Emission Mechanisms

Lesson 2

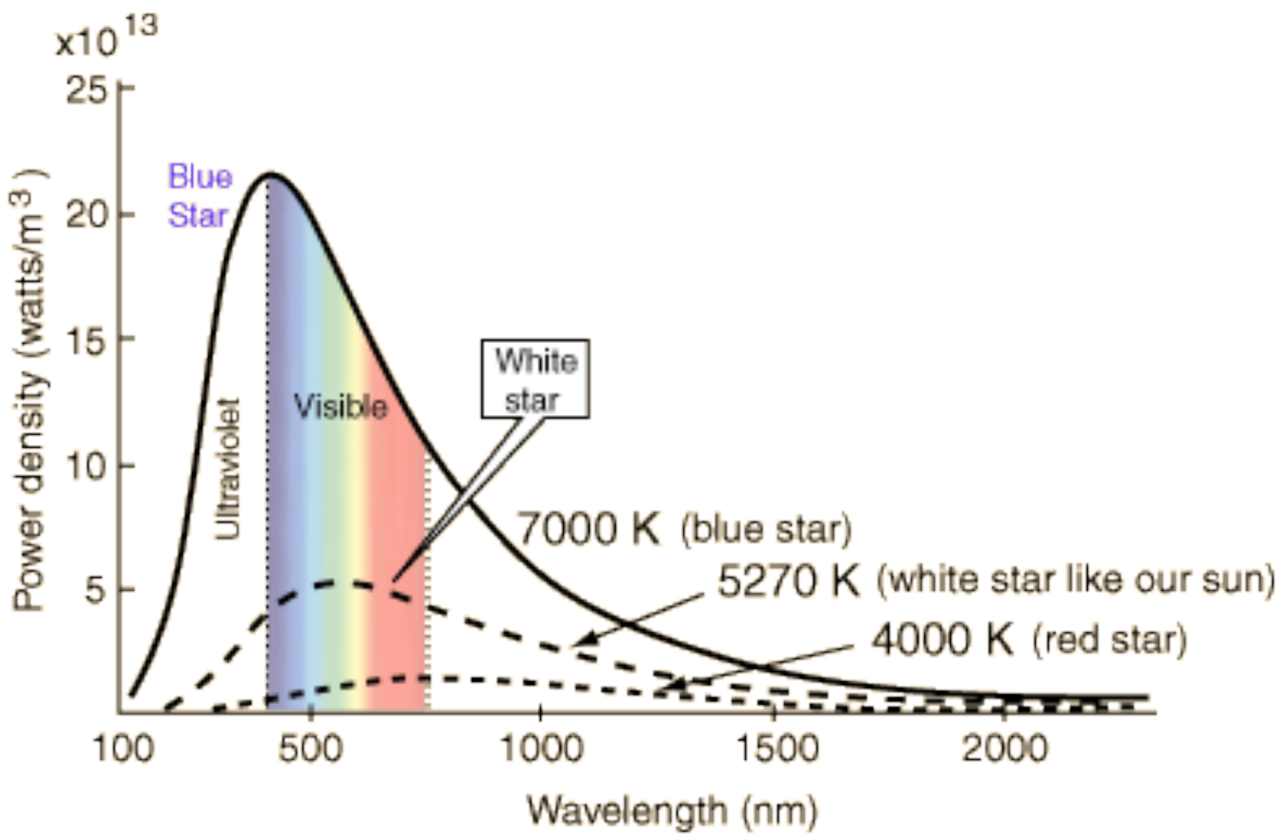
Chiara Ferrari

Reference Text:

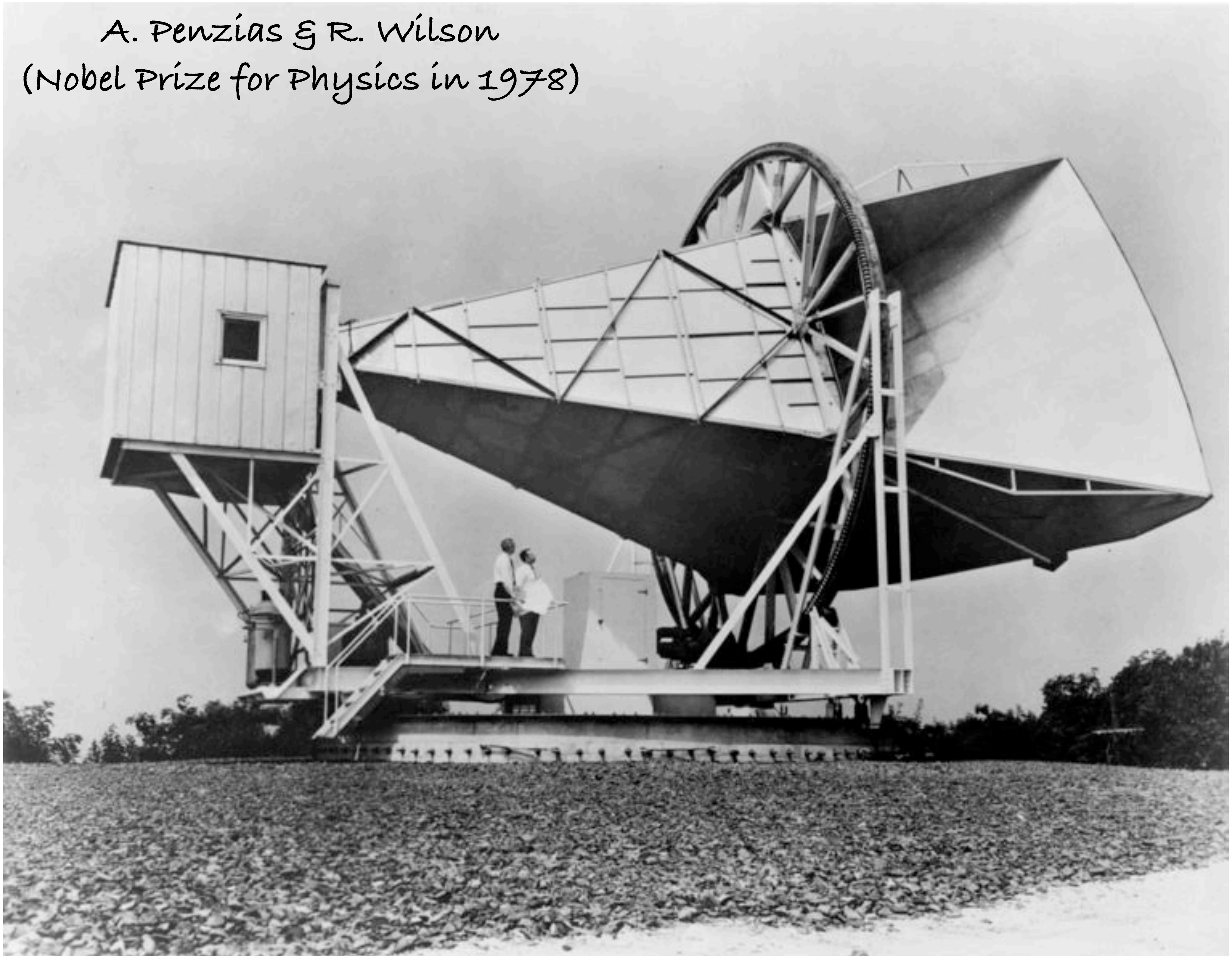
"Astrophysical Processes" by H. Bradt
Cambridge University Press (2008)

