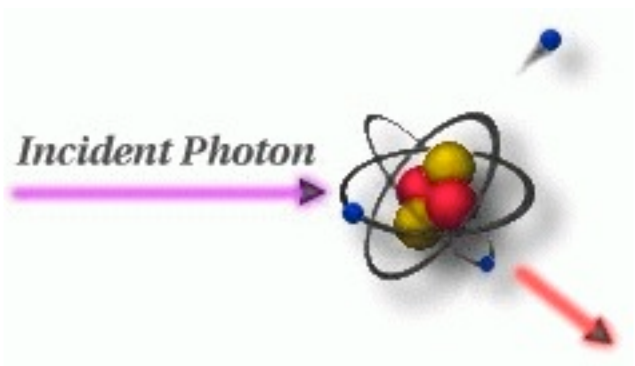
A significant fractional loss requires a high photon energy and a substantial scattering angle

Scattering from electrons at rest

Quantum effects



Thomson scattering



Compton scattering

Quantum effects appear in two ways:

- kinematics of the scattering process
- **alteration of the cross sections**

Compton scattering
Klein-Nishina cross section

$$\lambda_s - \lambda = \frac{h}{mc}(1 - \cos\theta)$$

$$\lambda_c = \frac{h}{mc} = 2.43 \times 10^{-12} \text{ m}$$

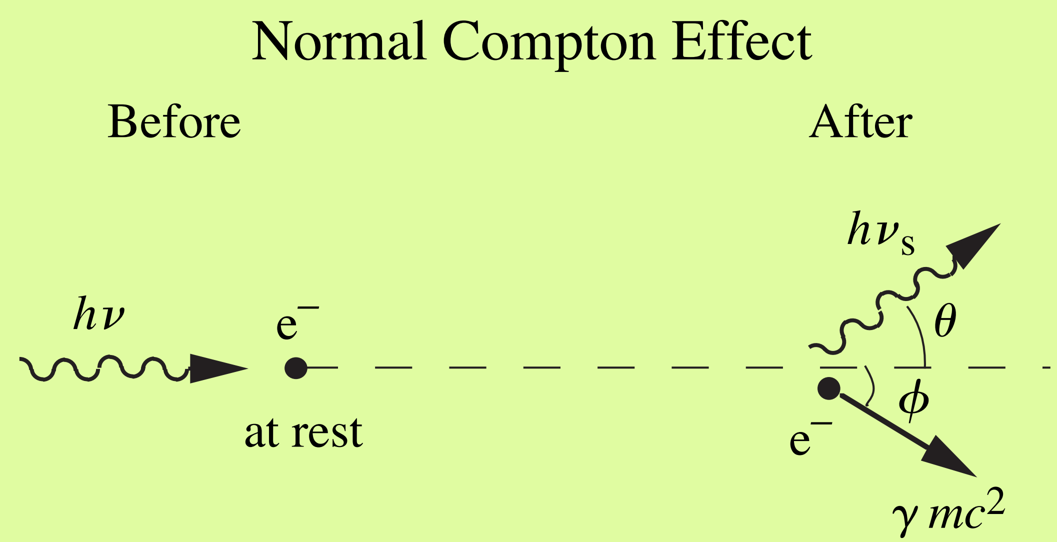


Fig.9.1: Astrophysics Processes (CUP), © H. Bradt 2008

→ wavelength change of the order of λ_C upon scattering

For long wavelengths ($\lambda >> \lambda_C$ i.e. $h\nu << mc^2$) the scattering is closely elastic. When this condition is satisfied, we can assume that there is no change in the photon energy in the rest frame of the electron (cf. rest of the lesson)

However, it can be shown in quantum electrodynamics that, as the photon energy becomes large ($h\nu >> mc^2$), the Compton scattering becomes less efficient (the cross section is reduced from its classical value - *Klein-Nishina formula*)

Compton scattering

Klein-Nishina cross section

$$\sigma = \sigma_T \times \frac{3}{4}\left[\frac{1+x}{x^3}\left[\frac{2x(1+x)}{1+2x} - \ln(1+2x)\right] + \frac{1}{2x}\ln(1+2x) - \frac{1+3x}{(1+2x)^2}\right]$$

$$\sigma \approx \sigma_T\left(1 - 2x + \frac{26x^2}{5} + \ldots\right), \quad x << 1$$

$$x \equiv h\nu/mc^2$$

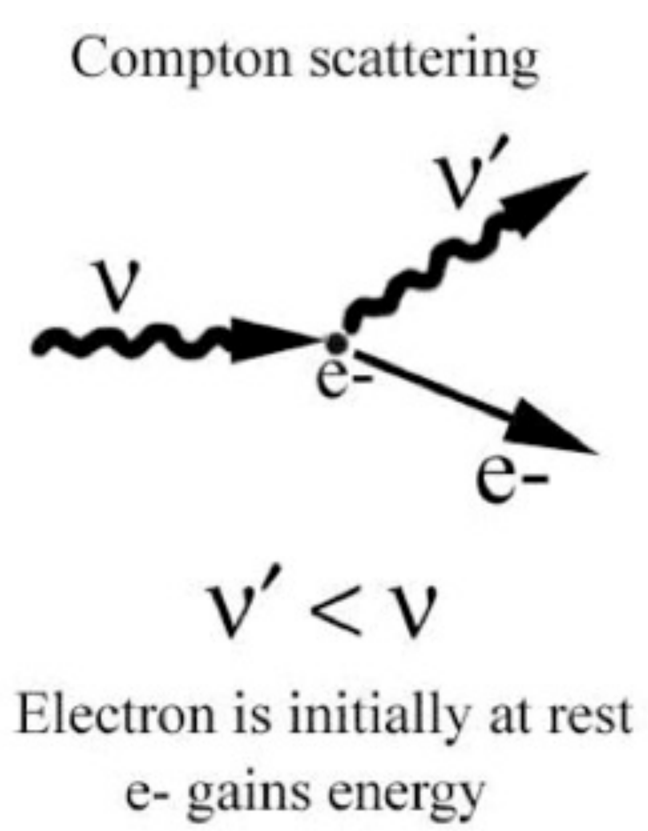
$$\sigma = \frac{3}{8}\sigma_T x^{-1}\left(\ln 2x + \frac{1}{2}\right), \quad x >> 1$$

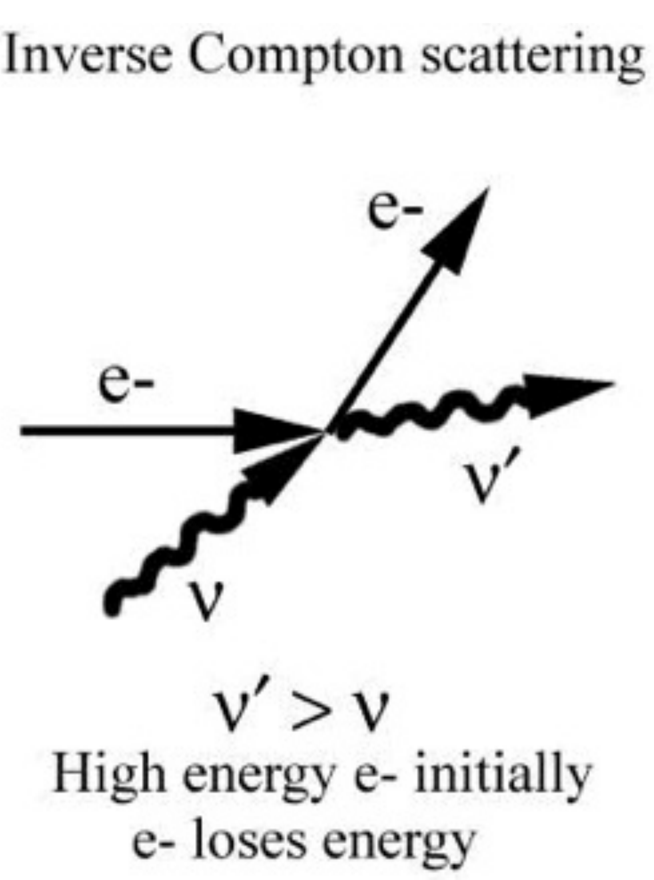
Inverse Compton scattering

It is the interaction of photon with a fastly moving electron !

$$h\nu << \gamma \, m_e c^2$$

If the moving electron has an energy significantly higher than the incoming photon (in the case of relativistic electrons), energy is transferred from the electron to the photon, i.e. it is the opposite of the Compton scattering





Inverse Compton scattering

We restrict our development to:

- Head-on collisions of electrons and photons
- Electrons that are highly relativistic ($v \sim c$)

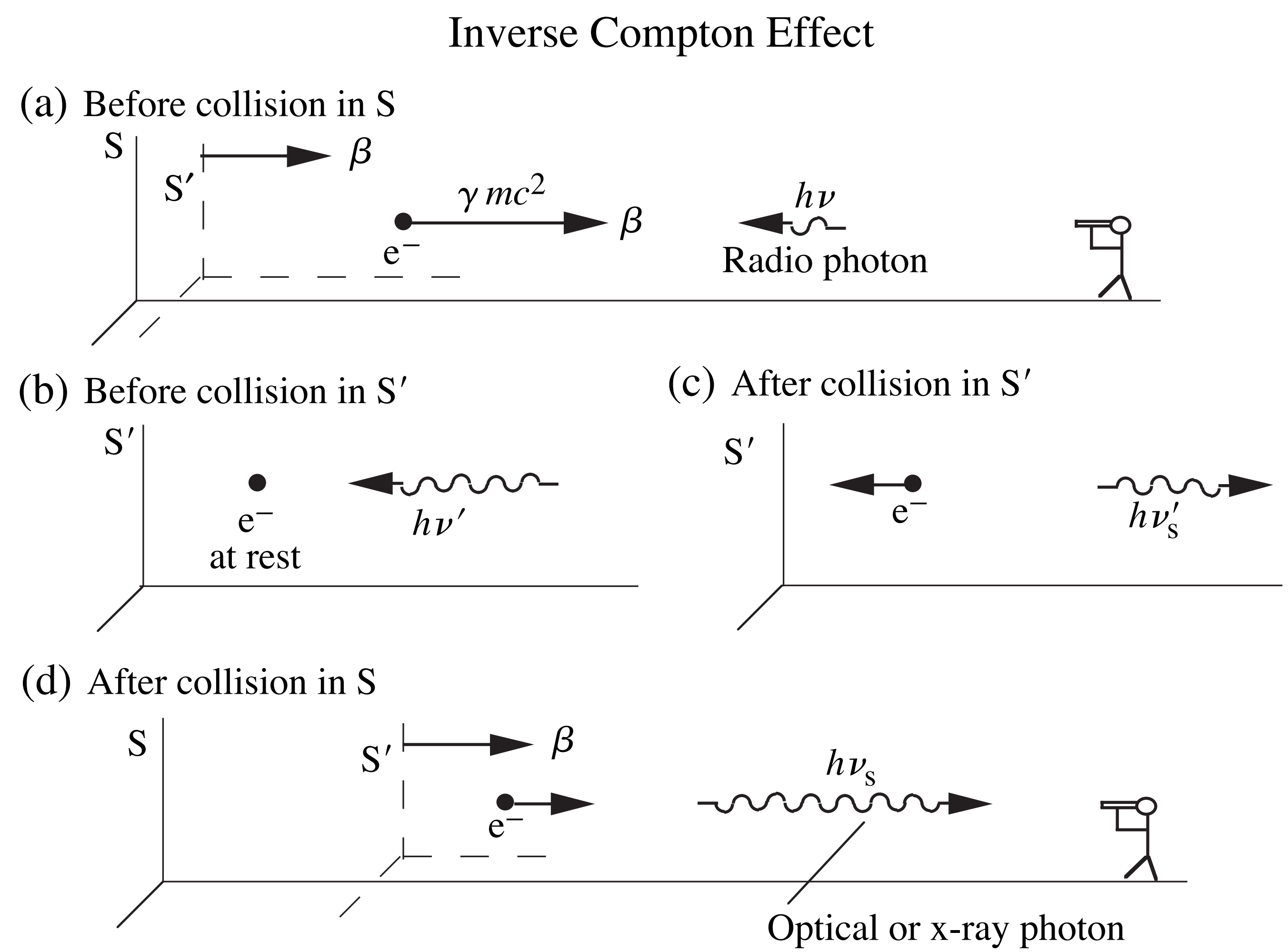


Fig.9.2: Astrophysics Processes (CUP), © H. Bradt 2008

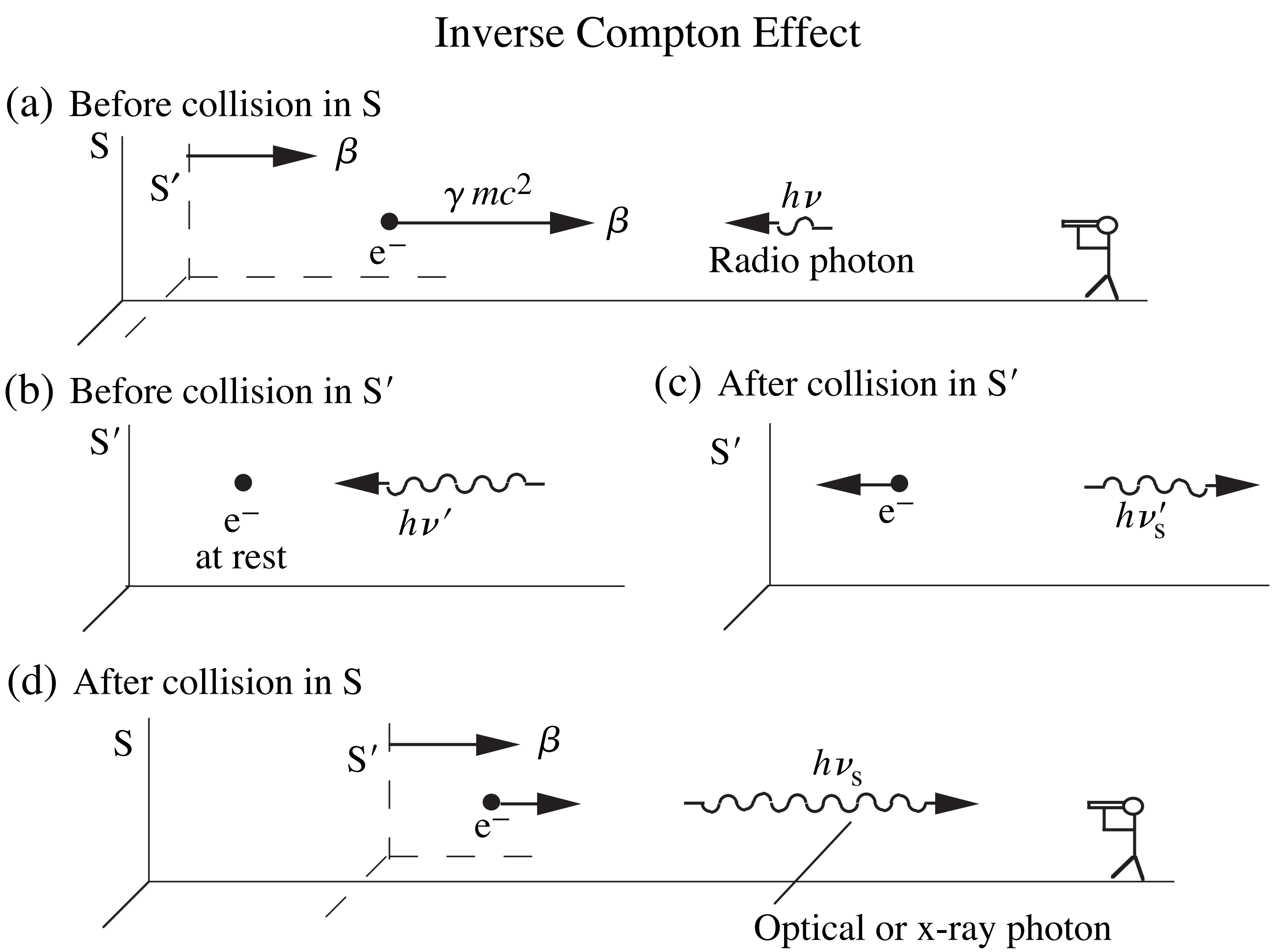
Inverse Compton scattering

$$\beta = v/c$$
$$E_{el} = \gamma mc^2$$

→ **Initial electron speed parameter and energy**

$$E_{ph} = h\nu$$

→ **Initial photon energy**



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→ **Initial electron speed parameter and energy**

$$E_{ph} = h\nu$$

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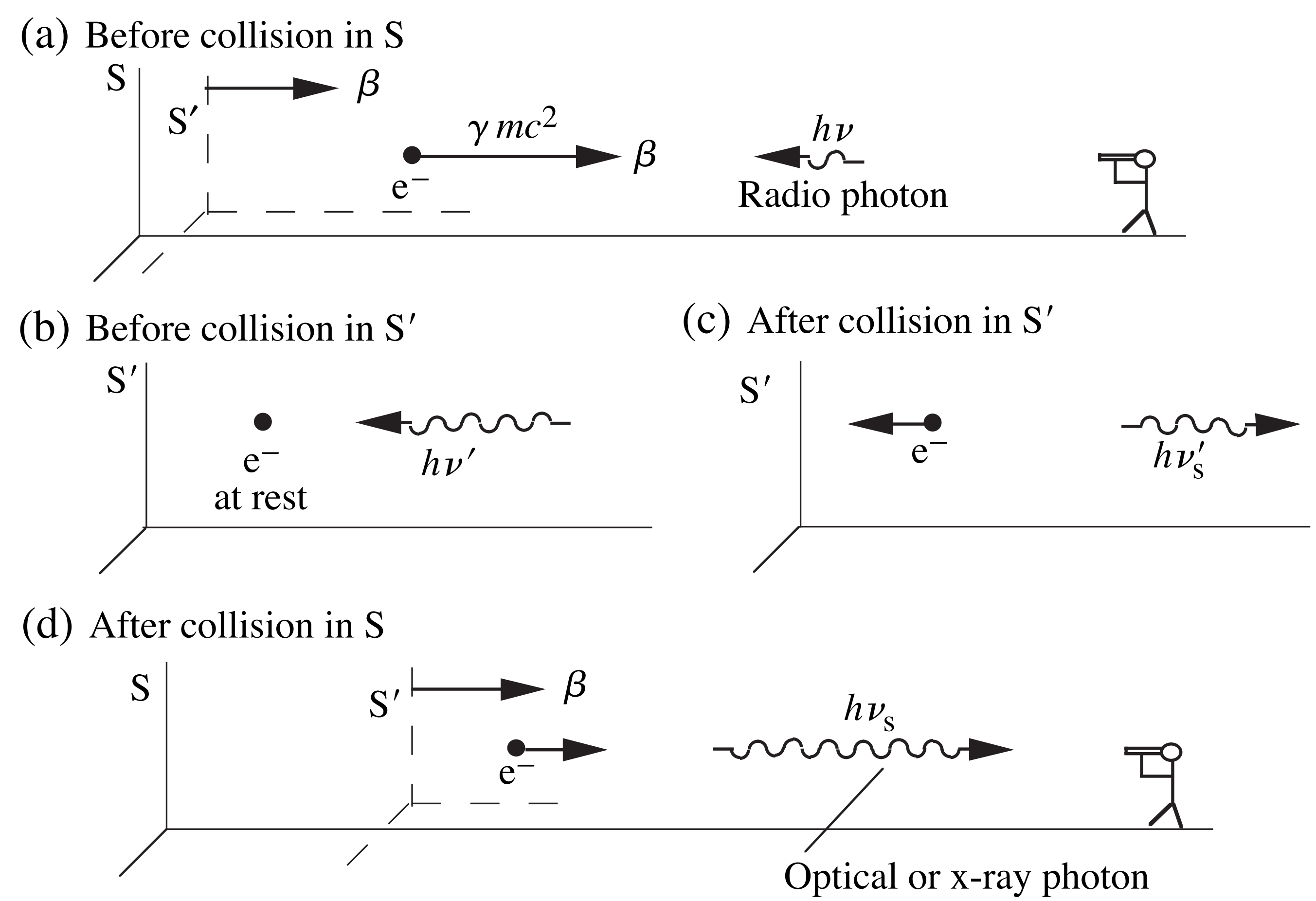
$$h\nu' = \left(\frac{1+\beta}{1-\beta}\right)^{1/2} h\nu$$