Blackbody radiation

e.g.: stars



Blackbody radiation
e.g.: Cosmic Microwave Background



A. Penzias & R. Wilson
(Nobel Prize for Physics in 1978)

Blackbody radiation
e.g.: Cosmic Microwave Background

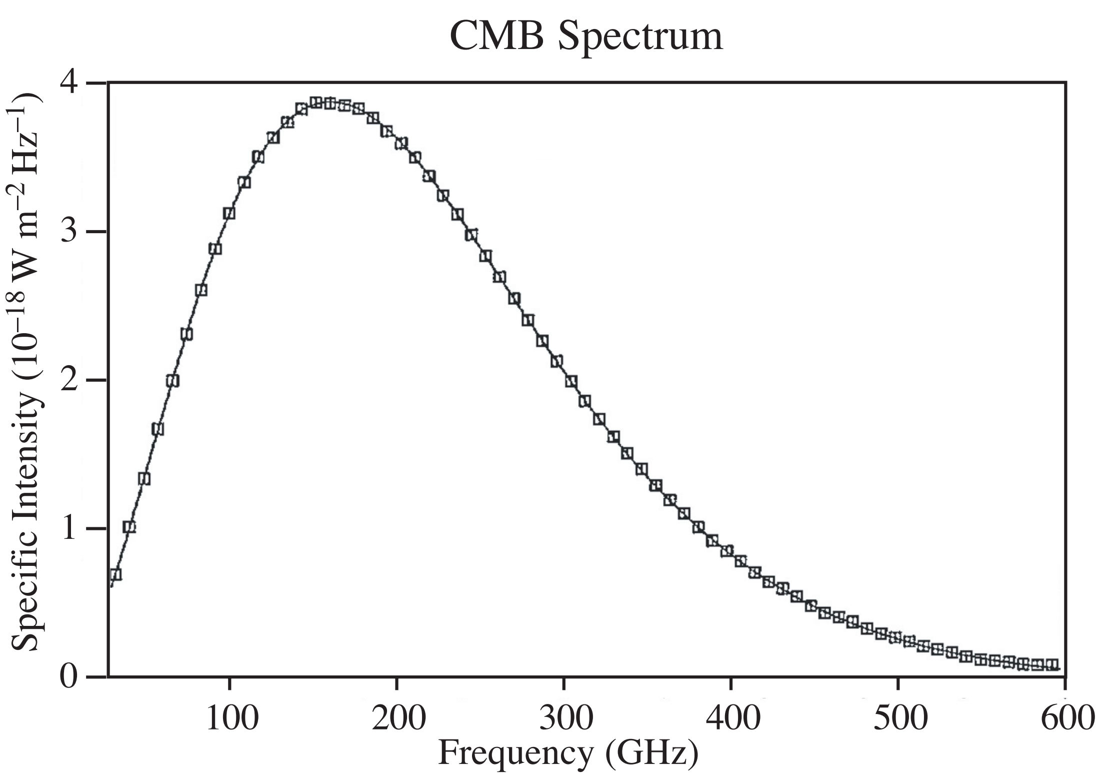


Fig. 6.1: Astrophysics Processes (CUP), H Bradt 2008
NASA/COBE, J. Mather et al., ApJ 354, 37 (1990)

The Independent

April 24, 1992

THE INDEPENDENT

How the universe began

SUMMARY

Bosnia ceasefire crumbles

Film maker Satyajit Ray dies

DO TIPSY ELEPHANTS SEE PINK PEOPLE?

De Klerk concession on hand-over

WORLD

Blackbody radiation
e.g.: Cosmic Microwave Background

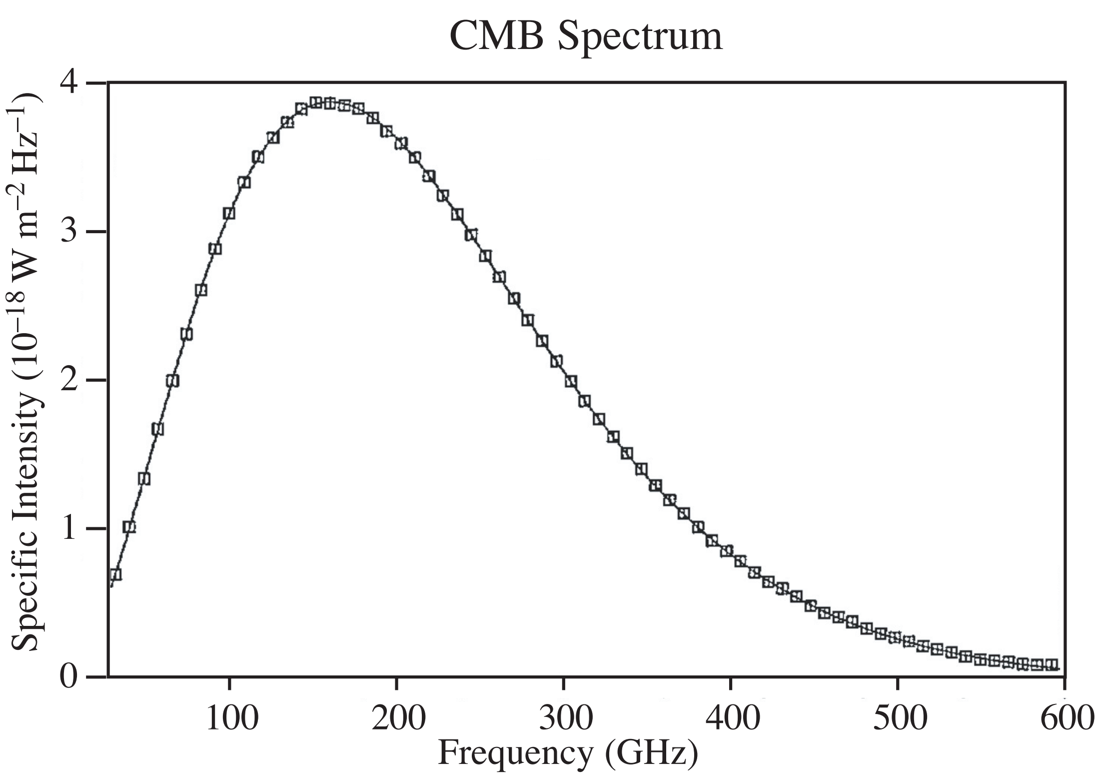
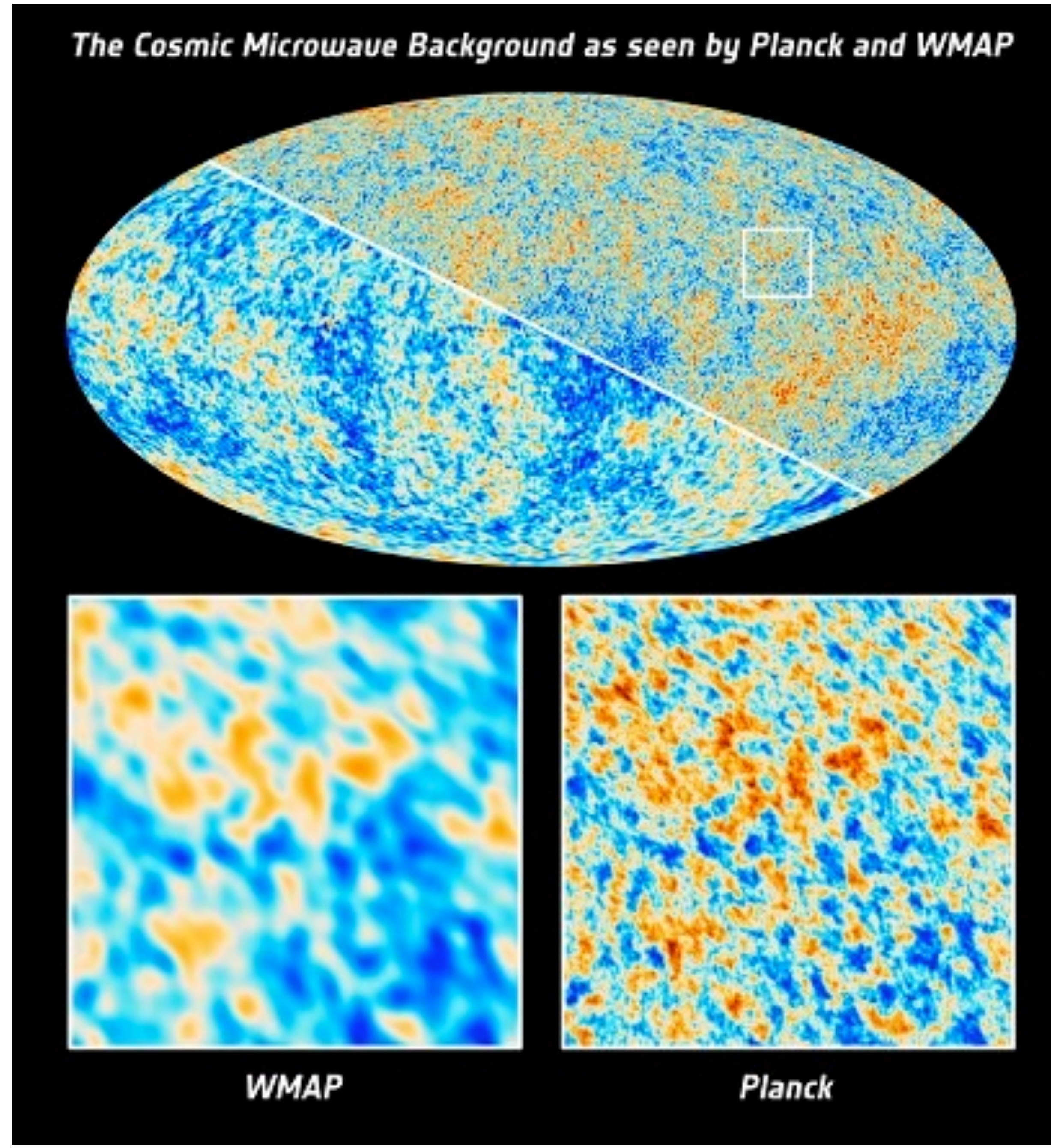
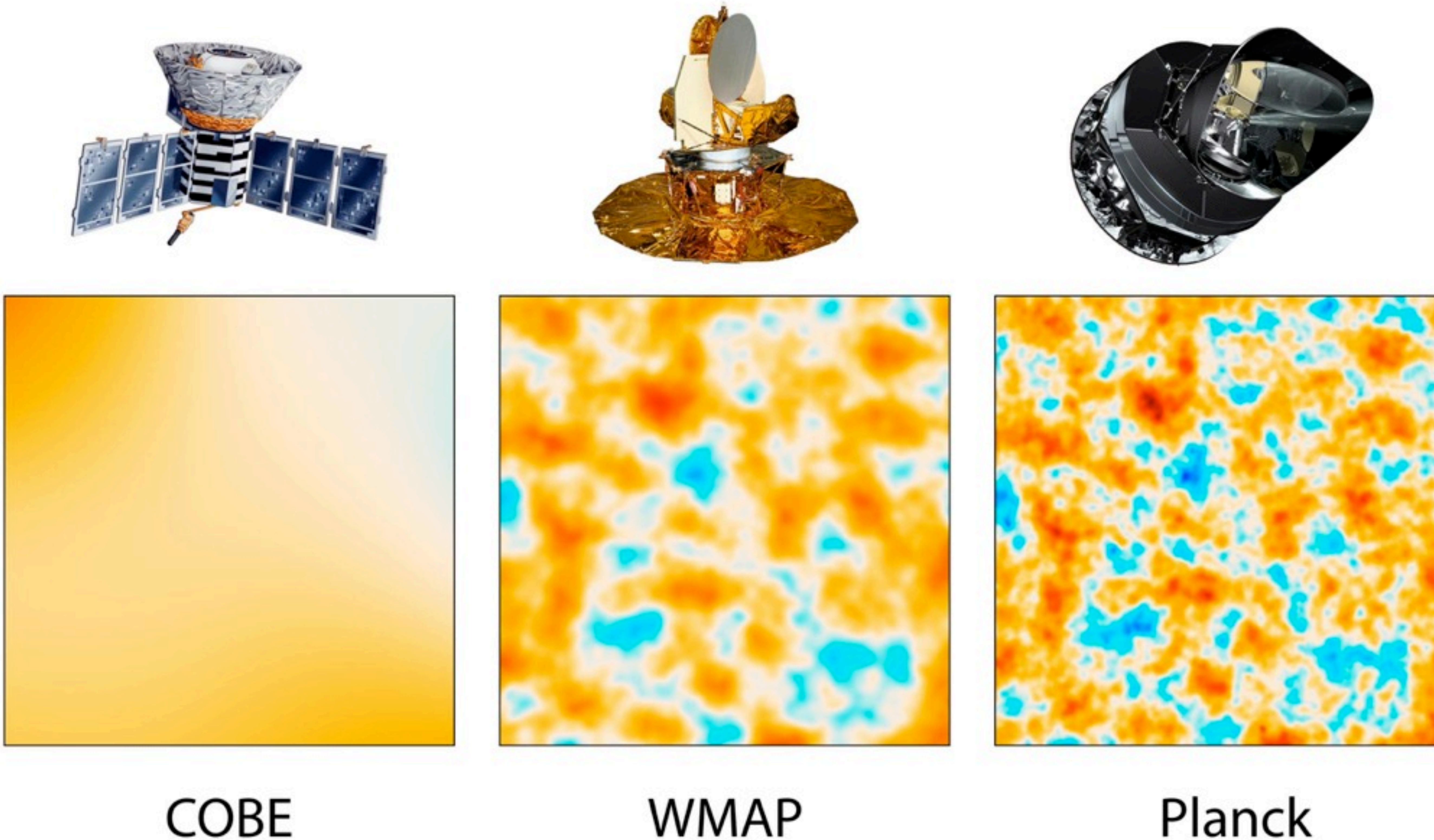


Fig. 6.1: Astrophysics Processes (CUP), H Bradt 2008
NASA/COBE, J. Mather et al., ApJ 354, 37 (1990)