First Doppler shift

Inverse Compton Effect



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First Doppler shift

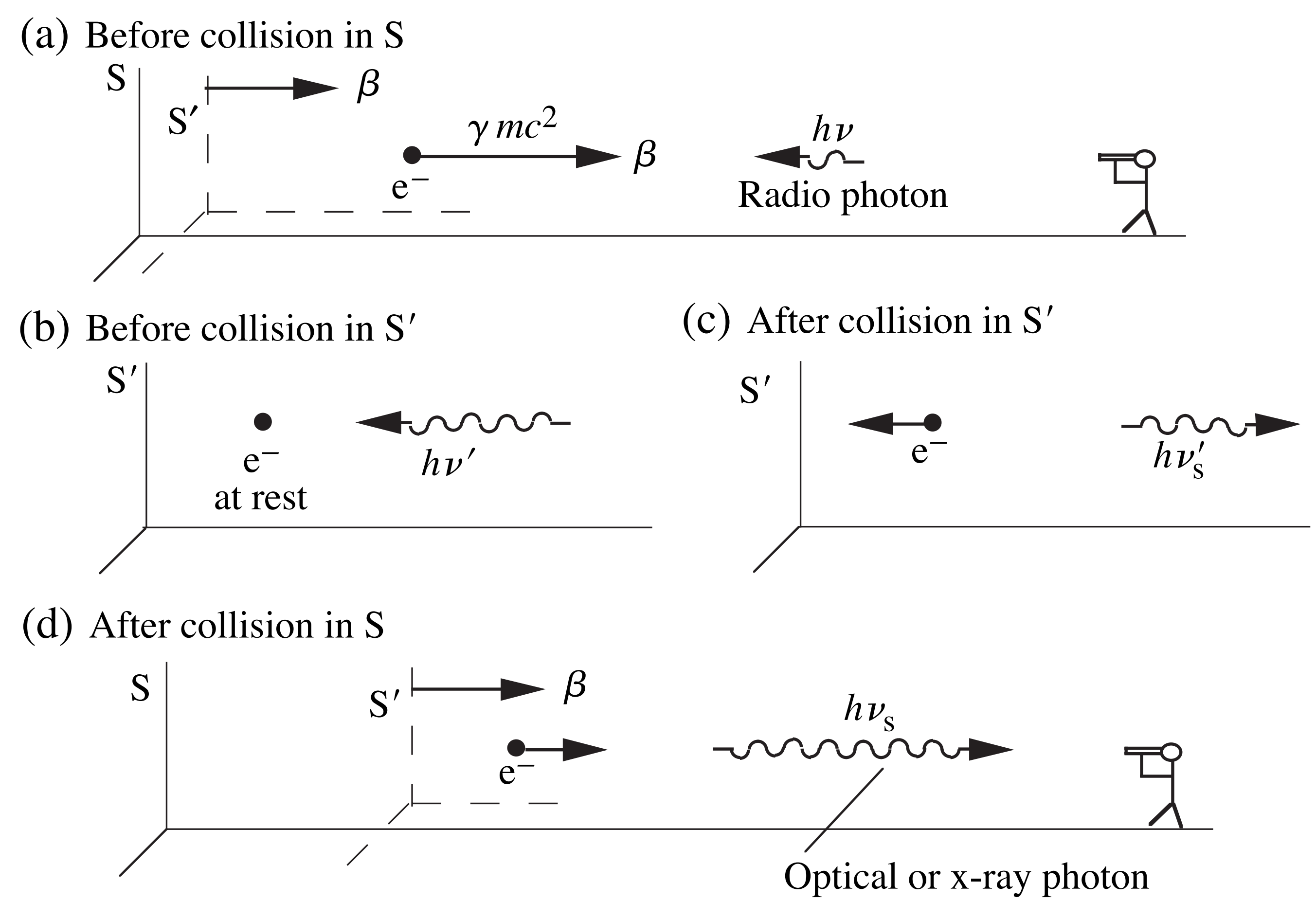
$$h\nu'_s = \frac{h\nu'}{1 + \frac{2h\nu'}{mc^2}}$$

Compton scatter in S'

$$h\nu_s = \frac{h\nu}{1 + \frac{h\nu}{mc^2}(1 - \cos\theta)}$$

$$\theta' = \pi$$

Inverse Compton Effect



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Inverse Compton scattering

$$\beta = v/c$$
$$E_{el} = \gamma mc^2$$
→ Initial electron speed parameter and energy

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First Doppler shift

$$h\nu'_s = \frac{h\nu'}{1 + \frac{2h\nu'}{mc^2}}$$
Compton scatter in S'

$$h\nu_s = \left(\frac{1+\beta}{1-\beta}\right)^{1/2} h\nu'_s$$
Second Doppler shift

Inverse Compton Effect

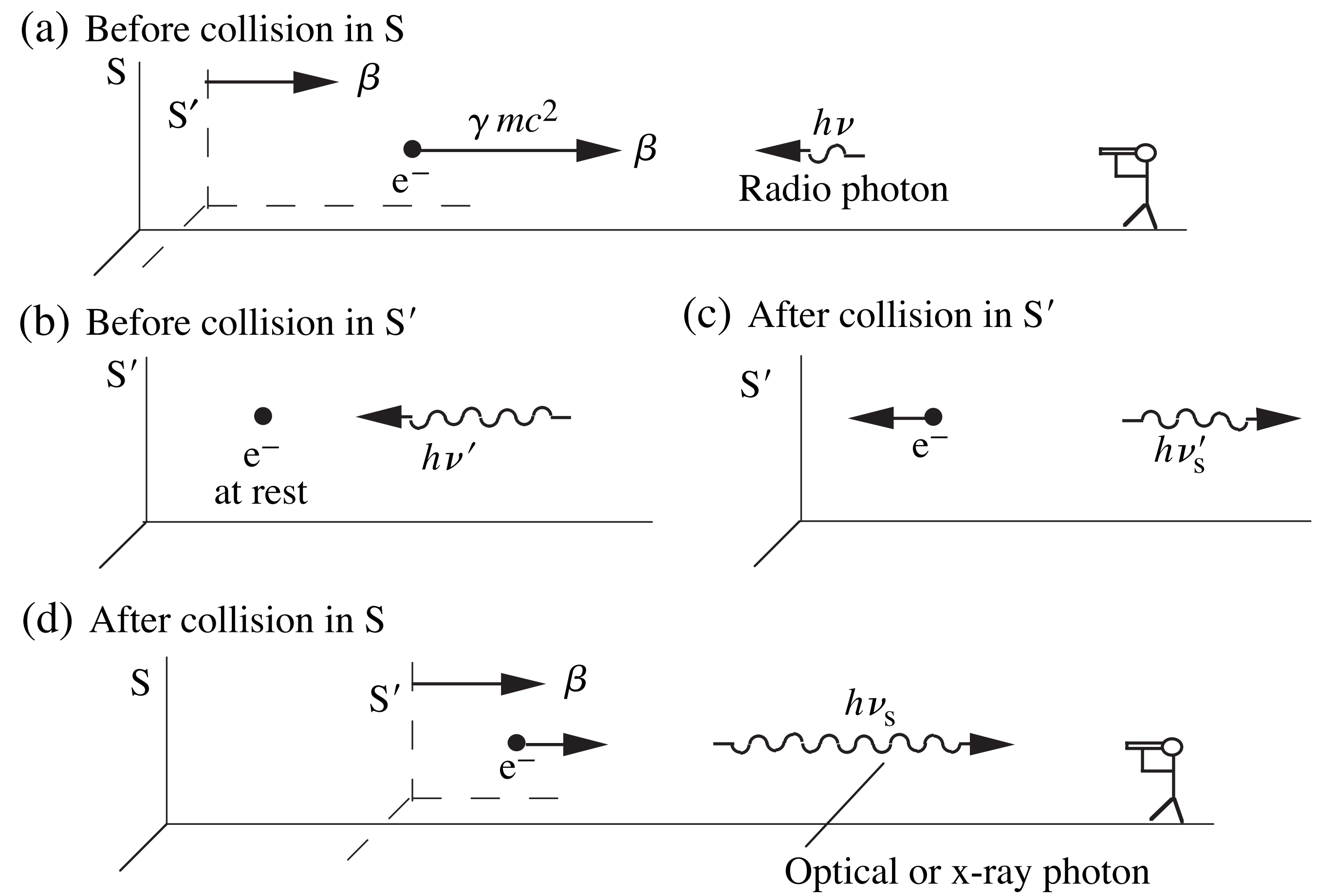

Fig.9.2: Astrophysics Processes (CUP), © H. Bradt 2008

Inverse Compton scattering

$$h\nu' = \left(\frac{1+\beta}{1-\beta}\right)^{1/2} h\nu$$

First Doppler shift

$$h\nu'_s = \frac{h\nu'}{1 + \frac{2h\nu'}{mc^2}}$$

Compton scatter in S’

$$h\nu_s = \left(\frac{1+\beta}{1-\beta}\right)^{1/2} h\nu'_s$$

Second Doppler shift

Inverse Compton scattering

$$h\nu' = \left(\frac{1+\beta}{1-\beta}\right)^{1/2} h\nu$$ First Doppler shift

$$h\nu'_s = \frac{h\nu'}{1+\frac{2h\nu'}{mc^2}}$$ Compton scatter in S'

$$h\nu_s = \left(\frac{1+\beta}{1-\beta}\right)^{1/2} h\nu'_s$$ Second Doppler shift

$$\left(\frac{1+\beta}{1-\beta}\right)^{1/2} = \left(\frac{1+\beta}{1-\beta}\right)^{1/2} \times \left(\frac{1+\beta}{1+\beta}\right)^{1/2} = \frac{1+\beta}{(1-\beta^2)^{1/2}} \xrightarrow[\beta \approx 1]{} 2\gamma$$

Inverse Compton scattering

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$$h\nu'_s = \frac{h\nu'}{1+\frac{2h\nu'}{mc^2}}$$ Compton scatter in S'

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$$\rightarrow h\nu_s \approx 2\gamma h\nu'_s = 2\gamma \frac{h\nu'}{1+\frac{2h\nu'}{mc^2}} \approx 4\gamma^2 \frac{h\nu}{1+\frac{4\gamma h\nu}{mc^2}}$$

Inverse Compton scattering

$$\left(\frac{1+\beta}{1-\beta}\right)^{1/2} = \left(\frac{1+\beta}{1-\beta}\right)^{1/2} \times \left(\frac{1+\beta}{1+\beta}\right)^{1/2} = \frac{1+\beta}{(1-\beta^2)^{1/2}} \xrightarrow[\beta \approx 1]{} 2\gamma$$

$$h\nu_s \approx 4\gamma^2 \frac{h\nu}{1+\frac{4\gamma h\nu}{mc^2}} \xrightarrow[4\gamma h\nu << mc^2]{} h\nu_s \approx 4\gamma^2 h\,\nu = 4\left(\frac{U}{mc^2}\right)^2 h\nu$$

$$U = \gamma mc^2$$

The energy of the photon is increased by γ^2

Each factor γ can be attributed to one Doppler-shift transformation

Inverse Compton scattering

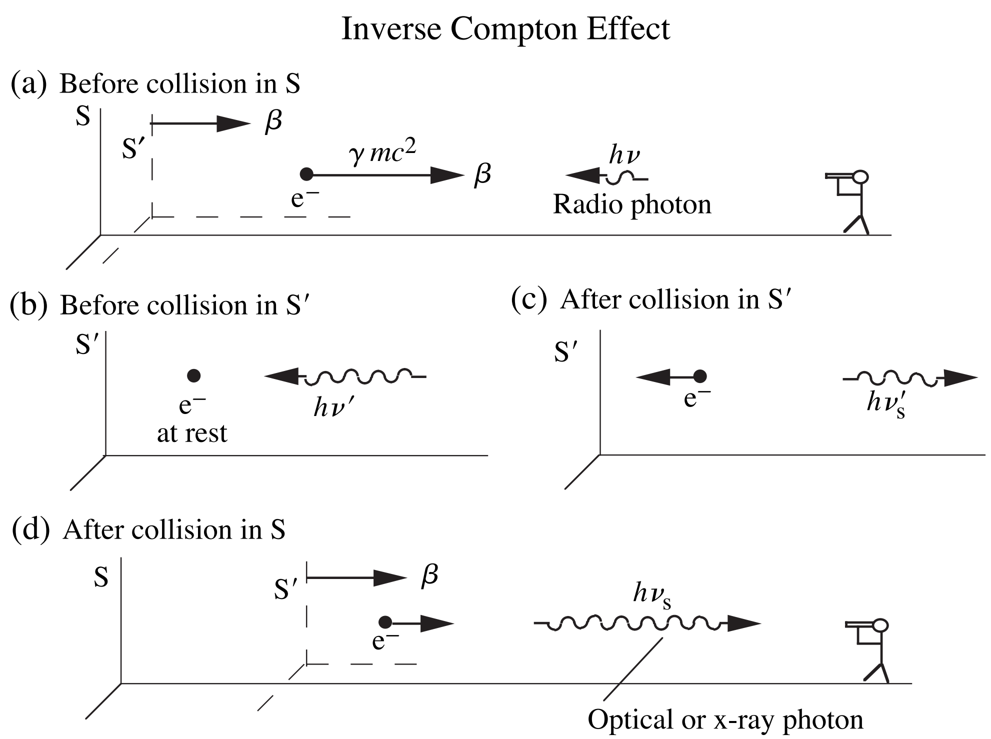
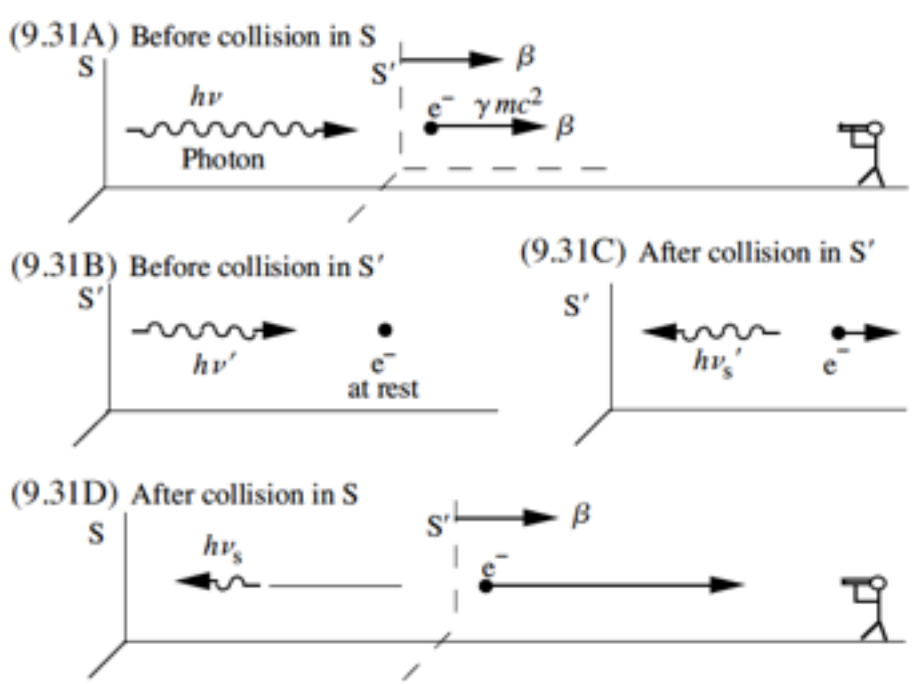


Fig.9.2: Astrophysics Processes (CUP), © H. Bradt 2008

$$h\nu_s \approx 4\gamma^2 h\ \nu$$



$$h\nu_s \approx \frac{h\nu}{4\gamma^2}$$

Inverse Compton scattering

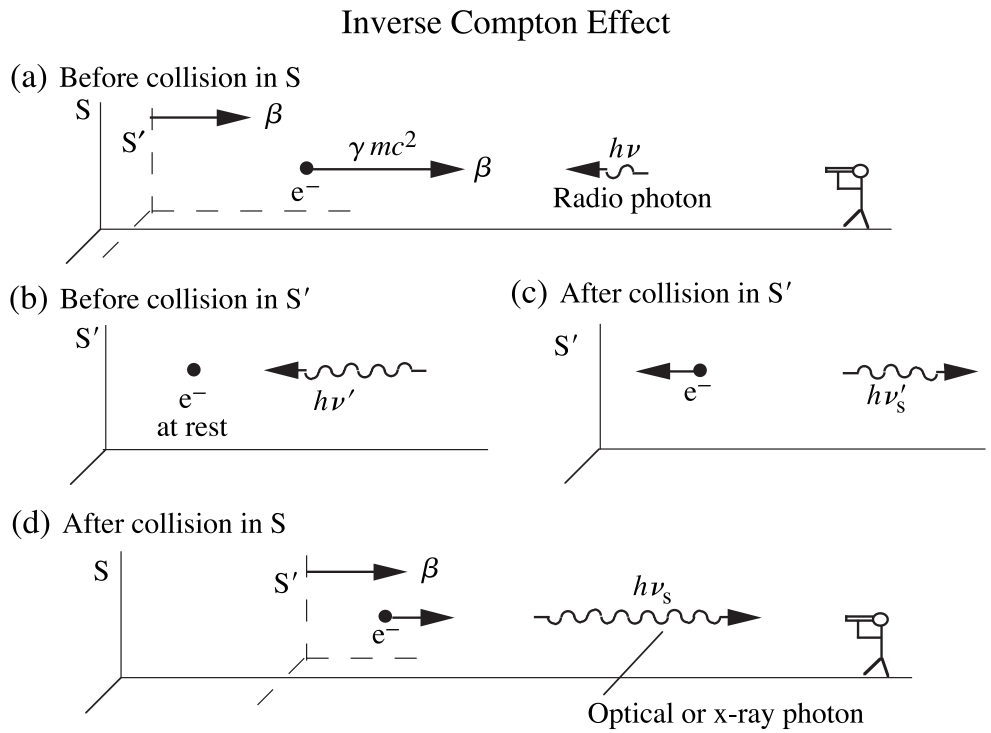
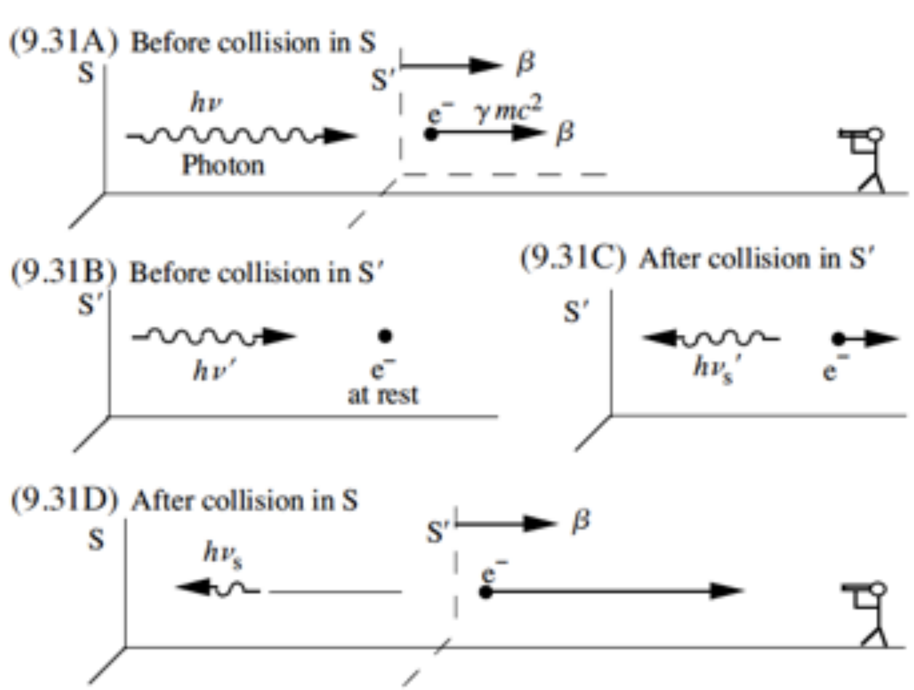