Courtesy: ESA

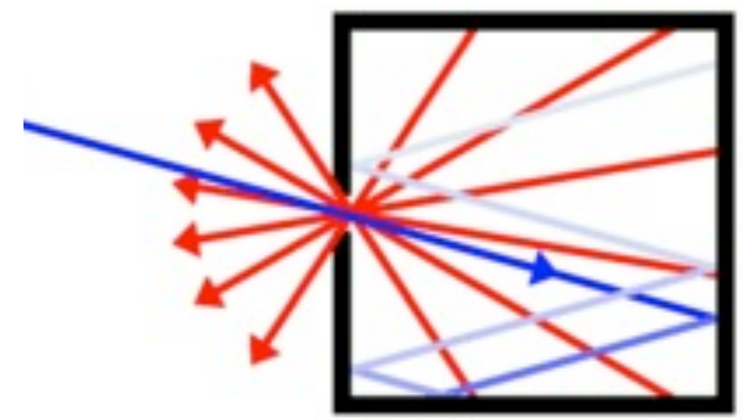
Blackbody radiation
e.g.: Cosmic Microwave Background



A Blackbody
Classical example

Blackbody radiation:
Matter is optically thick & photons scatter many times before encountering an observer

→ particles & photons share their kinetic energy: perfect thermodynamic equilibrium between radiation & container walls at temperature T



- Cavity with a small hole ≡ opaque & non-reflecting object
- Radiation enters the cavity through the hole & bounce off many times the walls before returning outside
- The hole will appear black
- A blackbody emits radiation since the atoms and molecules are continually oscillating (recall that a vibrating electric charge emits EM radiation)

Incoming rays absorbed completely

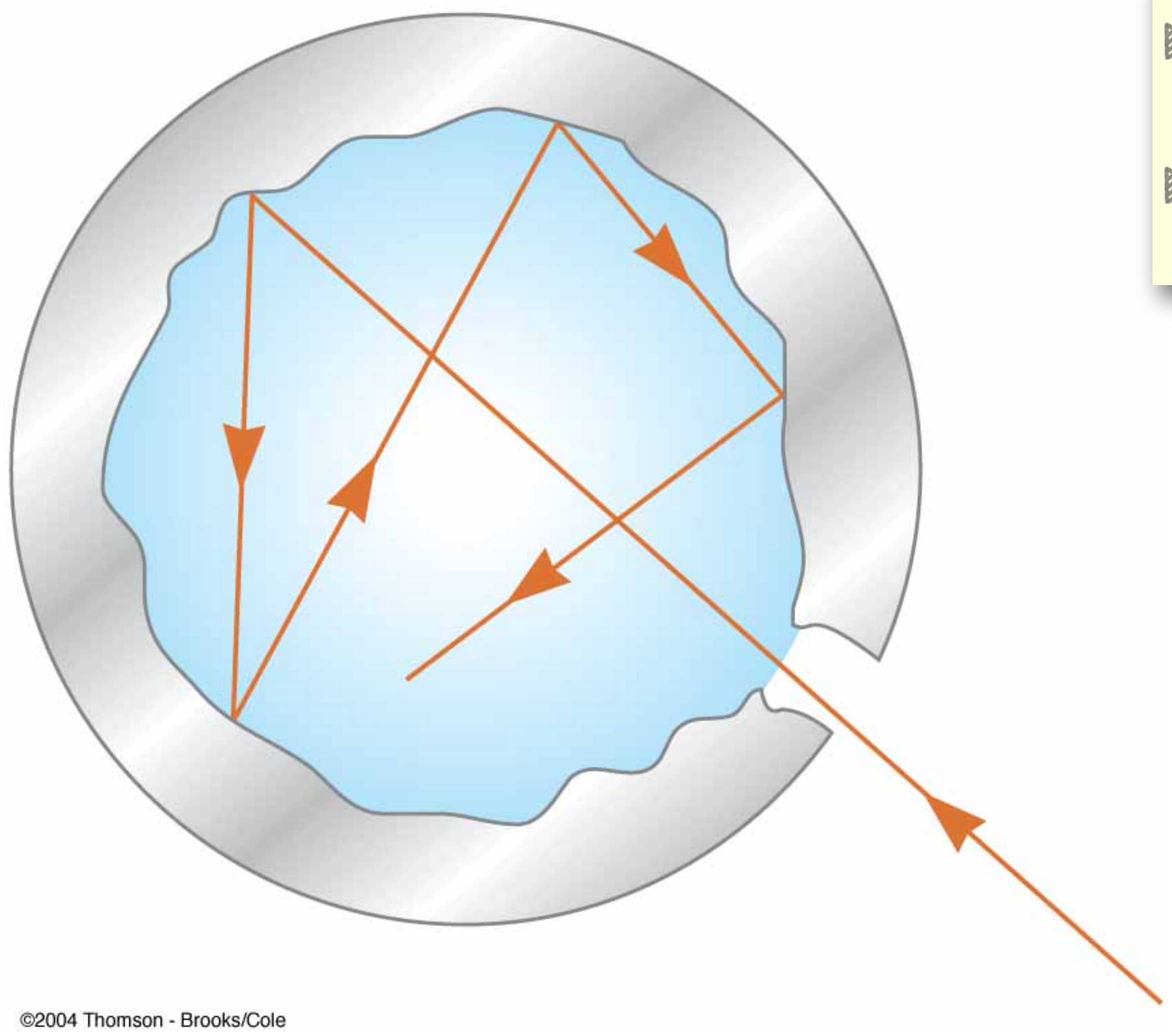
Red rays outgoing thermal radiation

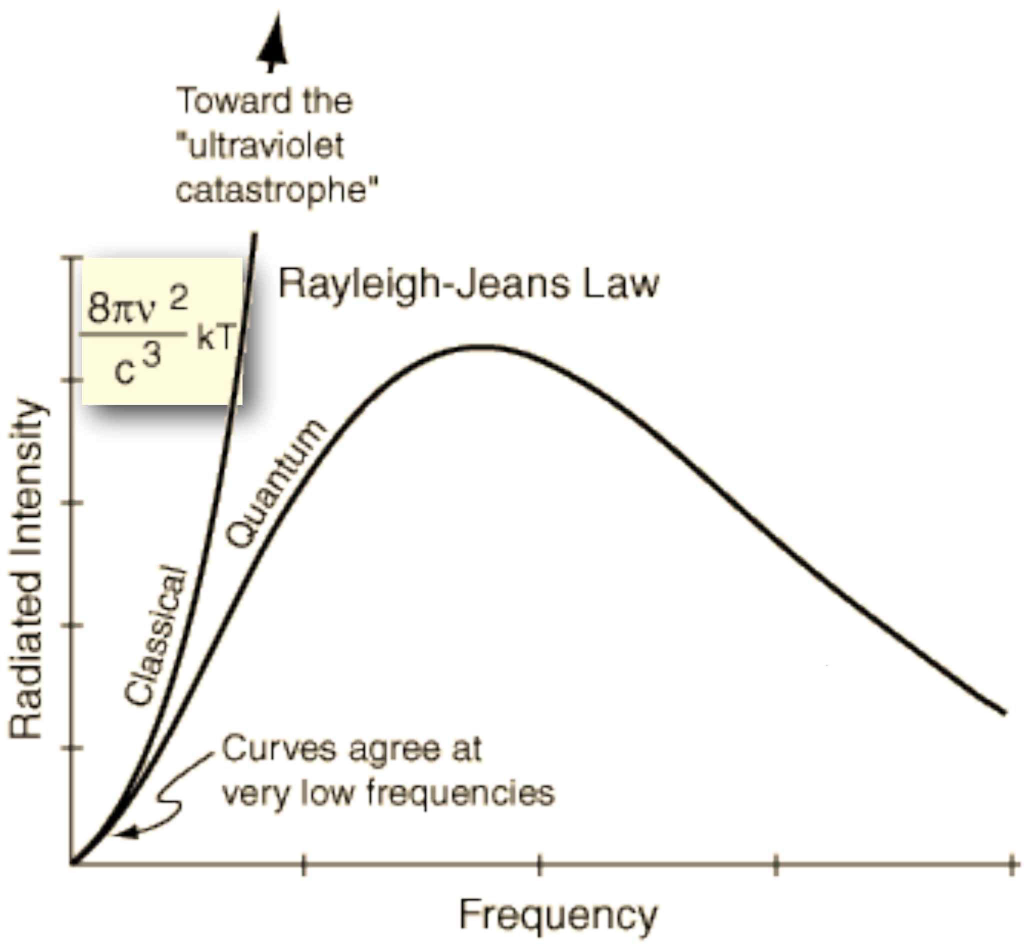
Blackbody spectrum
The Ultraviolet Catastrophe ...