Fig.9.2: Astrophysics Processes (CUP), © H. Bradt 2008

$$h\nu_s \approx 4\gamma^2 h\ \nu$$

Much more frequent !



$$h\nu_s \approx \frac{h\nu}{4\gamma^2}$$

Inverse Compton scattering

Inverse Compton Effect

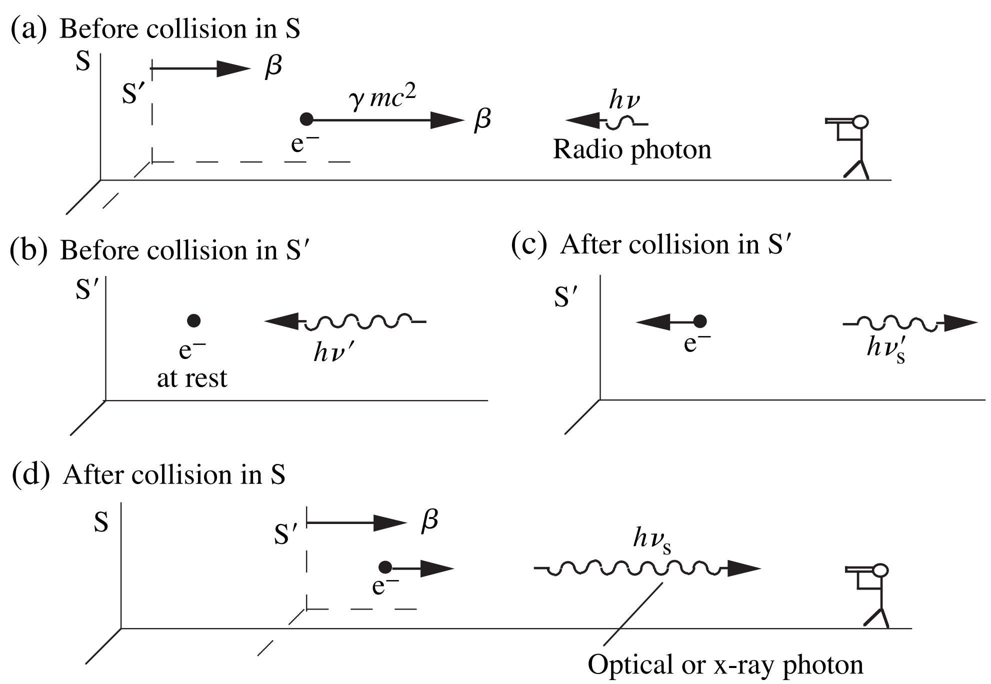
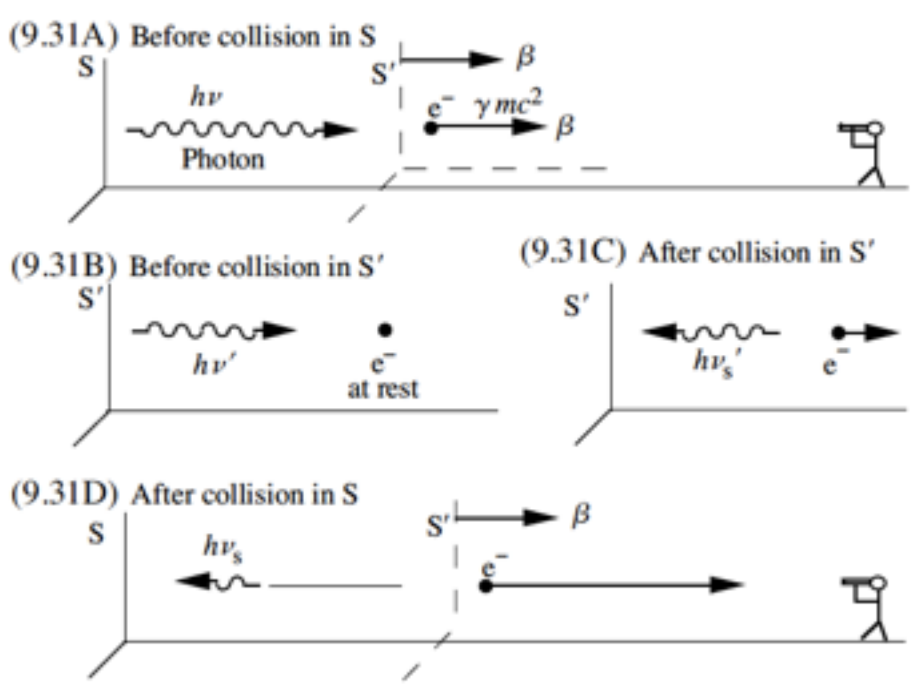


Fig.9.2: Astrophysics Processes (CUP), © H. Bradt 2008



$$h\nu_s \approx 4\gamma^2 h\ \nu$$

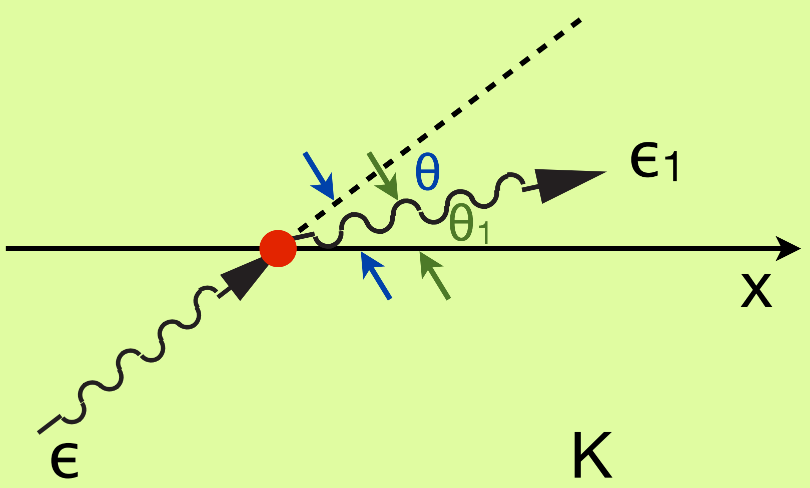
$$h\nu_{s,\mathrm{iso}} = \frac{4}{3}\gamma^2 h\ \nu$$

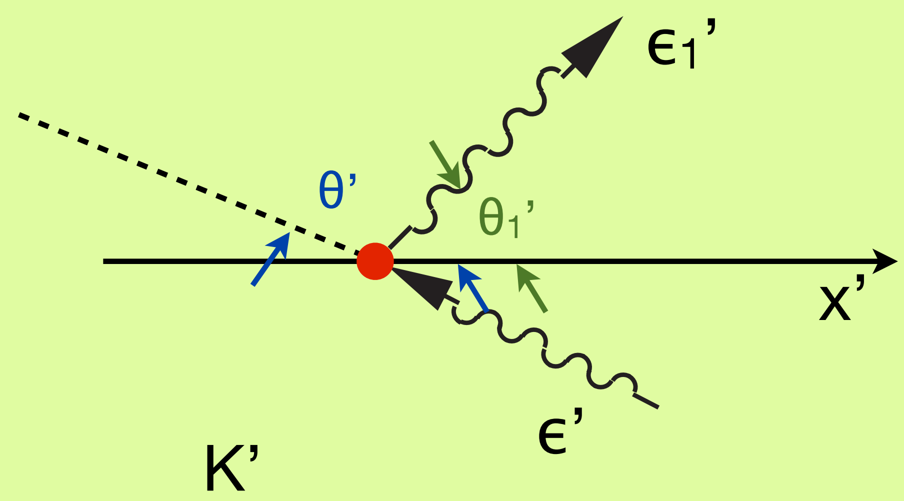
$4\gamma h\nu << mc^2$ $\quad \beta \approx 1$

$$h\nu_s \approx \frac{h\nu}{4\gamma^2}$$

Inverse Compton

Full derivation power for single scattering



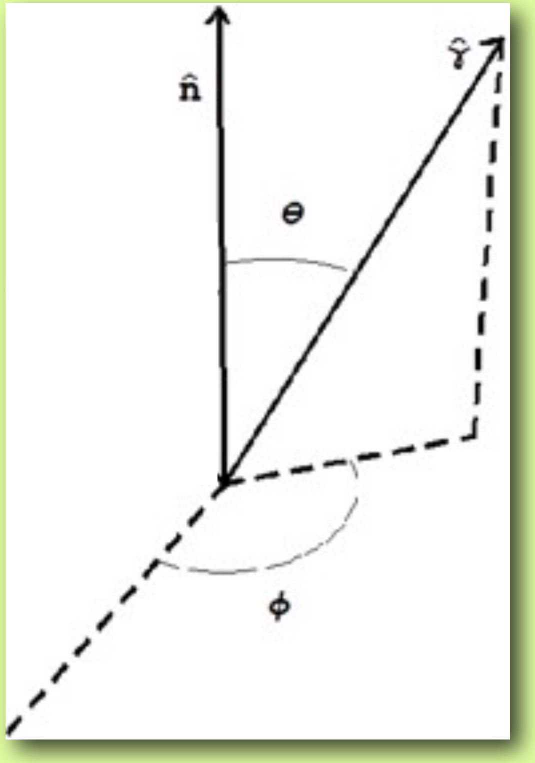


$$\epsilon' = \epsilon\gamma(1-\beta\cos\theta)$$

$$\epsilon_1 = \epsilon_1'\gamma(1+\beta\cos\theta_1')$$

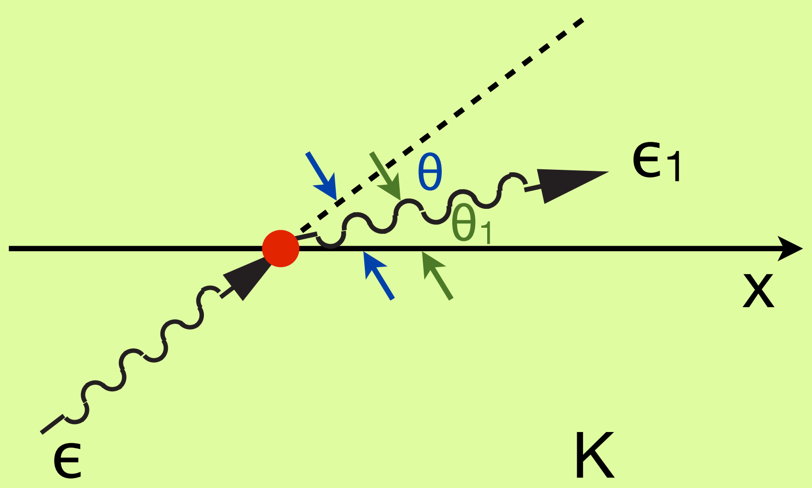
$$\epsilon_1' = \frac{\epsilon'}{1+\frac{\epsilon'}{mc^2}(1-\cos\Theta)} \approx \epsilon'\left[1-\frac{\epsilon'}{mc^2}(1-\cos\Theta)\right]$$

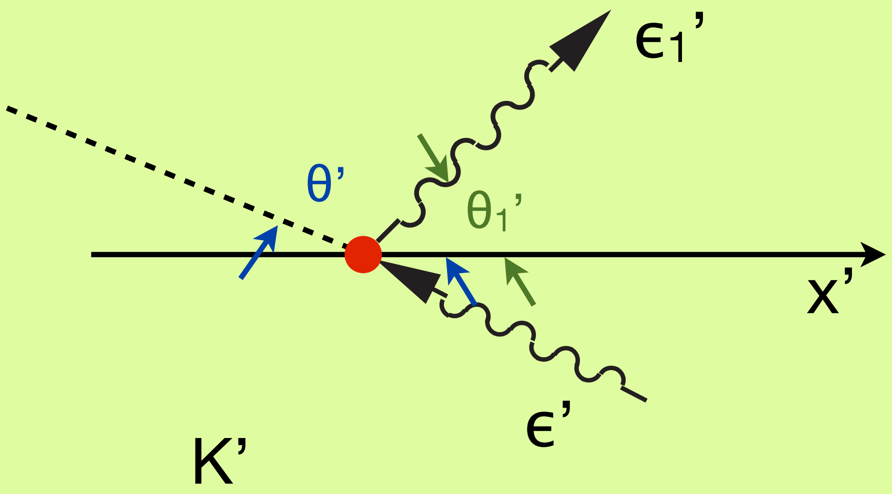
$$\cos\Theta = \cos\theta_1'\ \cos\theta' + \sin\theta_1'\ \sin\theta'\ \cos(\phi'-\phi_1')$$



Inverse Compton

Full derivation power for single scattering



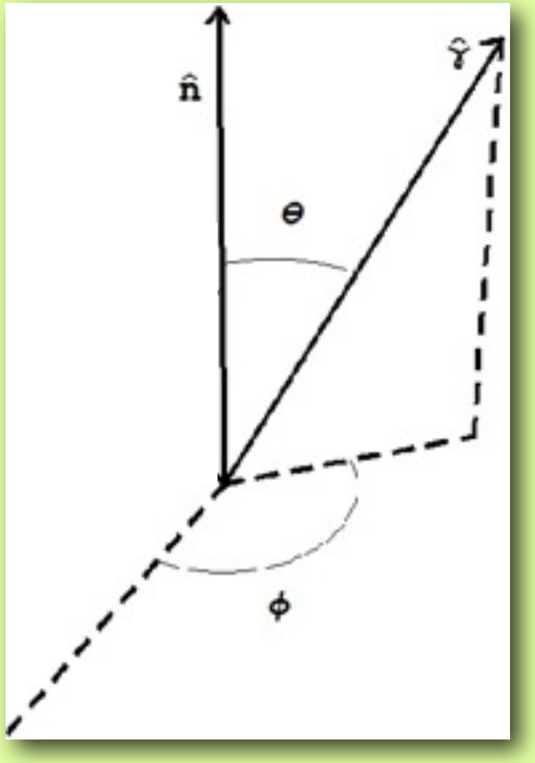


$$\epsilon' = \epsilon\gamma(1 - \beta\cos\theta)$$

$$\epsilon_1 = \epsilon_1'\gamma(1 + \beta\cos\theta_1')$$

$$\epsilon_1' = \frac{\epsilon'}{1 + \frac{\epsilon'}{mc^2}(1 - \cos\Theta)} \approx \epsilon'\left[1 - \frac{\epsilon'}{mc^2}(1 - \cos\Theta)\right]$$

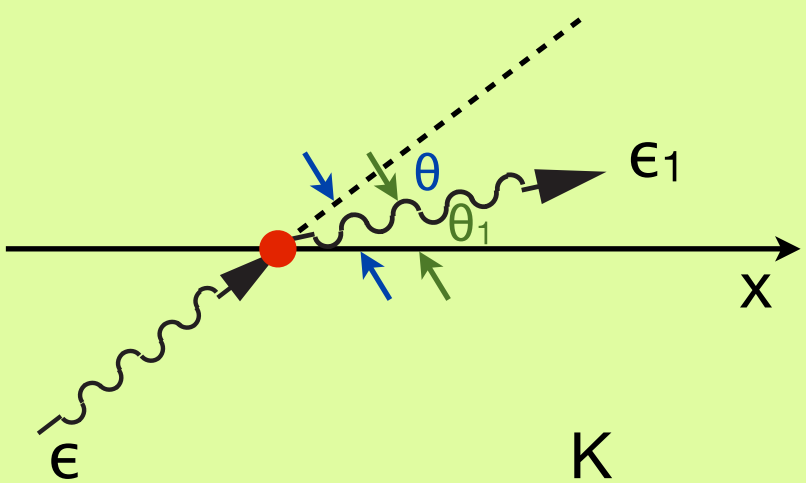
$$\cos\Theta = \cos\theta_1' \, \cos\theta' + \sin\theta_1' \, \sin\theta' \, \cos(\phi' - \phi_1')$$

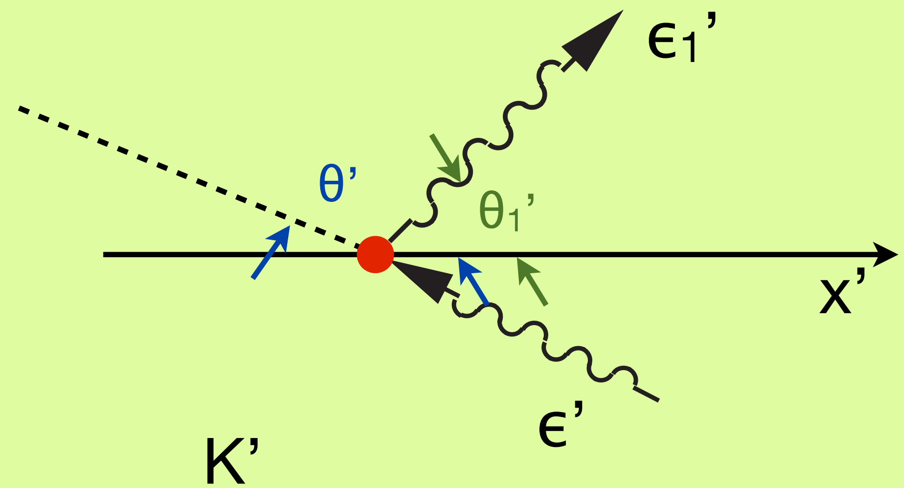


$$\epsilon' \approx \gamma\epsilon << mc^2$$ → $$\epsilon_1' \approx \epsilon'$$

Inverse Compton

Full derivation power for single scattering



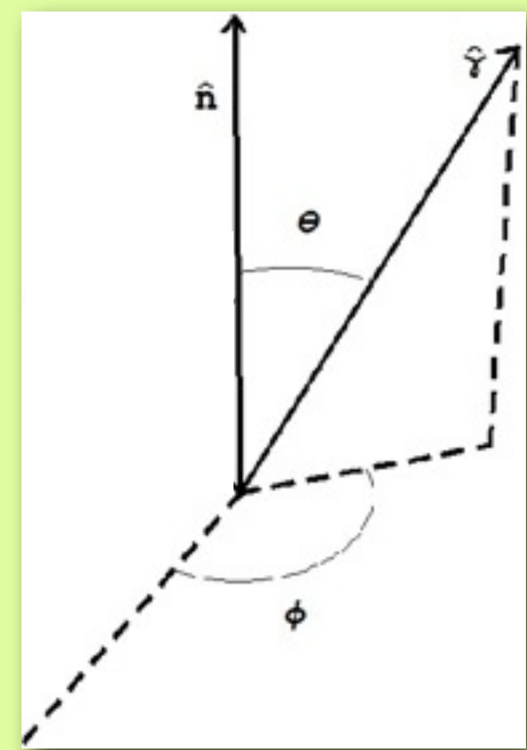


$$\epsilon' = \epsilon\gamma(1 - \beta\cos\theta)$$

$$\epsilon_1 = \epsilon_1'\gamma(1 + \beta\cos\theta_1')$$

$$\epsilon_1' = \frac{\epsilon'}{1 + \frac{\epsilon'}{mc^2}(1 - \cos\Theta)} \approx \epsilon' \left[1 - \frac{\epsilon'}{mc^2}(1 - \cos\Theta)\right]$$

$$\cos\Theta = \cos\theta_1' \, \cos\theta' + \sin\theta_1' \, \sin\theta' \, \cos(\phi' - \phi_1')$$



$$\epsilon' \approx \gamma\epsilon << mc^2$$ → $$\epsilon_1' \approx \epsilon'$$

$$\epsilon \; : \; \epsilon' \; : \; \epsilon_1 \equiv 1 \; : \; \gamma \; : \; \gamma^2$$

Inverse Compton

Full derivation power for single scattering

$$v \; d\epsilon = n \; dp^3$$

$$v \; d\epsilon \; = \; \text{density of photons having energy in the range } d\epsilon$$

$$n(p) \; = \; \text{photon phase distribution funtion}$$

$$\frac{v \; d\epsilon}{\epsilon} \; = \; \frac{v' \; d\epsilon'}{\epsilon'} \; = \; \text{Lorentz invariant}$$

Inverse Compton

Full derivation power for single scattering

$$\upsilon \, \mathrm{d}\epsilon \; = \; \text{density of photons having energy in the range } \mathrm{d}\epsilon$$

$$\mathcal{P} = \mathcal{F}_P \, \sigma_T = c \, U_{\mathrm{rad}} \, \sigma_T$$

Larmor's formula
Instantaneous emitted power
[W]

Poynting vector
Energy flux density
[W/m^2]

Energy density of
incident radiation
[W s /m^3]

$$\frac{\mathrm{d}E_1'}{\mathrm{d}t'} = c \, \sigma_T \, U_{\mathrm{rad}}' = c \, \sigma_T \int \epsilon_1' \upsilon' \mathrm{d}\epsilon'$$

Total power emitted (i.e. scattered) in the electron's rest frame

Inverse Compton

Full derivation power for single scattering

$$v \, \mathrm{d}\epsilon \; = \; \text{density of photons having energy in the range } \mathrm{d}\epsilon$$

$$\mathcal{P} = \mathcal{F}_P \, \sigma_T = c \, U_{\mathrm{rad}} \, \sigma_T$$

Larmor's formula
Instantaneous emitted power
[W]

Poynting vector
Energy flux density
[W/m^2]

Energy density of incident radiation
[W s /m^3]

$$\frac{\mathrm{d}E_1'}{\mathrm{d}t'} = c \, \sigma_T \, U_{\mathrm{rad}}' = c \, \sigma_T \int \epsilon_1' v' \mathrm{d}\epsilon'$$

Total power emitted (i.e. scattered) in the electron's rest frame

$$\epsilon_1' \approx \epsilon'$$

$(\epsilon' \approx \gamma\epsilon << mc^2)$ Thomson condition in the rest frame

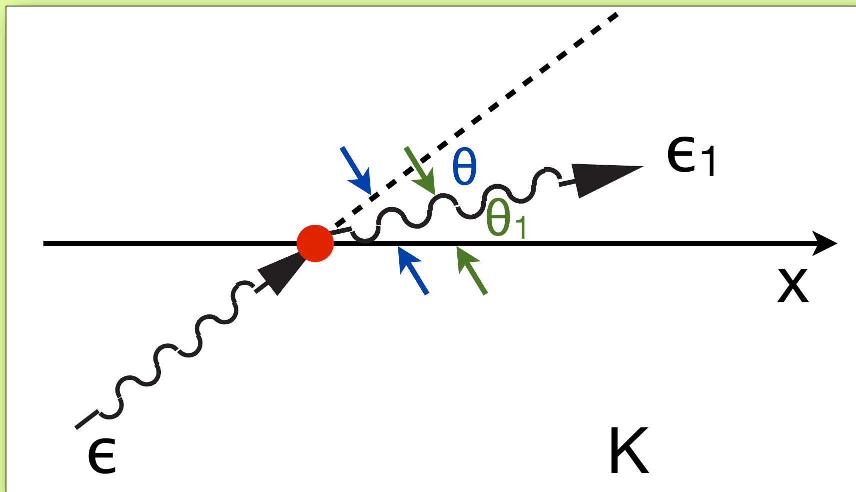
$(\gamma^2 - 1 >> \epsilon/mc^2)$ Photon energy change in the rest frame << Photon energy change in the lab frame

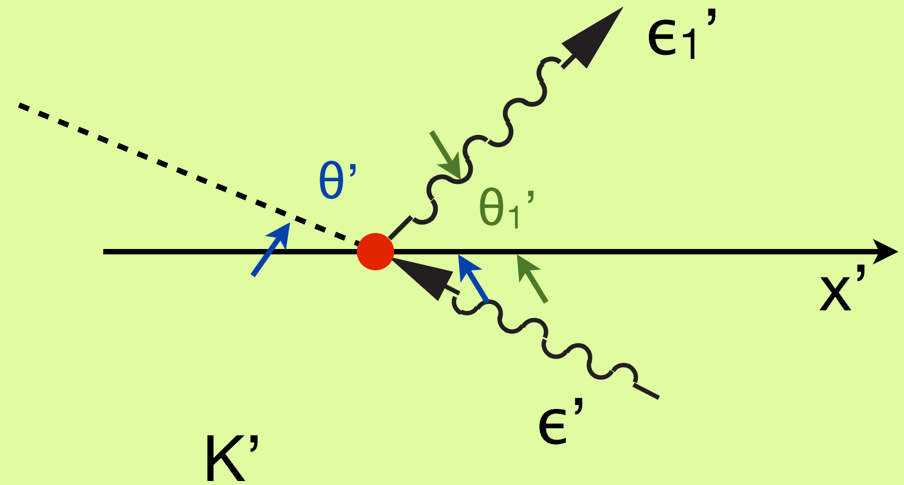
$$\frac{\mathrm{d}E_1}{\mathrm{d}t} = \frac{\mathrm{d}E_1'}{\mathrm{d}t'}$$

$$\frac{\mathrm{d}E_1}{\mathrm{d}t} = c \, \sigma_T \int \epsilon' v' \mathrm{d}\epsilon' = c \, \sigma_T \int \frac{\epsilon'}{\epsilon'} \, \epsilon' v' \mathrm{d}\epsilon' = c \, \sigma_T \int \epsilon'^2 \, \frac{v' \mathrm{d}\epsilon'}{\epsilon'} = c \, \sigma_T \int \epsilon'^2 \, \frac{v \mathrm{d}\epsilon}{\epsilon}$$

Inverse Compton

Full derivation power for single scattering





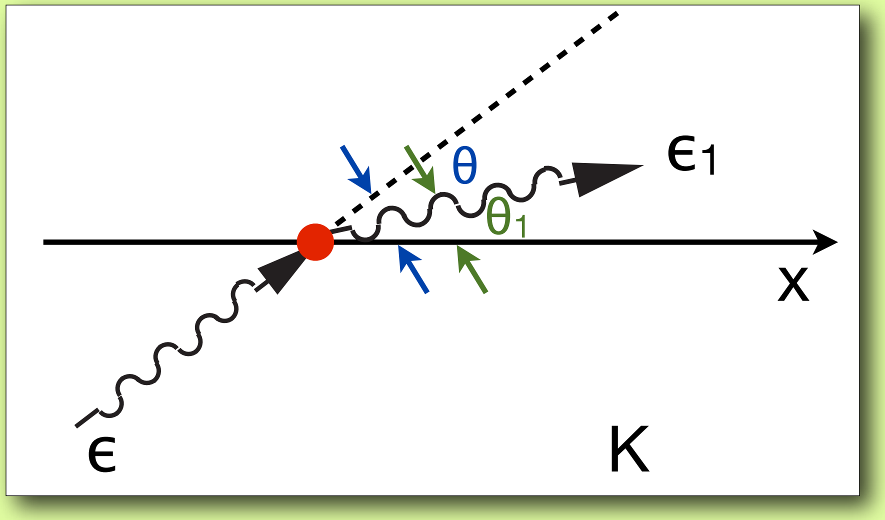
$$\frac{\mathrm{d}E_1}{\mathrm{d}t} = c\ \sigma_T \int \epsilon' \upsilon' \mathrm{d}\epsilon' = c\ \sigma_T \int \frac{\epsilon'}{\epsilon'}\ \epsilon' \upsilon' \mathrm{d}\epsilon' = c\ \sigma_T \int \epsilon'^2\ \frac{\upsilon' \mathrm{d}\epsilon'}{\epsilon'} = c\ \sigma_T \int \epsilon'^2\ \frac{\upsilon \mathrm{d}\epsilon}{\epsilon}$$

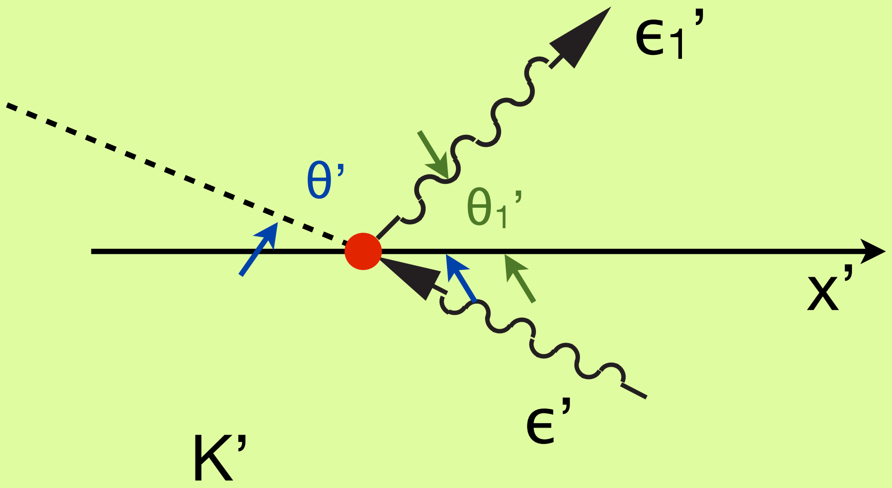
$$\epsilon' = \epsilon\gamma(1 - \beta\cos\theta)$$

$$\frac{\mathrm{d}E_1}{\mathrm{d}t} = c\ \sigma_T\ \gamma^2 \int (1 - \beta\ \cos\theta)^2 \epsilon\ \upsilon\ \mathrm{d}\epsilon$$

Inverse Compton

Full derivation power for single scattering





$$\frac{\mathrm{d}E_1}{\mathrm{d}t} = c\ \sigma_T \int \epsilon' \upsilon' \mathrm{d}\epsilon' = c\ \sigma_T \int \frac{\epsilon'}{\epsilon'}\ \epsilon' \upsilon' \mathrm{d}\epsilon' = c\ \sigma_T \int \epsilon'^2\ \frac{\upsilon' \mathrm{d}\epsilon'}{\epsilon'} = c\ \sigma_T \int \epsilon'^2\ \frac{\upsilon \mathrm{d}\epsilon}{\epsilon}$$

$$\epsilon' = \epsilon\gamma(1-\beta\cos\theta)$$

$$\frac{\mathrm{d}E_1}{\mathrm{d}t} = c\ \sigma_T\ \gamma^2 \int (1-\beta\ \cos\theta)^2 \epsilon\ \upsilon\ \mathrm{d}\epsilon$$

$$< \cos\theta >= 0 \qquad < \cos^2\theta >= \frac{1}{3} \qquad \text{(isotropic photon distribution)}$$

$$< (1-\beta\ \cos\theta)^2 >=< 1 - 2\ \beta\ \cos\theta + \beta^2\ \cos\theta^2 >= 1 + \frac{1}{3}\ \beta^2$$

Inverse Compton

Full derivation power for single scattering

$$\frac{\mathrm{d}E_1}{\mathrm{d}t} = c\ \sigma_T\ \gamma^2\ \int (1-\beta\ \cos\theta)^2 \epsilon\ \upsilon\ \mathrm{d}\epsilon \qquad U_{\mathrm{rad}} \equiv \int \epsilon\ \upsilon\ \mathrm{d}\epsilon$$

$$< (1-\beta\ \cos\theta)^2 > = < 1 - 2\ \beta\ \cos\theta + \beta^2\ \cos\theta^2 > = 1 + \frac{1}{3}\ \beta^2$$

$$\frac{\mathrm{d}E_1}{\mathrm{d}t} = c\ \sigma_T\ \gamma^2\ \left(1 + \frac{1}{3}\beta^2\right) U_{\mathrm{rad}}$$

Total power in the radiation field after IC upscattering of low-energy photons

Inverse Compton

Full derivation power for single scattering

$$\frac{\mathrm{d}E_1}{\mathrm{d}t} = c\ \sigma_T\ \gamma^2 \int (1 - \beta\ \cos\theta)^2 \epsilon\ \upsilon\ \mathrm{d}\epsilon \qquad U_{\mathrm{rad}} \equiv \int \epsilon\ \upsilon\ \mathrm{d}\epsilon$$

$$< (1 - \beta\ \cos\theta)^2 > = < 1 - 2\ \beta\ \cos\theta + \beta^2\ \cos\theta^2 > = 1 + \frac{1}{3}\ \beta^2$$

$$\frac{\mathrm{d}E_1}{\mathrm{d}t} = c\ \sigma_T\ \gamma^2 \left(1 + \frac{1}{3}\beta^2\right) U_{\mathrm{rad}} \longrightarrow$$

Total power in the radiation field after IC upscattering of low-energy photons

$$\frac{\mathrm{d}E_1}{\mathrm{d}t} = -c\ \sigma_T\ U_{\mathrm{rad}} \longrightarrow$$

Initial power of photons in the radiation field

$$\frac{\mathrm{d}E_{\mathrm{rad}}}{\mathrm{d}t} = c\ \sigma_T\ U_{\mathrm{rad}} \left[\gamma^2 \left(1 + \frac{1}{3}\beta^2\right) - 1\right]$$

Net power lost by the electron and converted into increased radiation

Inverse Compton

Full derivation power for single scattering

$$\frac{\mathrm{d}E_{\rm rad}}{\mathrm{d}t} = c\ \sigma_T\ U_{\rm rad}\left[\gamma^2 + \frac{1}{3}\gamma^2\beta^2 - 1\right] = c\ \sigma_T\ U_{\rm rad}\left[(\gamma^2 - 1) + \frac{1}{3}\gamma^2\beta^2\right] = c\ \sigma_T\ U_{\rm rad}\left[\gamma^2\beta^2 + \frac{1}{3}\gamma^2\beta^2\right]$$

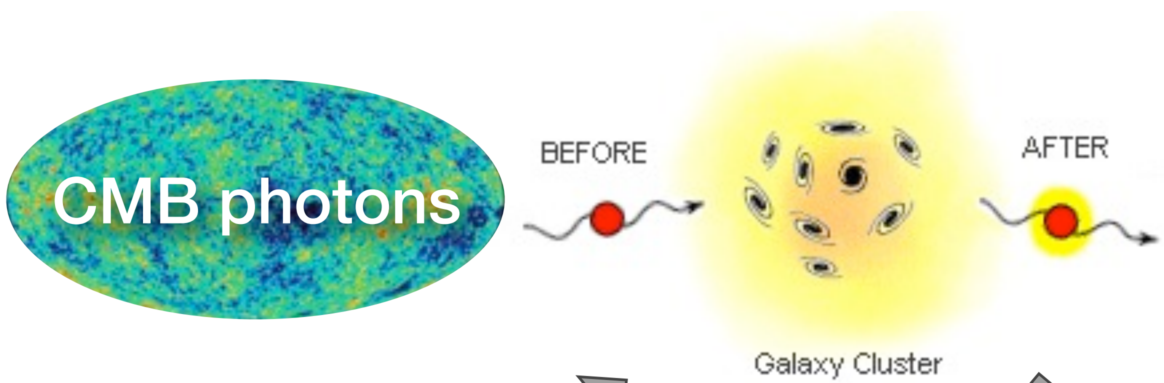
$$\frac{\mathrm{d}E_{\rm rad}}{\mathrm{d}t} = P_{\rm compt} = \frac{4}{3}\sigma_T\ c\ \gamma^2\ \beta^2\ U_{\rm rad}$$

Net power lost by the relativistic electron
and converted into increased radiation

$$\frac{\mathrm{d}E_{\rm rad}}{\mathrm{d}t} = \frac{4}{3}\sigma_T\ c\ \gamma^2 \int \epsilon\ \upsilon\ \mathrm{d}\epsilon \quad (\beta \approx 1)$$

$$\frac{\mathrm{d}E_{\rm rad}}{\mathrm{d}t} = \frac{4}{3}\sigma_T\ c\ \gamma^2\ < \epsilon > \int \upsilon\ \mathrm{d}\epsilon = \sigma_T\ c\ < \epsilon_1 > \int \upsilon\ \mathrm{d}\epsilon$$