Classical statistical mechanics:

equipartition of energy

- For a system in thermal equilibrium at T, each degree of freedom has average energy 1/2 kT
- The number of electromagnetic modes in a 3D cavity per unity frequency is $\propto \nu^2$



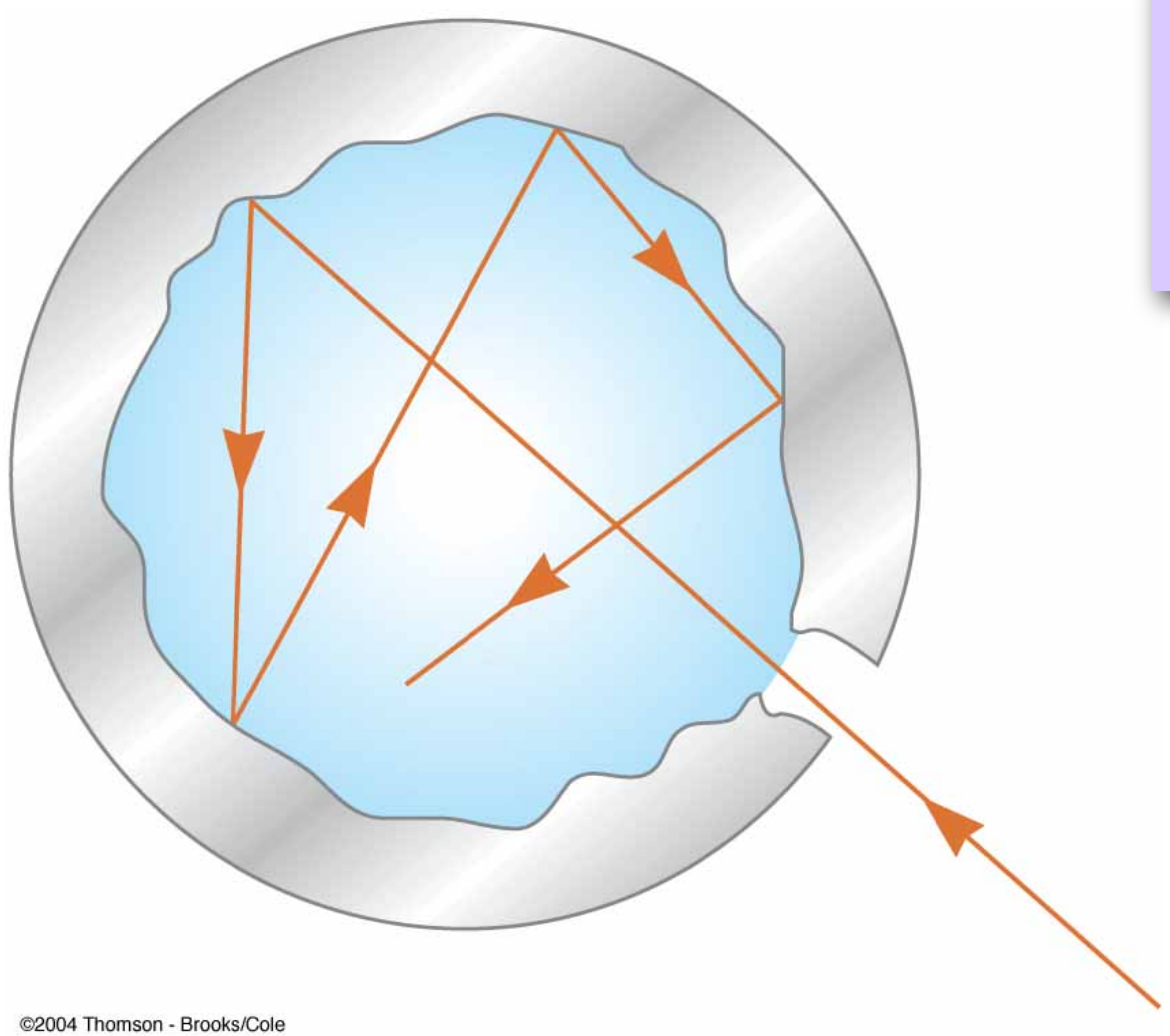


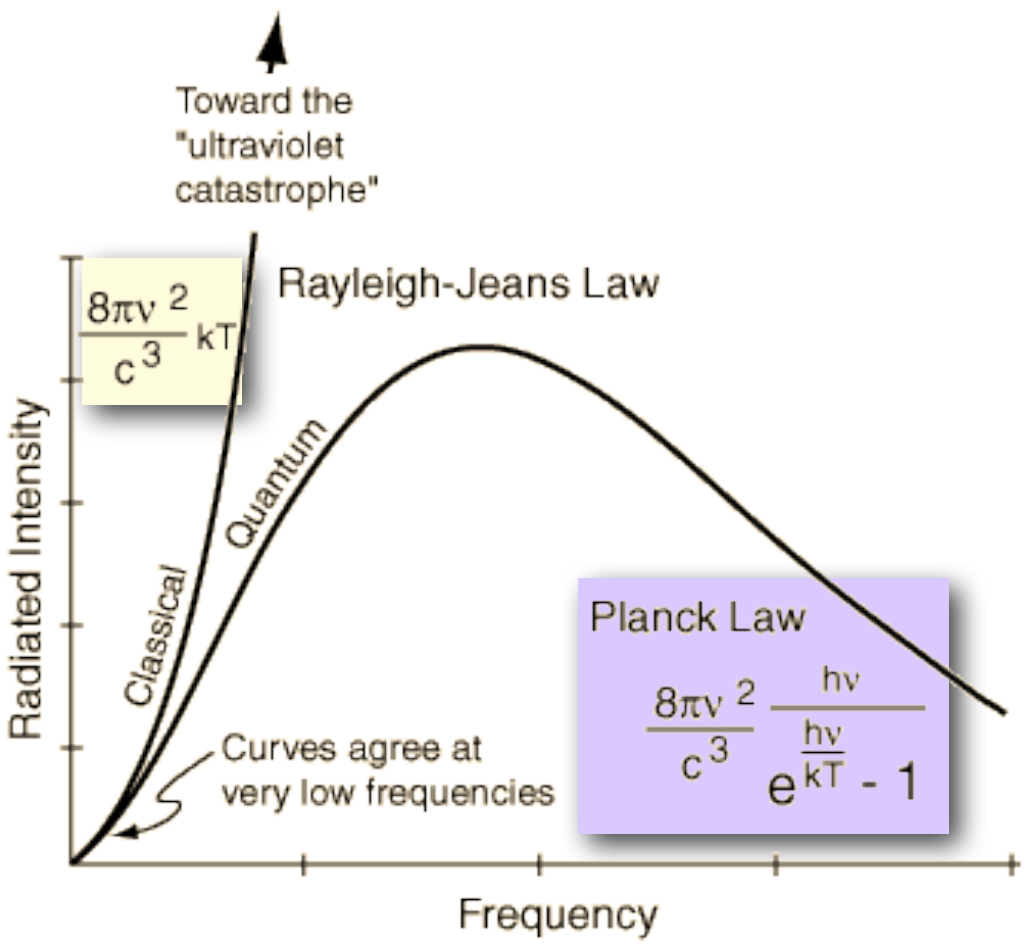
Blackbody spectrum
… and Planck's Solution