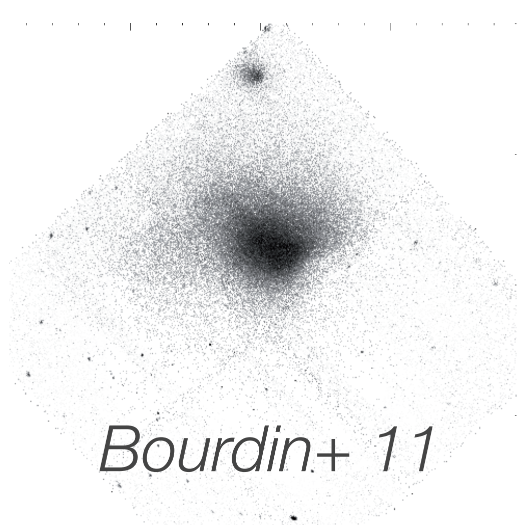
Thermal Sunyaev-Zel'dovich effect



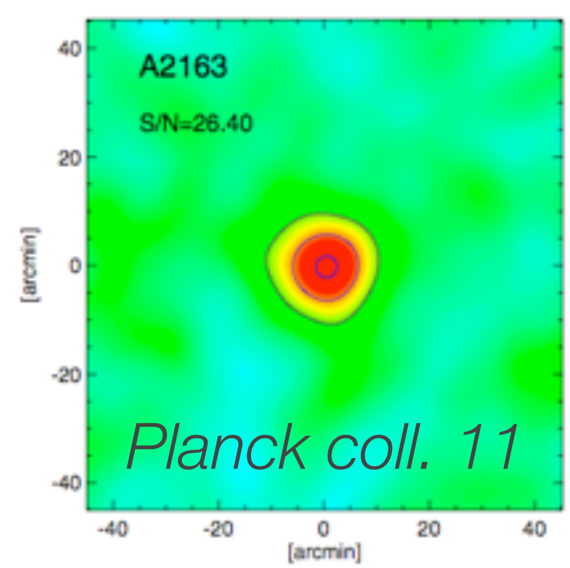
Inverse Compton effect

X-ray emission:
thermal intracluster medium

Thermal SZ effect

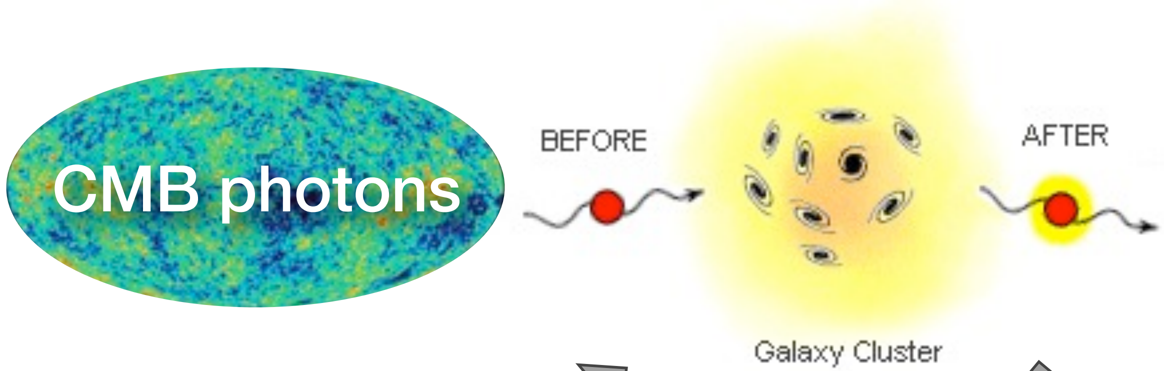


Bourdin+ 11



Planck coll. 11

Thermal Sunyaev-Zel'dovich effect



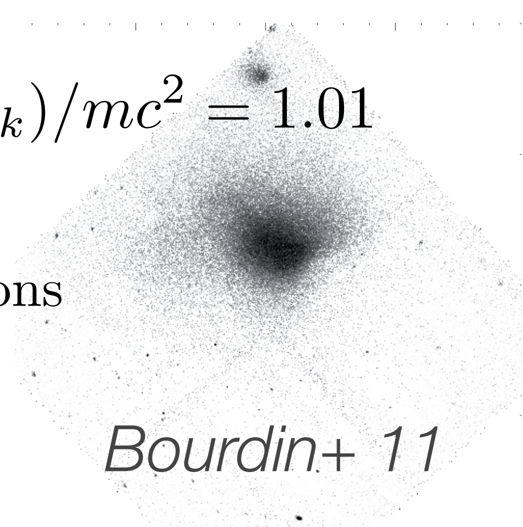
Inverse Compton effect

X-ray emission:
thermal intracluster medium

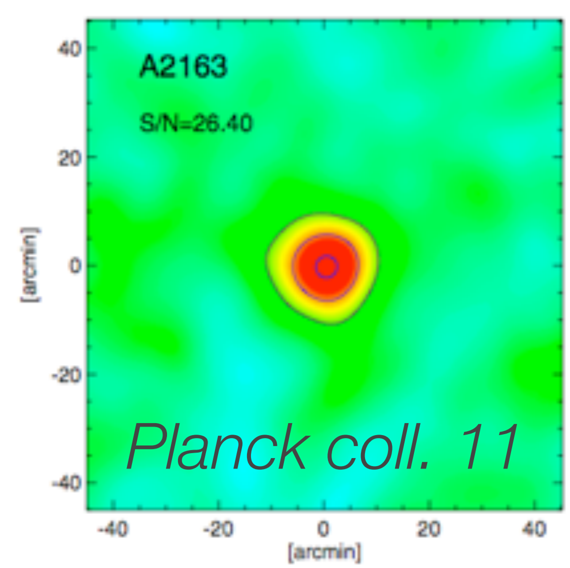
Thermal SZ effect

$$\gamma = (mc^2 + E_k)/mc^2 = 1.01$$

≈ 5 keV electrons



Bourdin+ 11



Planck coll. 11

$$T_r = 2.73 \text{ K}$$

$$h\nu_{\rm av} = 2.70\ kT = 6.4 \times 10^{-4}\text{eV}$$

Inverse Compton scattering

$$h\nu' = \left(\frac{1+\beta}{1-\beta}\right)^{1/2} h\nu$$ First Doppler shift

$$h\nu'_s = \frac{h\nu'}{1+\frac{2h\nu'}{mc^2}}$$ Compton scatter in S'

$$h\nu_s = \left(\frac{1+\beta}{1-\beta}\right)^{1/2} h\nu'_s$$ Second Doppler shift

$$\left(\frac{1+\beta}{1-\beta}\right)^{1/2} = \left(\frac{1+\beta}{1-\beta}\right)^{1/2} \times \left(\frac{1+\beta}{1+\beta}\right)^{1/2} = \frac{1+\beta}{(1-\beta^2)^{1/2}} \xrightarrow[\beta \approx 1]{} 2\gamma$$

$$\rightarrow h\nu_s \approx 2\gamma h\nu'_s = 2\gamma \frac{h\nu'}{1+\frac{2h\nu'}{mc^2}} \approx 4\gamma^2 \frac{h\nu}{1+\frac{4\gamma h\nu}{mc^2}}$$

Thermal Sunyaev-Zel'dovich effect

Sunyaev-Zel'dovich Effect

Isotropy of CMB →

total number of photons arriving at the observer unchanged

BUT

some of them have undergone scattering

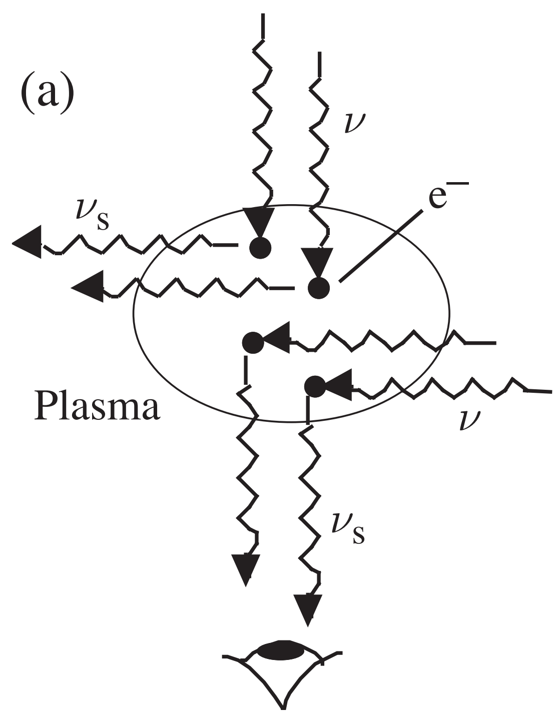


Fig.9.7: Astrophysics Processes (CUP), © H. Bradt 2008
(b,d) R. Sunyaev & Y. Zel'dovich in ARAA 18, 537 (1980)
(c) E. L. Wright, pvt. comm.

Energy transfer in thermal ICM
Compton parameter

$y \equiv$ (average fractional energy change per scattering) $\times$ (mean numbers of scattering)

$$\frac{\Delta\epsilon}{\epsilon} = -\frac{\epsilon}{mc^2} + \frac{\alpha kT}{mc^2}$$

$$\tau_{\rm ICM} \int_0^L n_e \sigma_T \mathrm{d}l$$

Energy transfer in thermal ICM
Compton parameter

$y \equiv$ (average fractional energy change per scattering) $\times$ (mean numbers of scattering)

$$\frac{\Delta\epsilon}{\epsilon} = -\frac{\epsilon}{mc^2} + \frac{\alpha kT}{mc^2}$$

$$\epsilon_1 = \frac{\epsilon}{1 + \frac{\epsilon}{mc^2}(1 - \cos\theta)} \approx \epsilon \left[1 - \frac{\epsilon}{mc^2}(1 - \cos\theta)\right]$$

$$\frac{\epsilon_1 - \epsilon}{\epsilon} = \frac{\epsilon \left[1 - \frac{\epsilon}{mc^2}(1 - \cos\theta)\right] - \epsilon}{\epsilon} = 1 - \frac{\epsilon}{mc^2}(1 - \cos\theta) - 1$$

$$< \cos\theta >= 0 \quad \rightarrow \quad \left\langle \frac{\Delta\epsilon}{\epsilon} \right\rangle = -\frac{\epsilon}{mc^2}$$

Energy transfer in thermal ICM
Compton parameter

$y \equiv$ (average fractional energy change per scattering) $\times$ (mean numbers of scattering)

$$\frac{\Delta\epsilon}{\epsilon} = -\frac{\epsilon}{mc^2} + \frac{\alpha kT}{mc^2}$$

Head-on collisions

$$h\nu' = \left(\frac{1+\beta}{1-\beta}\right)^{1/2} h\nu$$ First Doppler shift

$$h\nu'_s = \frac{h\nu'}{1 + \frac{2h\nu'}{mc^2}}$$ Compton scatter in S'

$\epsilon' \approx \gamma\epsilon << mc^2$ → $\epsilon'_1 \approx \epsilon'$

$$h\nu_s = \left(\frac{1+\beta}{1-\beta}\right)^{1/2} h\nu'_s$$ Second Doppler shift

$$\left(\frac{\Delta\nu}{\nu}\right)_+ = \frac{\nu_S - \nu}{\nu} = \frac{2\beta}{1-\beta}$$

Energy transfer in thermal ICM

Compton parameter

$y \equiv$ (average fractional energy change per scattering) $\times$ (mean numbers of scattering)

$$\frac{\Delta\epsilon}{\epsilon} = -\frac{\epsilon}{mc^2} + \frac{\alpha kT}{mc^2}$$

Overtaking collisions

$$h\nu' = \left(\frac{1-\beta}{1+\beta}\right)^{1/2} h\nu$$

First Doppler shift

$$h\nu'_s = \frac{h\nu'}{1 + \frac{2h\nu'}{mc^2}}$$

Compton scatter in S'

$\epsilon' \approx \gamma\epsilon << mc^2$ → $\epsilon'_1 \approx \epsilon'$

$$h\nu_s = \left(\frac{1-\beta}{1+\beta}\right)^{1/2} h\nu'_s$$

Second Doppler shift

$$\left(\frac{\Delta\nu}{\nu}\right)_- = \frac{\nu_S - \nu}{\nu} = -\frac{2\beta}{1+\beta}$$

Energy transfer in thermal ICM
Compton parameter

$y \equiv$ (average fractional energy change per scattering) $\times$ (mean numbers of scattering)

$$\frac{\Delta\epsilon}{\epsilon} = -\frac{\epsilon}{mc^2} + \frac{\alpha kT}{mc^2}$$

Mean between head-on & overtaking collisions

$$\left(\frac{\Delta\nu}{\nu}\right)_+ = \frac{\nu_S - \nu}{\nu} = \frac{2\beta}{1-\beta} \qquad \left(\frac{\Delta\nu}{\nu}\right)_- = \frac{\nu_S - \nu}{\nu} = -\frac{2\beta}{1+\beta}$$

$$\left(\frac{\Delta\nu}{\nu}\right)_{\rm av} = \frac{1}{2}\left[\left(\frac{\Delta\nu}{\nu}\right)_+ + \left(\frac{\Delta\nu}{\nu}\right)_-\right] = \cdots = 2\frac{\beta^2}{1-\beta^2} \leftarrow 2\beta^2 \quad (\beta^2 <<< 1)$$

$$\beta^2 = \frac{v^2}{c^2} \approx \frac{kT/m}{c^2} \qquad (< mv^2/2 >= 3kT/2)$$

Energy transfer in thermal ICM
Compton parameter

$y \equiv$ (average fractional energy change per scattering) $\times$ (mean numbers of scattering)

$$\frac{\Delta\epsilon}{\epsilon} = -\frac{\epsilon}{mc^2} + \frac{\alpha kT}{mc^2}$$

Photons & electrons are in thermal equilibrium & interact only through scatter:
no net energy transferred from photons to electrons < Δε>=0

$$< \epsilon >= \frac{\int \epsilon \frac{\mathrm{d}N}{\mathrm{d}\epsilon}\mathrm{d}\epsilon}{\int \frac{\mathrm{d}N}{\mathrm{d}\epsilon}\mathrm{d}\epsilon} = 3kT$$

$N(E)$ for thermal distribution of ultrarelativistic particles

$$< \epsilon^2 >= \frac{\int \epsilon^2 \frac{\mathrm{d}N}{\mathrm{d}\epsilon}\mathrm{d}\epsilon}{\int \frac{\mathrm{d}N}{\mathrm{d}\epsilon}\mathrm{d}\epsilon} = 12\ (k\ T)^2$$

$$< \Delta\epsilon >= -\frac{< \epsilon^2 >}{mc^2} + \frac{\alpha kT}{mc^2} < \epsilon >= \frac{3kT}{mc^2}(\alpha - 4)kT = 0 \rightarrow \alpha = 4$$

Energy transfer in thermal ICM
Compton parameter

$y \equiv$ (average fractional energy change per scattering) $\times$ (mean numbers of scattering)

$$\frac{\Delta\epsilon}{\epsilon} = -\frac{\epsilon}{mc^2} + \frac{\alpha kT}{mc^2}$$

$$(\Delta\epsilon)_{\rm NR} = \frac{\epsilon}{mc^2}(4kT - \epsilon)$$

$kT >> \epsilon$ — **Energy transferred from electrons to photons**

$kT << \epsilon$ — **Energy transferred from photons to electrons**

Energy transfer in thermal ICM
Compton parameter

$y \equiv$ (average fractional energy change per scattering) $\times$ (mean numbers of scattering)

$$(\Delta\epsilon)_{\mathrm{R}} \sim \frac{4}{3}\gamma^2\epsilon$$

$$<\gamma^2>=\frac{<\epsilon^2>}{(mc^2)^2}=12\left(\frac{kT}{mc^2}\right)^2 \qquad \left(<\epsilon^2>=\frac{\int \epsilon^2 \frac{\mathrm{d}N}{\mathrm{d}\epsilon}\mathrm{d}\epsilon}{\int \frac{\mathrm{d}N}{\mathrm{d}\epsilon}\mathrm{d}\epsilon}=12\ (k\ T)^2\right)$$

$$(\Delta\epsilon)_{\mathrm{R}} \sim 16\epsilon\left(\frac{kT}{mc^2}\right)^2$$

Energy transfer in thermal ICM

Compton parameter

$y \equiv$ (average fractional energy change per scattering) $\times$ (mean numbers of scattering)

$$\frac{\Delta\epsilon}{\epsilon} = -\frac{\epsilon}{mc^2} + \frac{\alpha kT}{mc^2}$$

$$(\Delta\epsilon)_{\rm NR} = \frac{\epsilon}{mc^2}(4kT - \epsilon)$$

$kT >> \epsilon$

$kT << \epsilon$

Energy transferred from electrons to photons

Energy transferred from photons to electrons

Energy transfer in thermal ICM
Compton parameter

$y \equiv$ (average fractional energy change per scattering) $\times$ (mean numbers of scattering)

$$\frac{\Delta\epsilon}{\epsilon} = -\frac{\epsilon}{mc^2} + \frac{\alpha kT}{mc^2} \qquad \tau_{\mathrm{ICM}} \int_0^L n_e \sigma_T \mathrm{d}l$$

Thermal Sunyaev-Zel'dovich effect

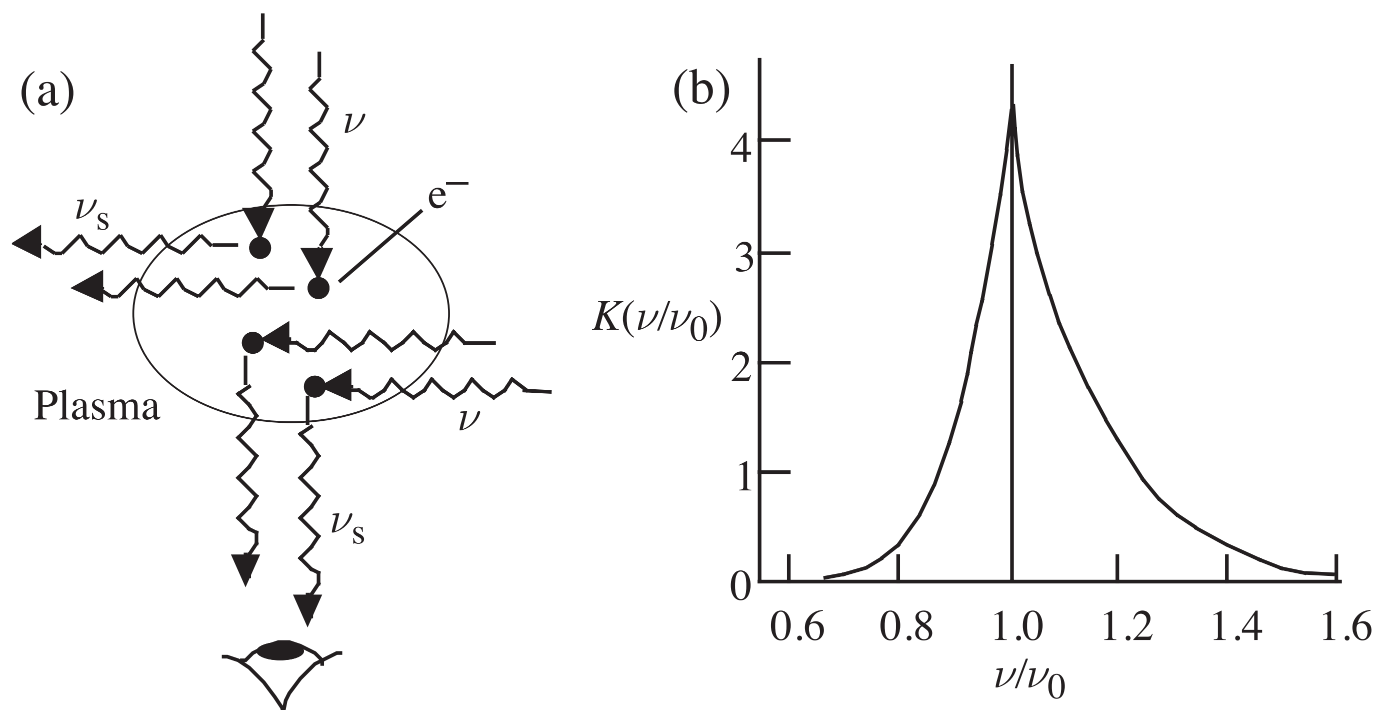
Isotropy of CMB →

total number of photons arriving at the observer unchanged

BUT

some of them have undergone scattering

Sunyaev-Zel'dovich Effect



$$\left(\frac{\Delta\nu}{\nu}\right)_{\rm av} = \frac{4kT_e}{mc^2}$$

Average fractional **frequency shift**

$$\left(\frac{\Delta\nu}{\nu}\right)_{\rm av} = \frac{4kT_e}{mc^2}\tau$$

Average fractional **photon frequency shift** in Maxwellian electron gas

$\tau \equiv$ probability of a scatter while photon in cluster for $\tau << 1$

Fig.9.7: Astrophysics Processes (CUP), © H. Bradt 2008
(b,d) R. Sunyaev & Y. Zel'dovich in ARAA 18, 537 (1980)
(c) E. L. Wright, pvt. comm.

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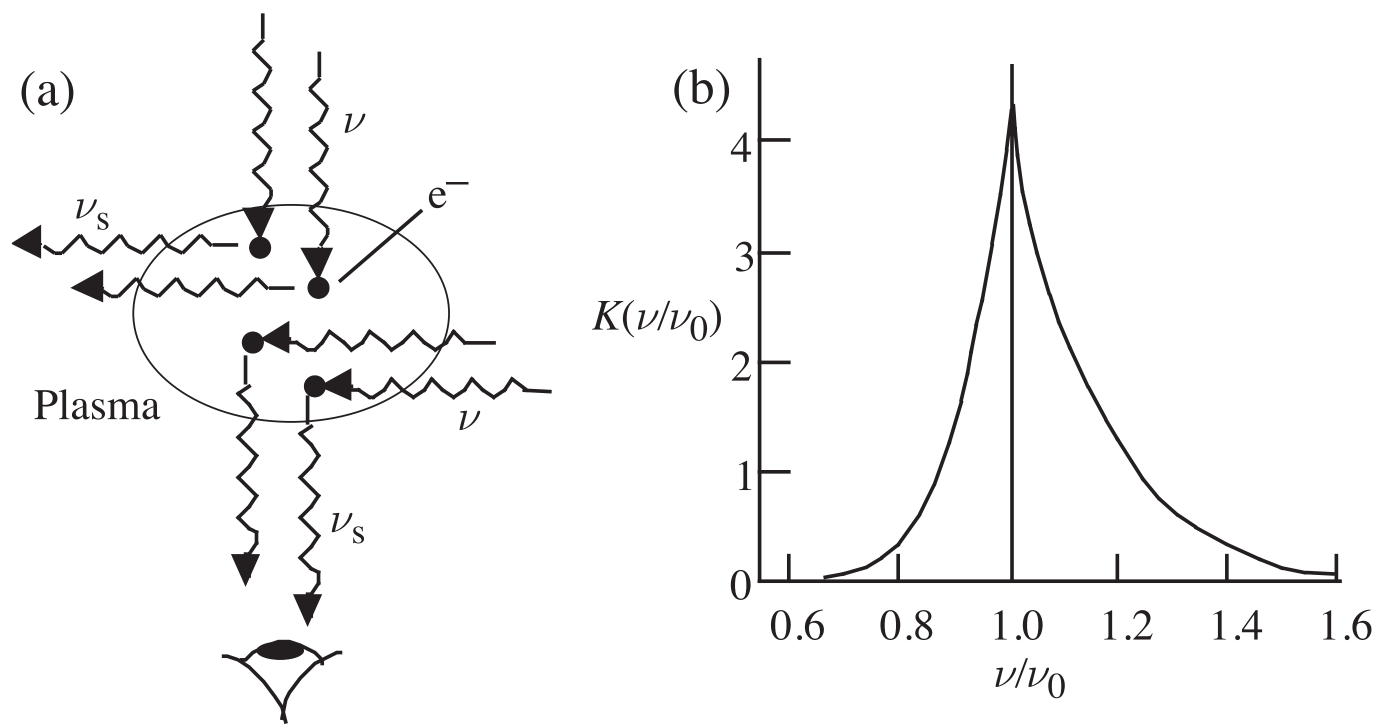

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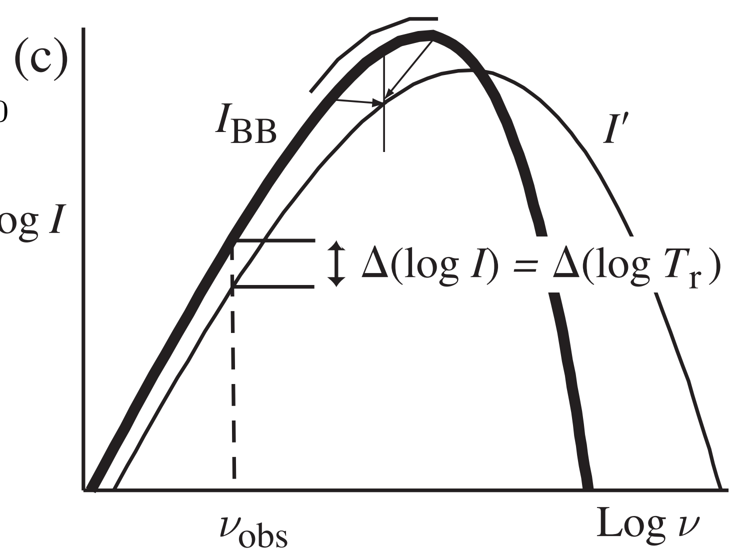
Average fractional **photon frequency shift** in Maxwellian electron gas

$$N_{\rm scatt}(\nu) = \int_0^\infty N(\nu_0) K\left(\frac{\nu}{\nu_0}\right) {\rm d}\nu_0$$

Conservation of the **photon number**

<u>*AND*</u>

distribution of them in **different frequency bins**



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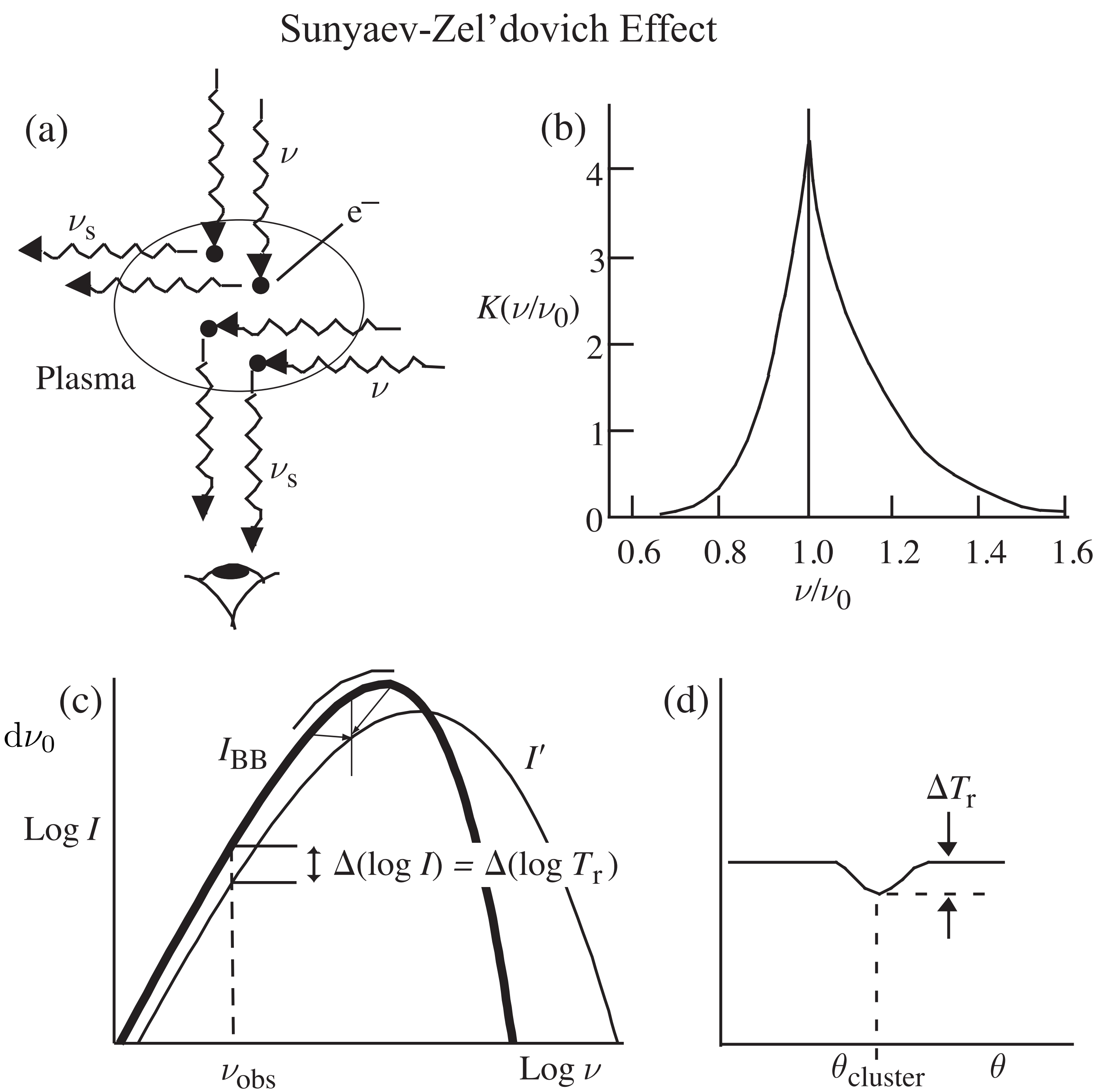

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AND

distribution of them in **different frequency bins**

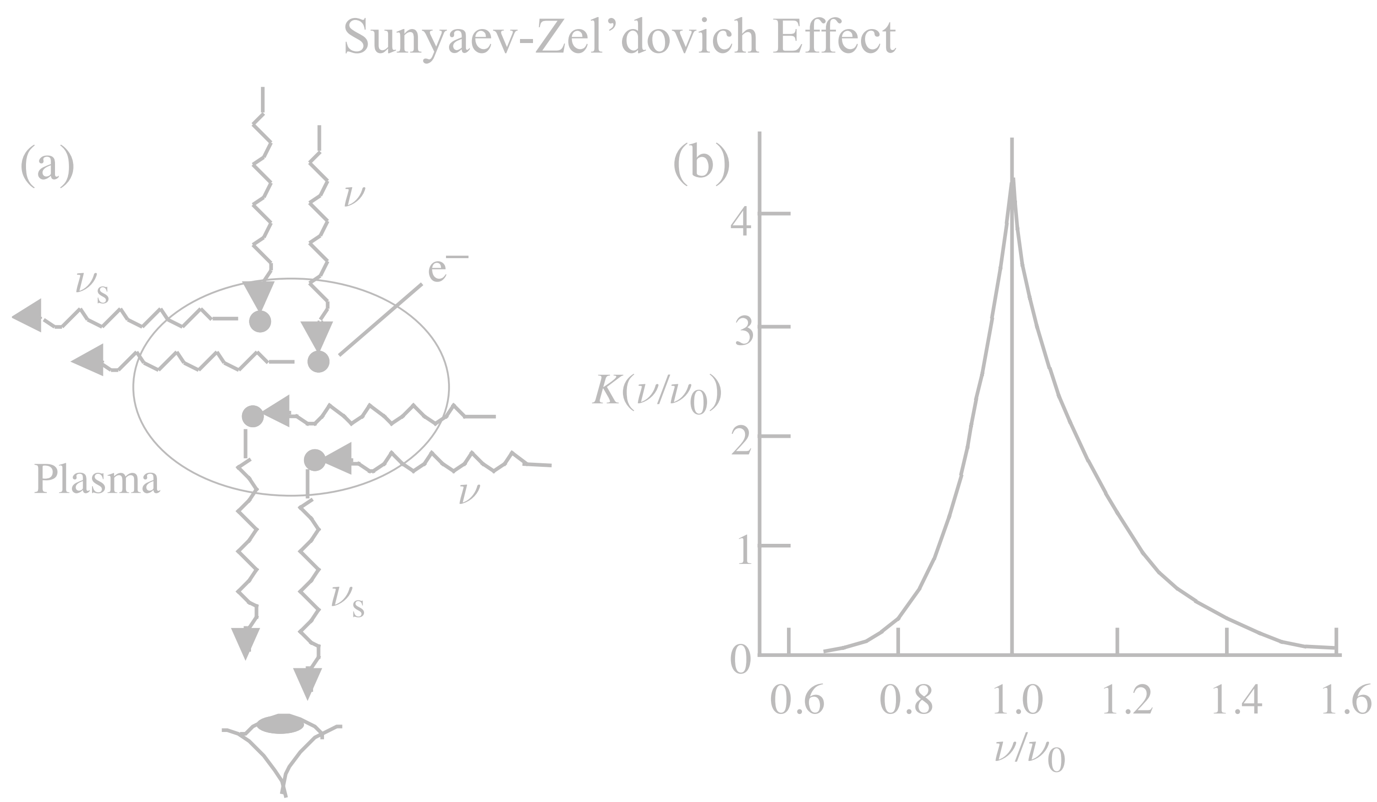
$$\frac{\Delta T_r}{T_r} \approx -2\frac{kT_e}{mc^2}\tau$$

$$\text{RJ regime}: \ h\nu << kT_r$$

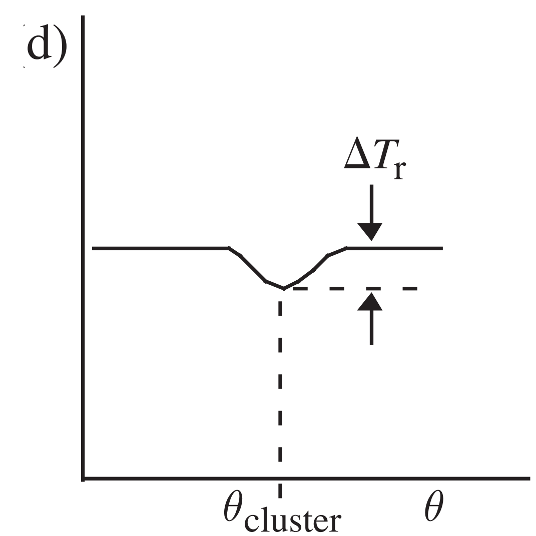
$$h\nu << kT_e << mc^2$$

$$\frac{\Delta T_r}{T_r} \approx \frac{\Delta I}{I}$$

Thermal Sunyaev-Zel'dovich effect



$$\frac{\Delta T_r}{T_r} \approx -2\frac{k\sigma_T}{mc^2}\int_0^L T_e n_e \mathrm{d}l$$



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$$\mathrm{h}\nu << \mathrm{kT_e} << \mathrm{mc}^2$$

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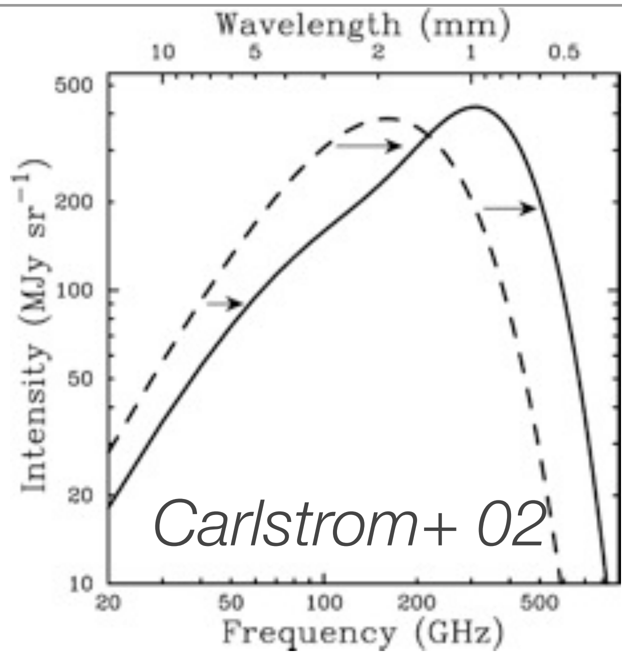
SZE:
IMPORTANCE FOR CLUSTER STUDIES



$$\frac{\Delta T_{SZE}}{T_{CMB}} = f(\nu)\, y$$

$$y \propto \int n_e T \, \mathrm{d}l$$

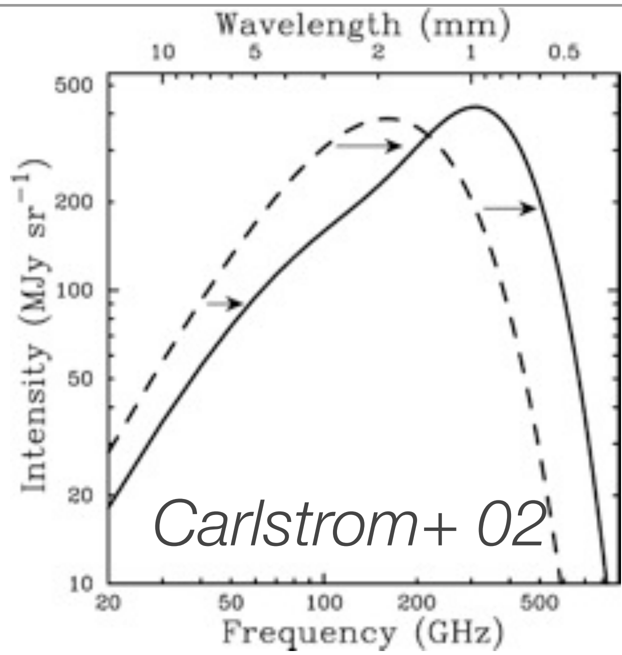
General case

(not only in RJ regime)