Planck's solution of the problem

- The energies of the oscillation of electrons which give rise to radiation must be proportional to integral multiples of the frequency:

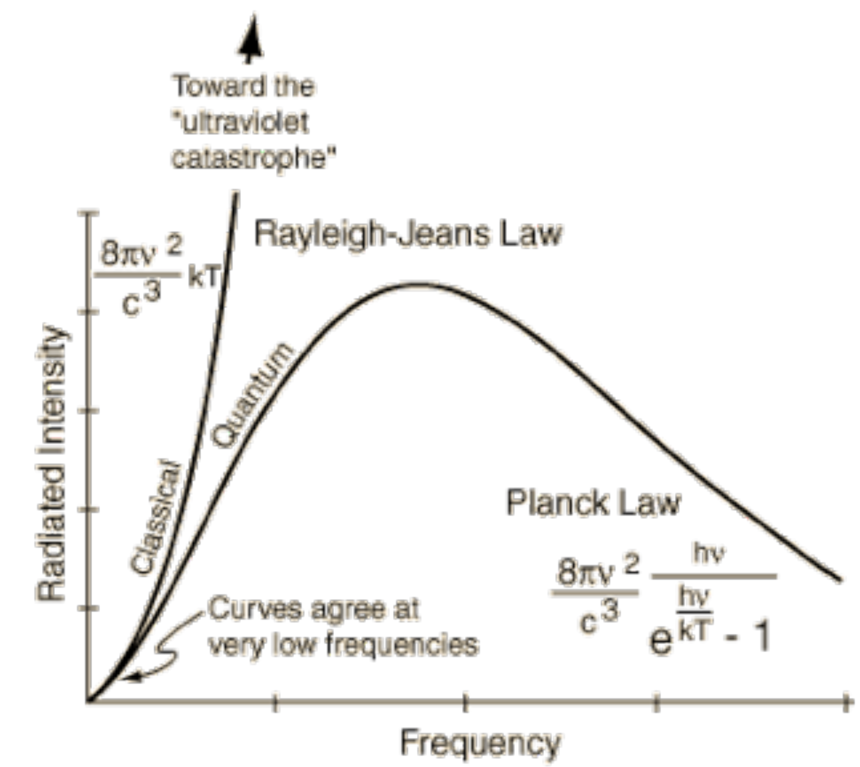
$$E = nh\nu \qquad h = 6.626 \times 10^{-34}\ \mathrm{Js}$$

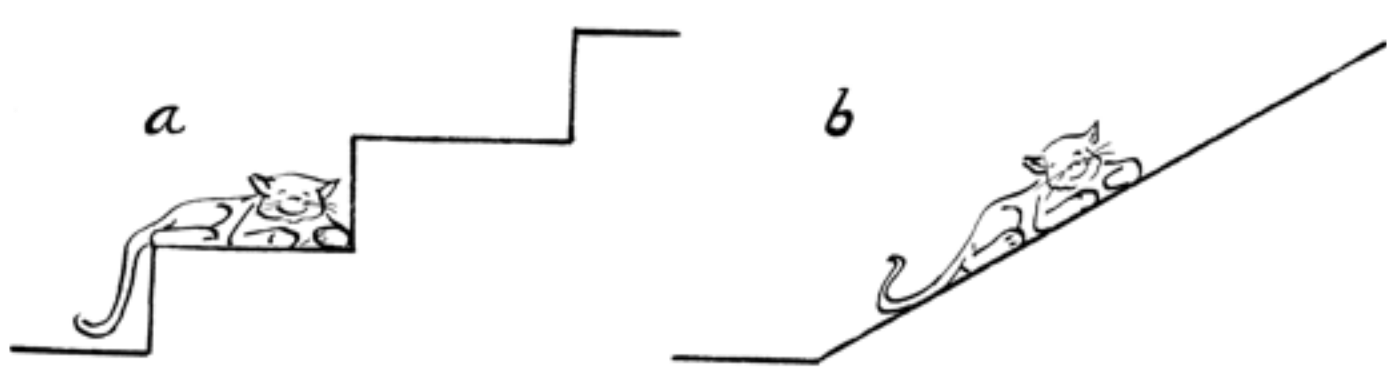




Understanding the Blackbody Curve

- Planck concentrated on modeling the oscillating charges that exist in the oven walls
- They radiate heat inwards
- In thermodynamic equilibrium, themselves are driven by the radiation field
- Planck found that he could account for the observed curve if he required these oscillators not to radiate energy continuously
- They could only lose / gain energy in quanta of size hν





The Planck function can be derived with Bose-Einstein statistics, that apply to bosons

Unlike fermions there is no a priori limit to the number of particles allowed in any given state

There is a limit on the total energy available for the photons to share

Distribution function of massless bosons

$$f = \frac{2}{h^3(e^{h\nu/kT} - 1)} \qquad [(\mathrm{Js})^{-3}]$$

Bose-Einstein statistics used to find the most probable distribution of photons as a function of their energy hν