Cluster detection

Courtesy: M. Douspis
& Planck collaboration

SZE:
IMPORTANCE FOR CLUSTER STUDIES



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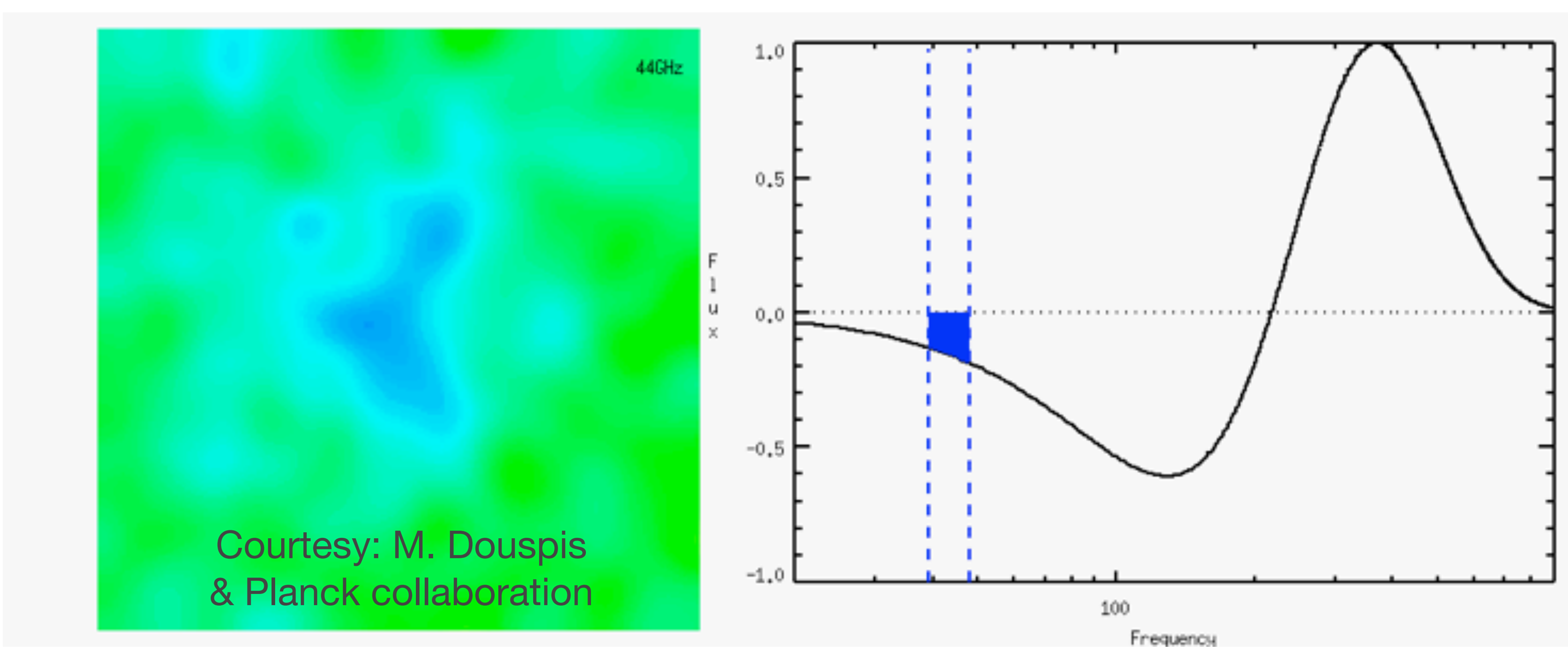
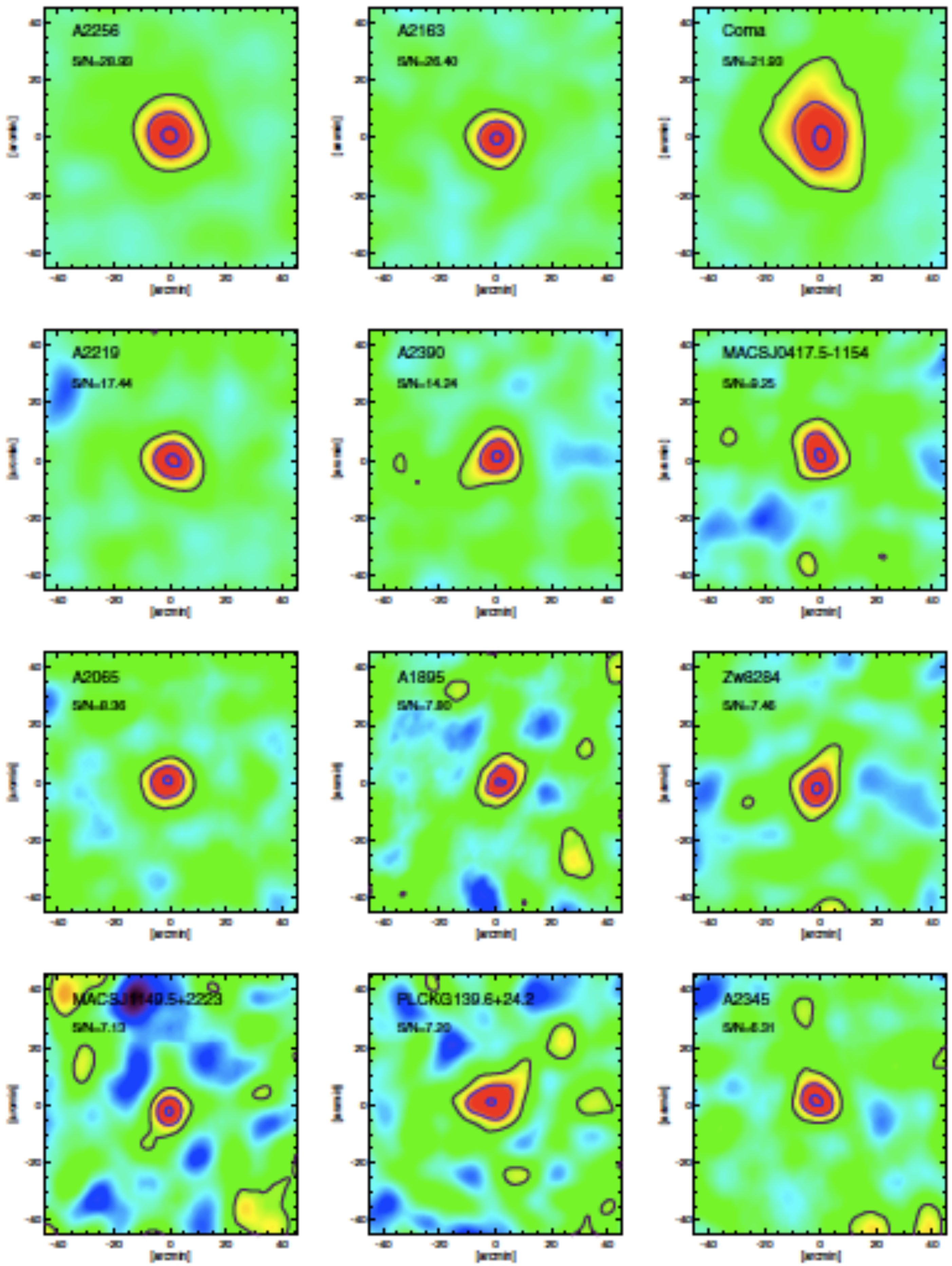
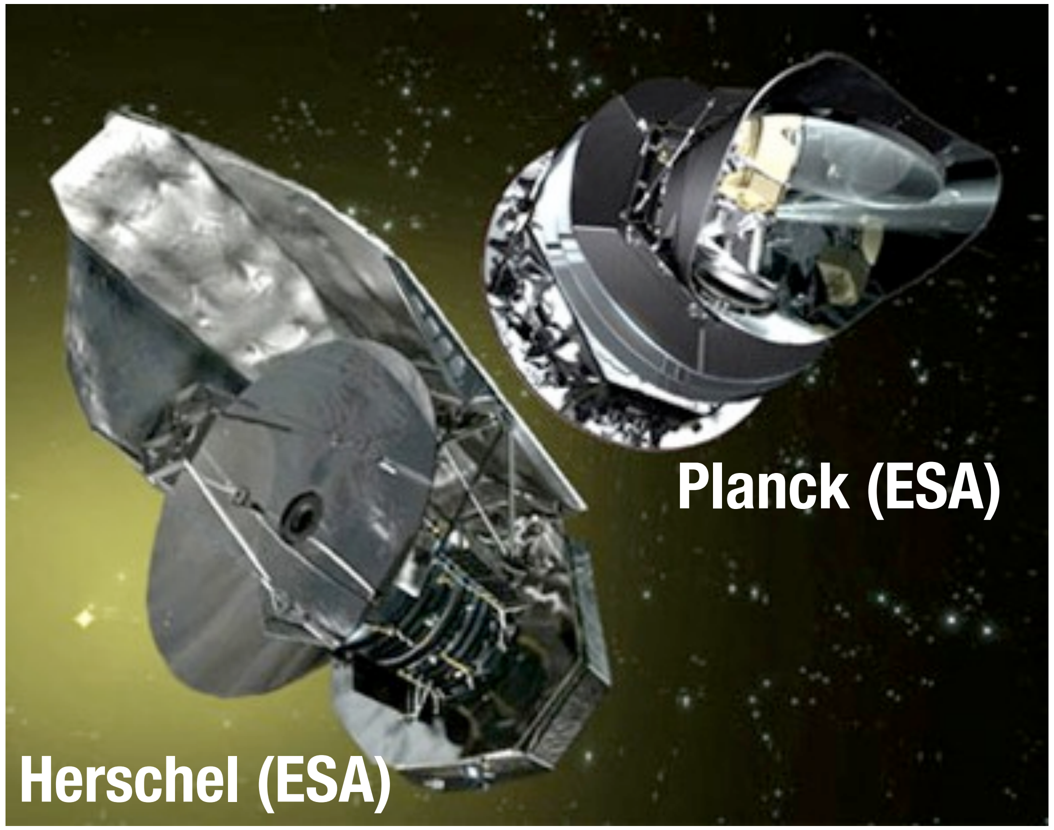




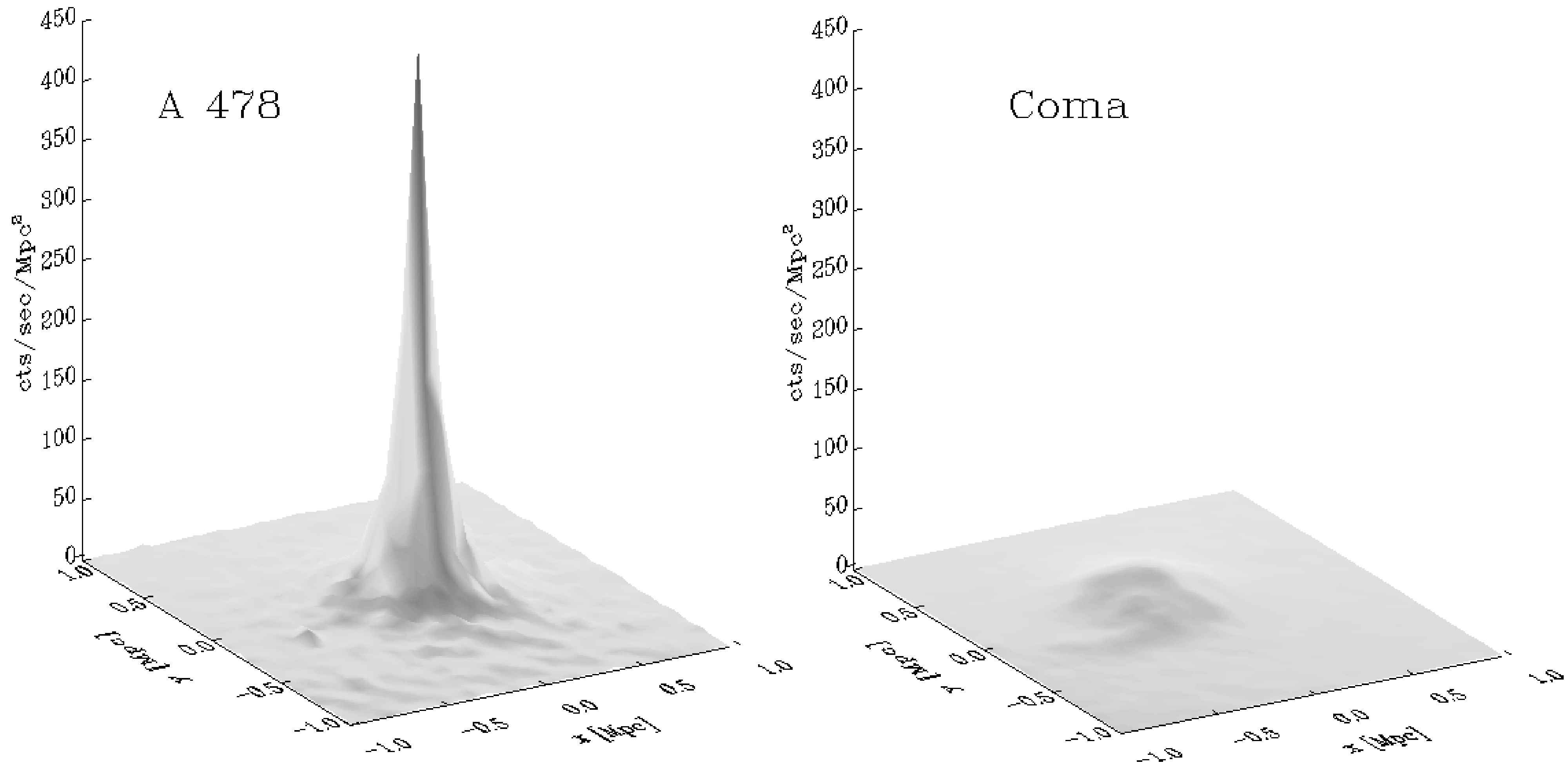


Planck collaboration 11 (A&A, 536, 8)

SZE:
importance for cluster studies

$$\text{X-ray emissivity} \propto \int n_e^2 T^{1/2} dl$$

ICM density



Fabian & Sanders 09

SZE: IMPORTANCE FOR CLUSTER STUDIES

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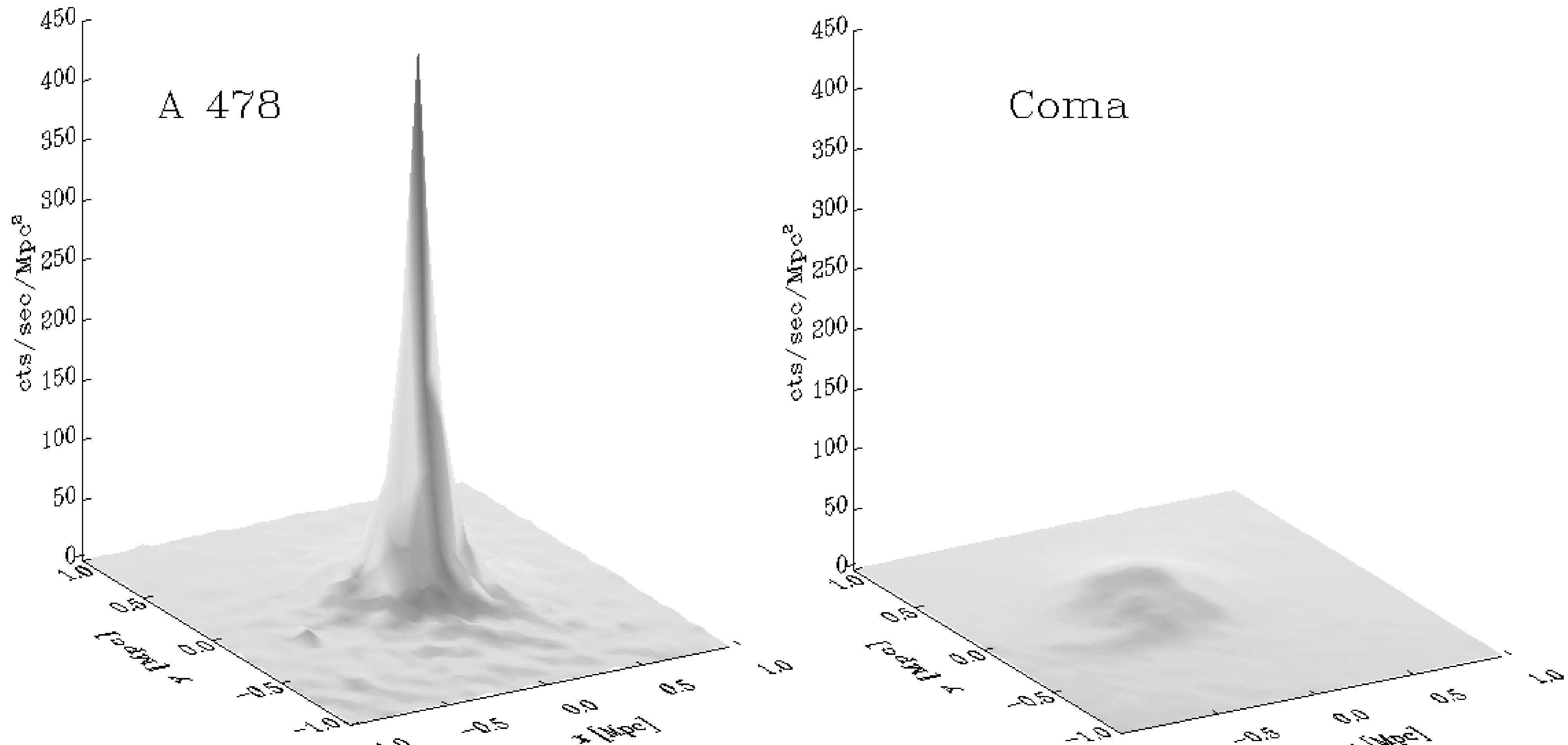
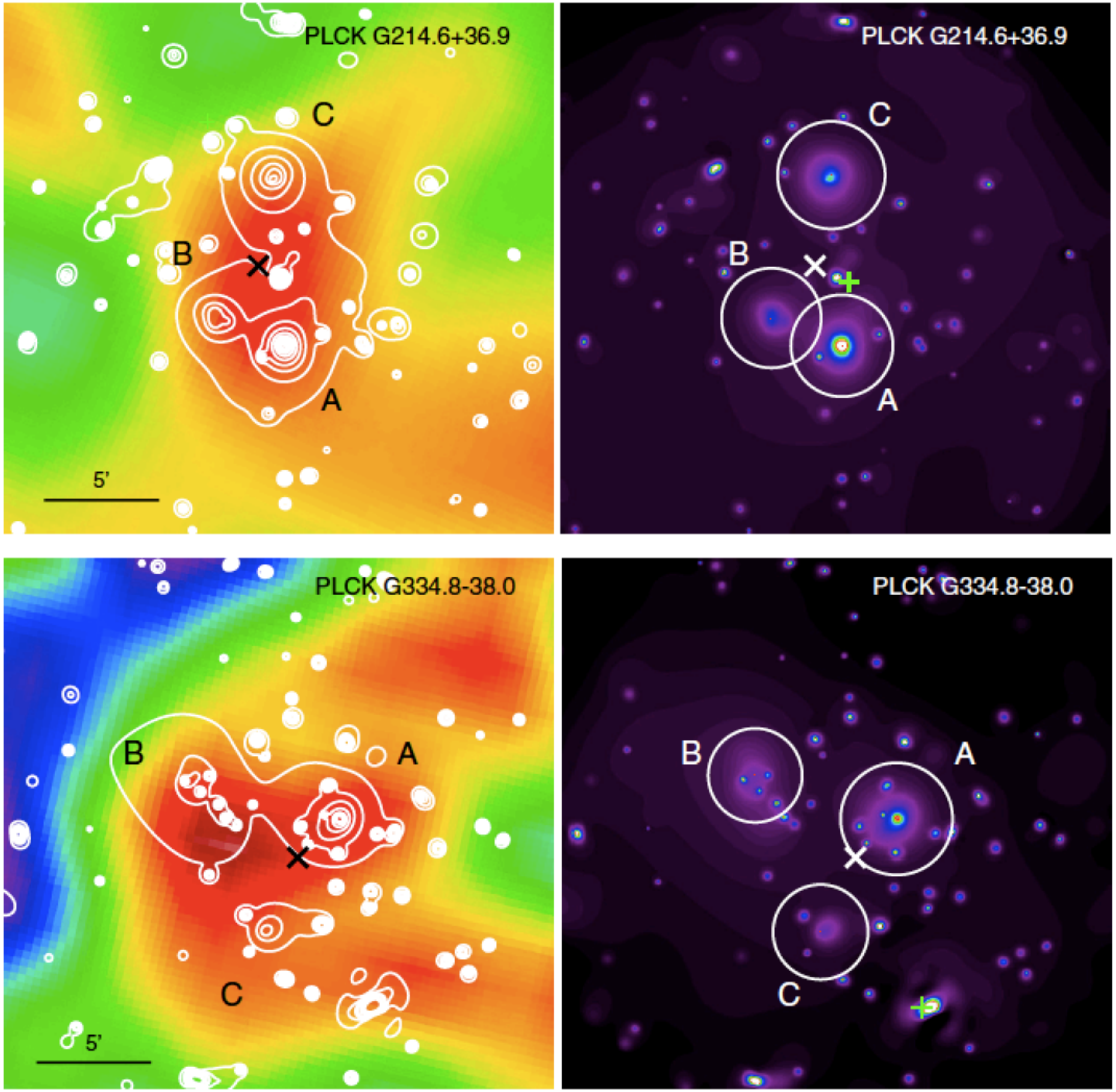
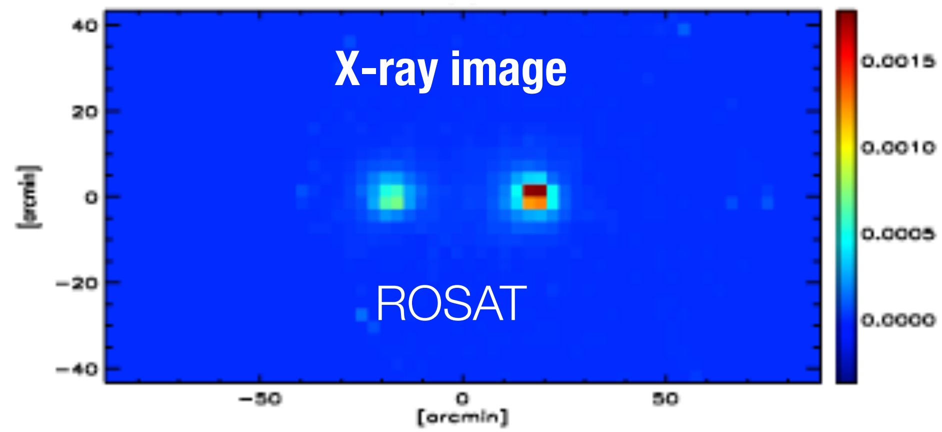
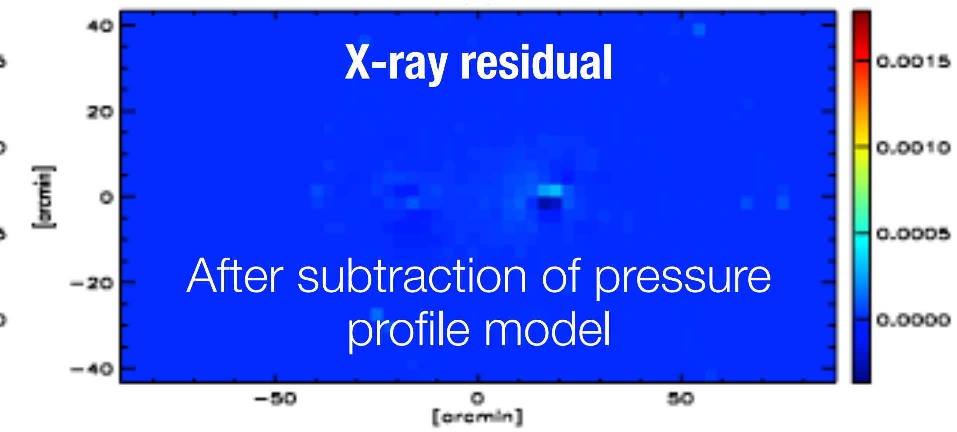


Fabian & Sanders 09

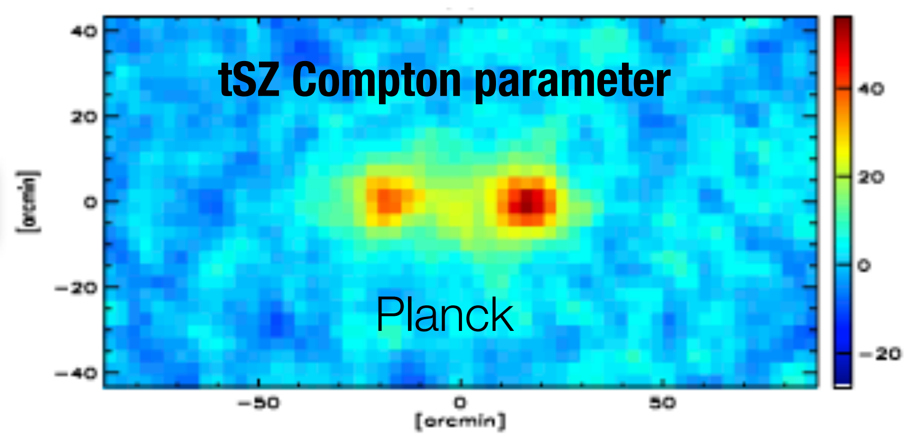


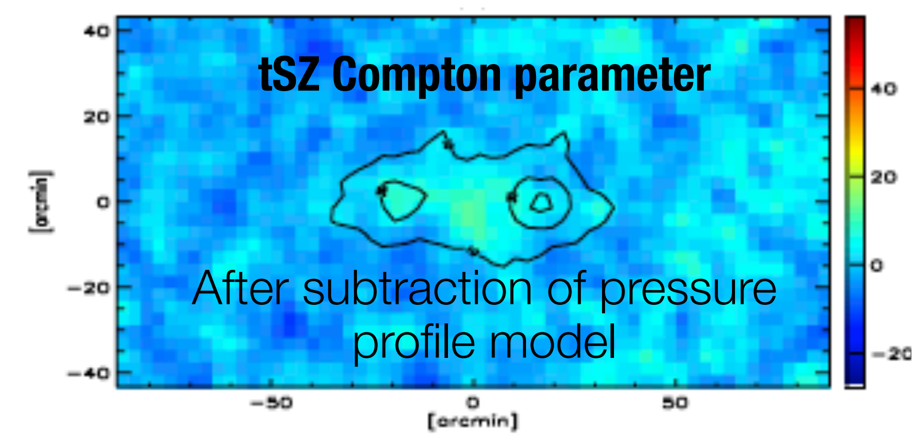
Planck collaboration 11 (A&A, 536, 9)

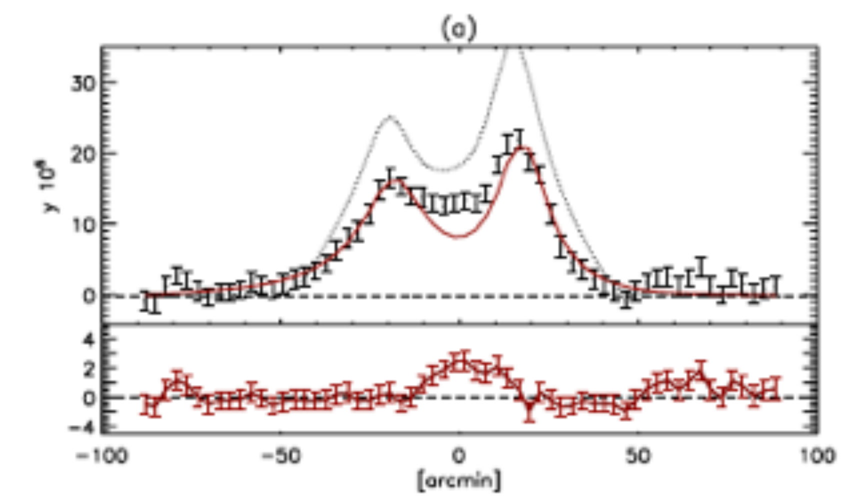


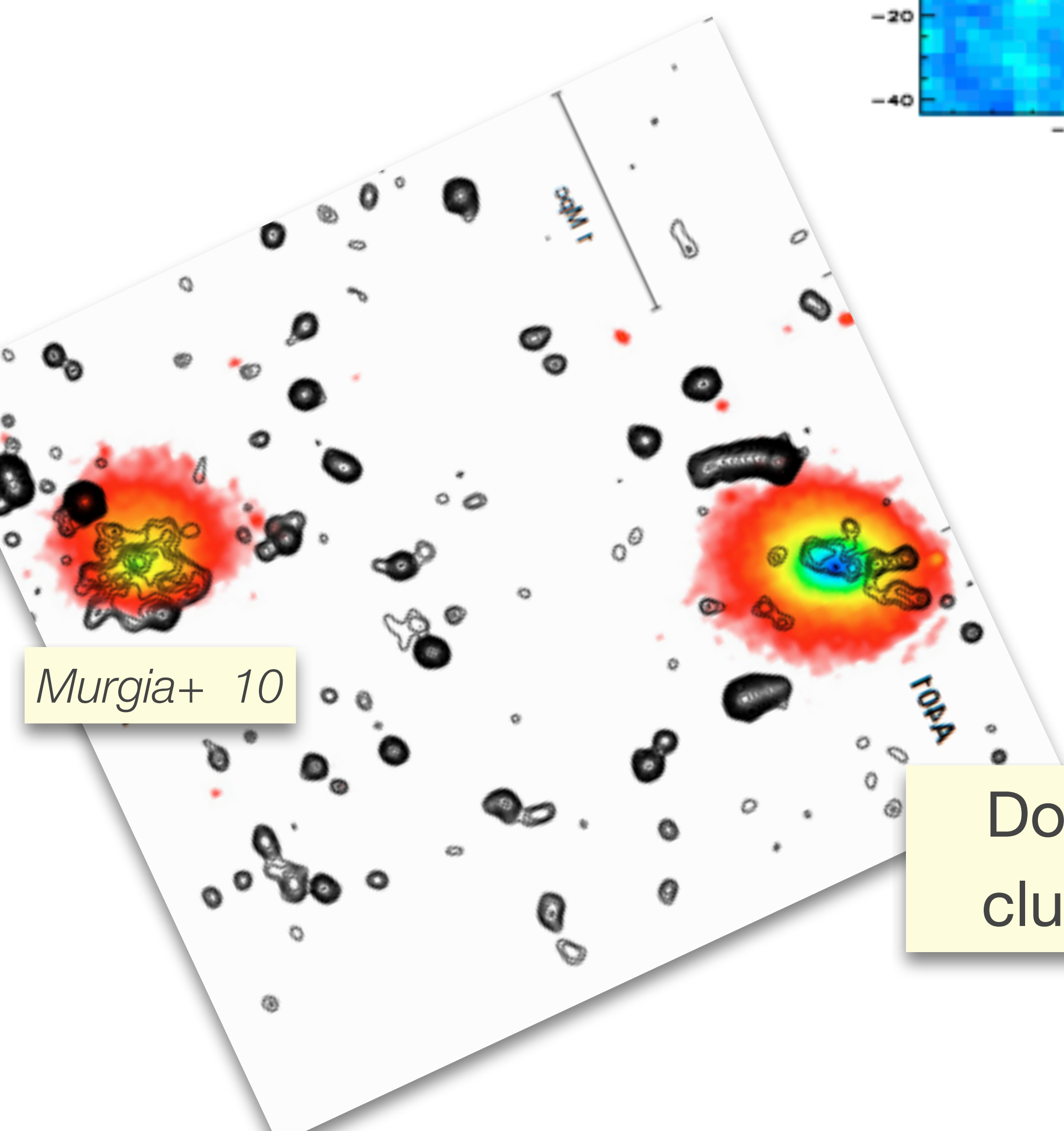


Planck collaboration 13 (paper VIII)







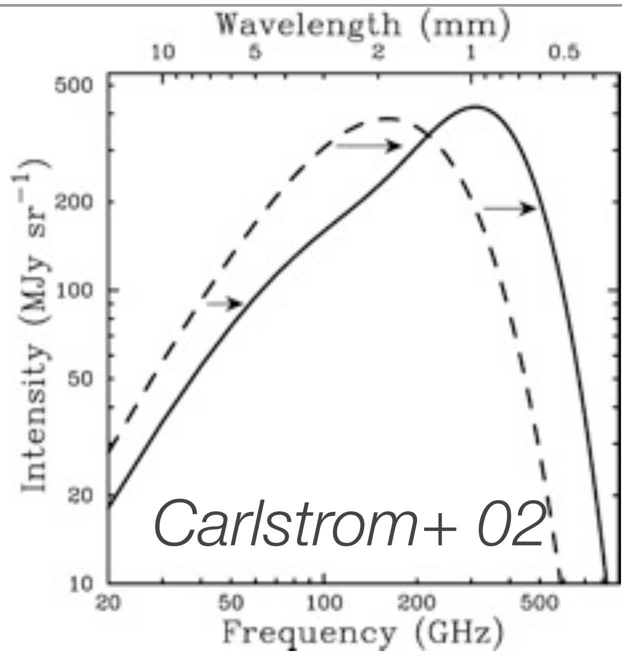


Murgia+ 10

Double galaxy cluster system

First SZ detection of hot and diffuse intra-cluster gas

SZE:
IMPORTANCE FOR CLUSTER STUDIES



Carlstrom+ 02

$$\frac{\Delta T_{SZE}}{T_{CMB}} = f(\nu)\ y$$

$$y \propto \int n_e T\ \mathrm{d}l$$

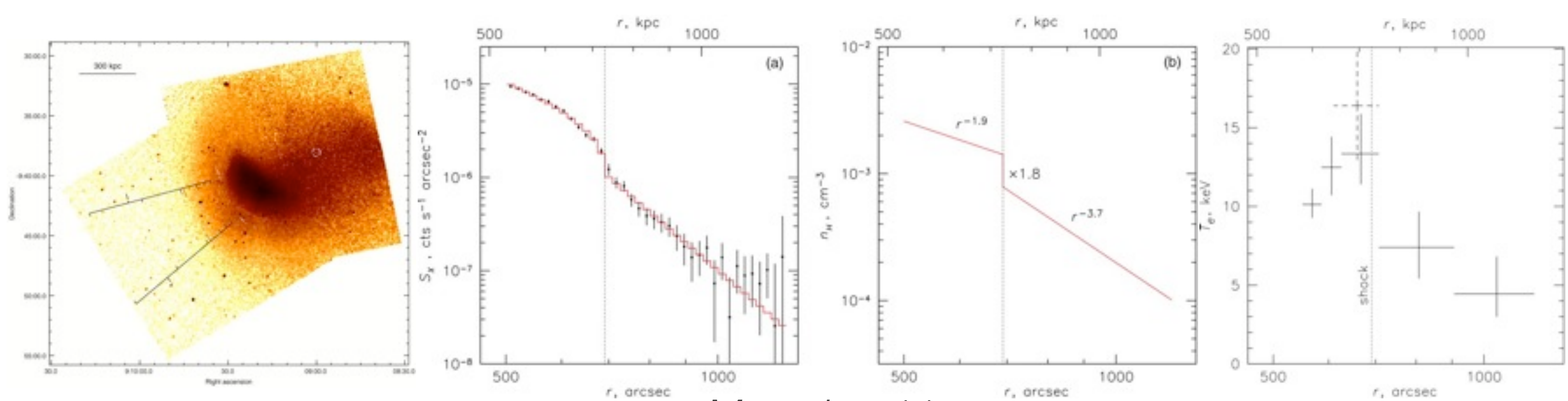
ICM shock detection

X-ray Image

Surface brightness profile

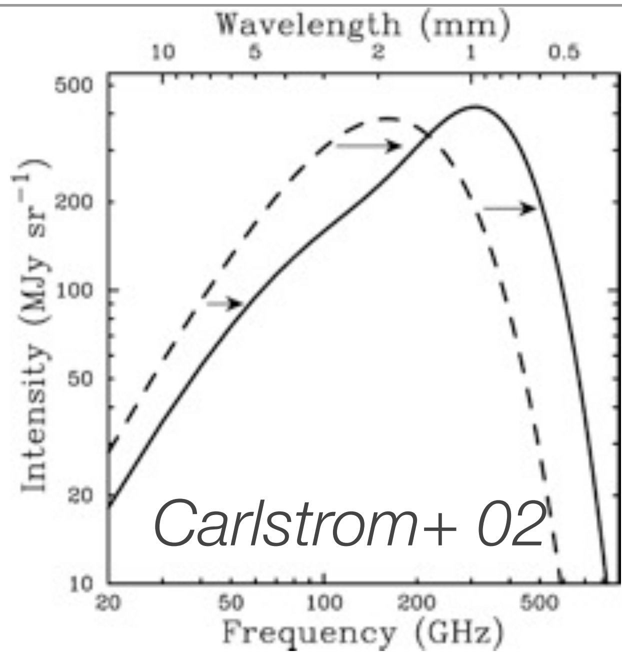
Density profile

Temperature profile



Macario+ 11

SZE:
IMPORTANCE FOR CLUSTER STUDIES



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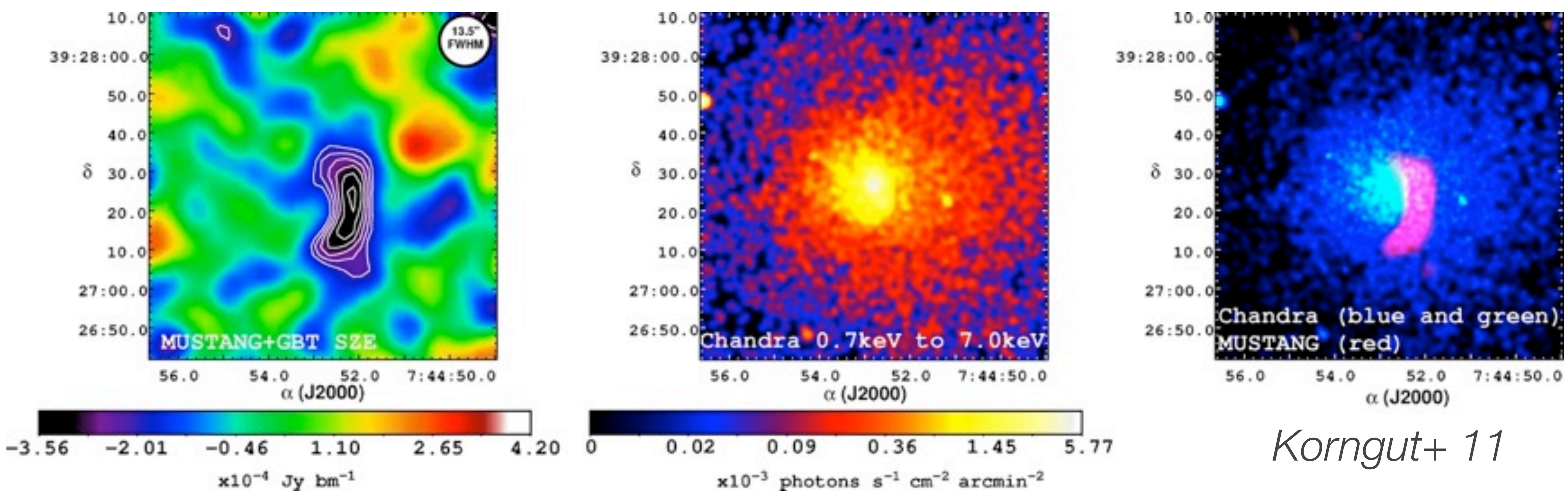

Korngut+ 11