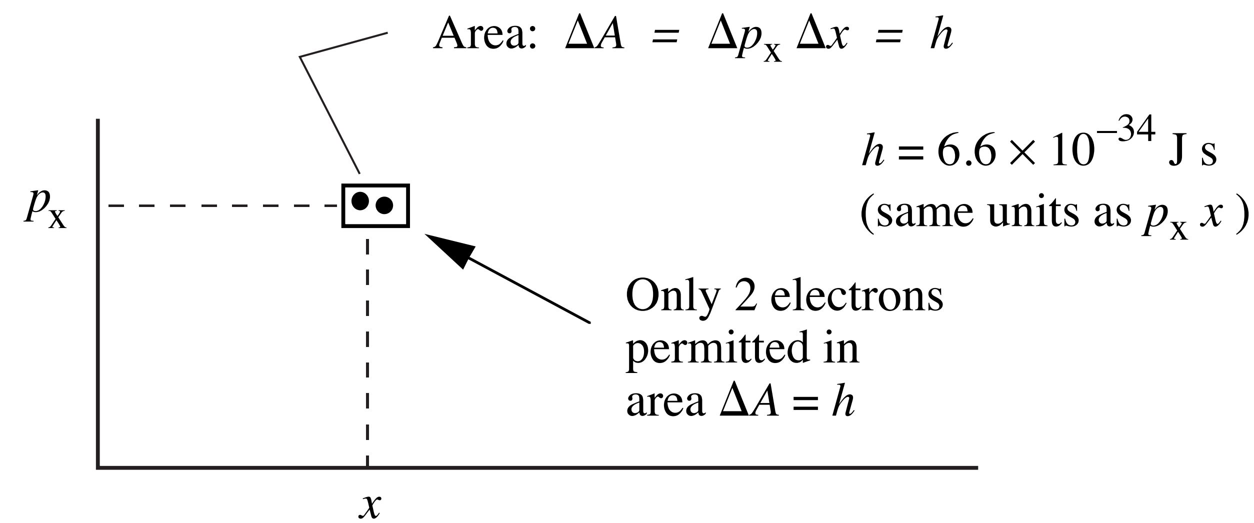
$$f = nP(\mathbf{p})$$

Probability distribution

Particle number density

Average number of photons in one 6-D phase-space cell divided by the volume h^3 of the cell as a function of the photon energy hν

Phase Space Element



$$2/(e^{h\nu/kT} - 1)$$ → Average number of photons in each cell as a function of the photon energy hν → $\to \infty$ at $\nu = 0$; $\to 0$ at $\nu = \infty$

Characteristics of the Radiation

$$I(\nu) = (h^4\nu^3/c^2)f$$

Specific intensity of propagating photons from distribution function
(see Chapter 3 in Astrophysics Process)

$$f = \frac{2}{h^3(e^{h\nu/kT} - 1)} \qquad [(\mathrm{Js})^{-3}]$$

Planck radiation law as a function of frequency

$$I(\nu, T) = \frac{2h\nu^3}{c^2}\frac{1}{e^{h\nu/kT} - 1} \qquad (\mathrm{Wm^{-2}Hz^{-1}sr^{-1}})$$

Blackbody spectrum
as a function of frequency

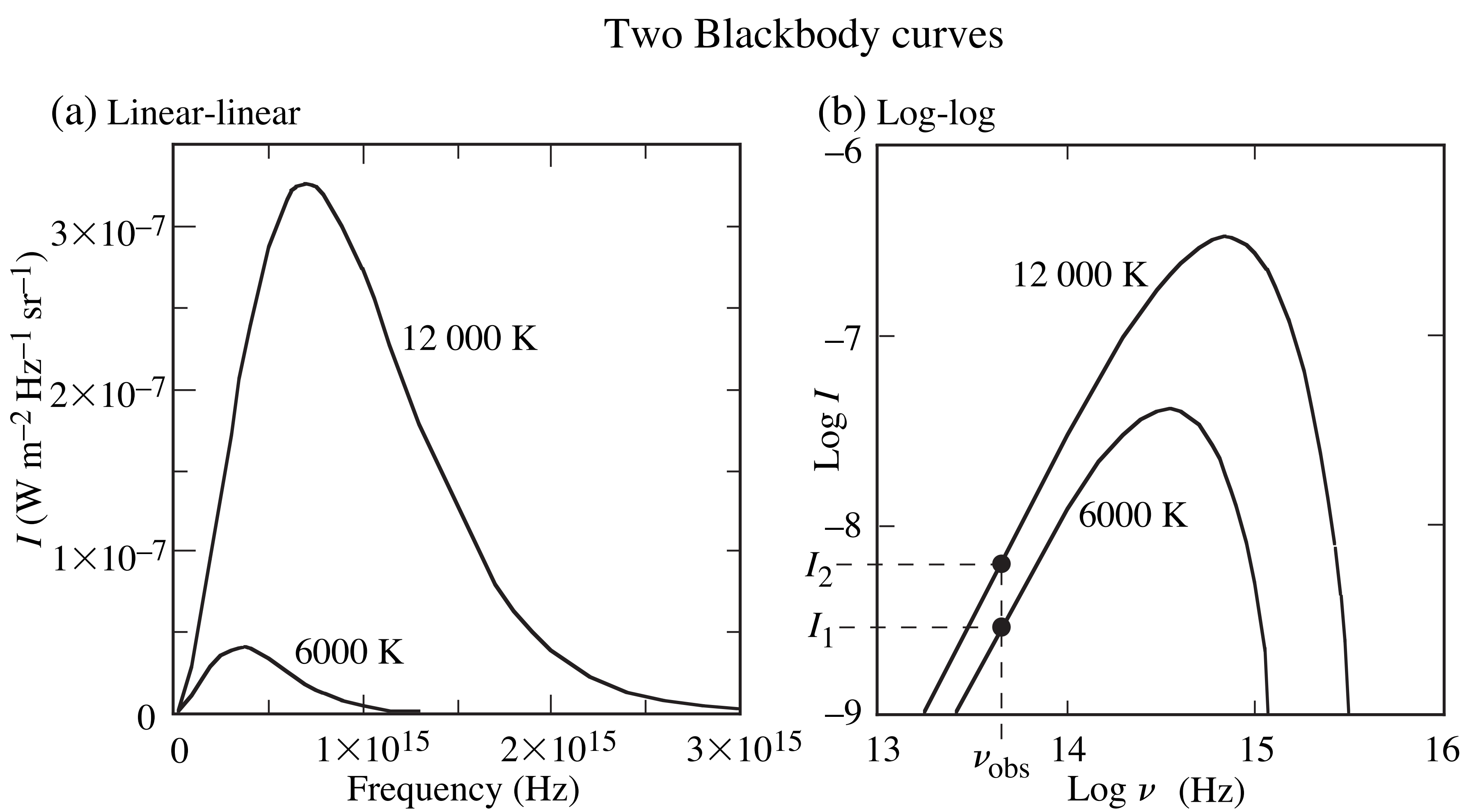


Fig. 6.2: Astrophysics Processes (CUP), © H Bradt 2008

$$I(\nu, T) = \frac{2h\nu^3}{c^2}\frac{1}{e^{h\nu/kT}-1} \qquad (\mathrm{Wm^{-2}Hz^{-1}sr^{-1}})$$

Blackbody spectrum
Approximations

$$I(\nu,T) = \frac{2h\nu^3}{c^2}\frac{1}{e^{h\nu/kT}-1}$$

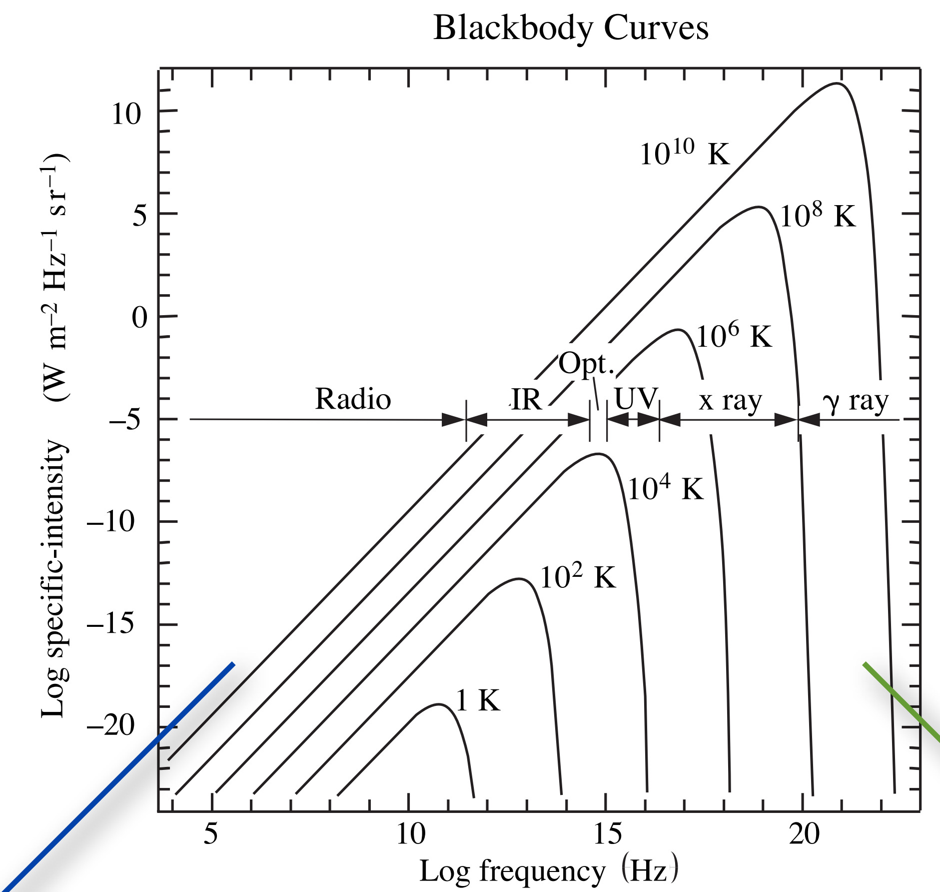


Fig. 6.3: Astrophysics Processes (CUP), © H Bradt 2008

Rayleigh-Jeans approximation

$$e^{h\nu/kT} = 1 + \frac{h\nu}{kT} + \ldots$$

$$I(\nu,T) \approx \frac{2\nu^2 kT}{c^2} \propto \nu^2 T \qquad (h\nu << kT)$$

Wien approximation

$$e^{h\nu/kT} >> 1$$

$$I(\nu,T) \approx \frac{2h\nu^3}{c^2} e^{-h\nu/kT} \qquad (h\nu >> kT)$$

Blackbody spectrum
Peak frequency

$$h\nu_{\text{peak}} = 2.82\ k\ T$$

Wien displacement law

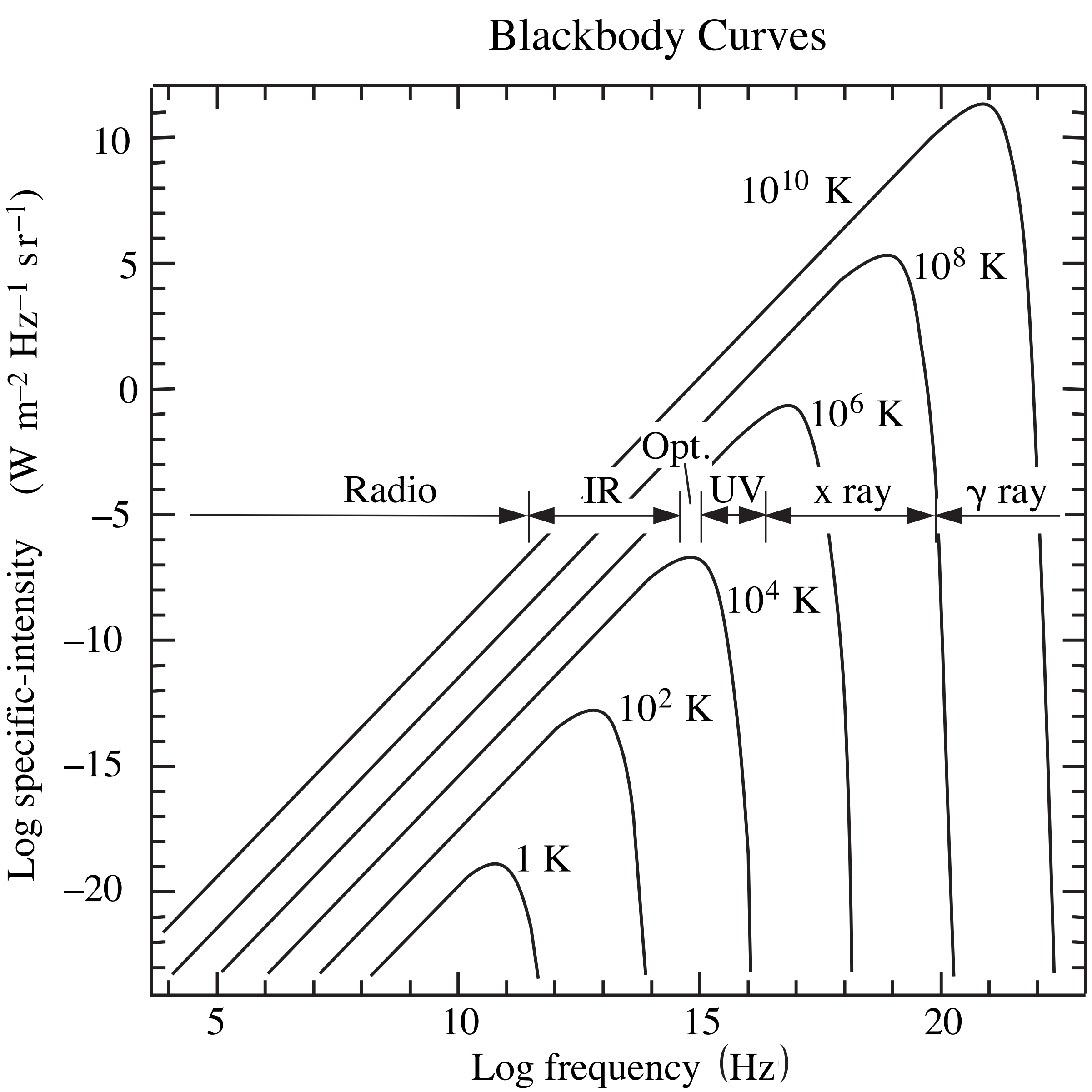


Fig. 6.3: Astrophysics Processes (CUP), © H Bradt 2008

Blackbody spectrum
Peak frequency

The BB specific intensity in frequency space is, from (6),

$$I(\nu,T) = \frac{2h\nu^3}{c^2}\frac{1}{e^{h\nu/kT}-1}$$

$$= \frac{2h}{c^2}\left(\frac{kT}{h}\right)^3\frac{x^3}{e^x-1}$$

where we changed the variable to $x = h\nu/kT$.
To find the maximum of I, take the derivative and set it to zero.

$$\frac{dI}{dx} = \frac{2h}{c^2}\left(\frac{kT}{h}\right)^3\left[\frac{3x^2}{e^x-1} + \frac{-x^3e^x}{(e^x-1)^2}\right] = 0$$

which gives, after canceling common terms, the transcendental equation (75) where $n = 3$,

$$3 = \frac{x}{1-e^{-x}}$$

which has solution $x = 2.82$, from Table 6.2. Thus the photon energy at maximum intensity is, from our definition $x = h\nu/kT$,

Blackbody spectrum
as a function of wavelength

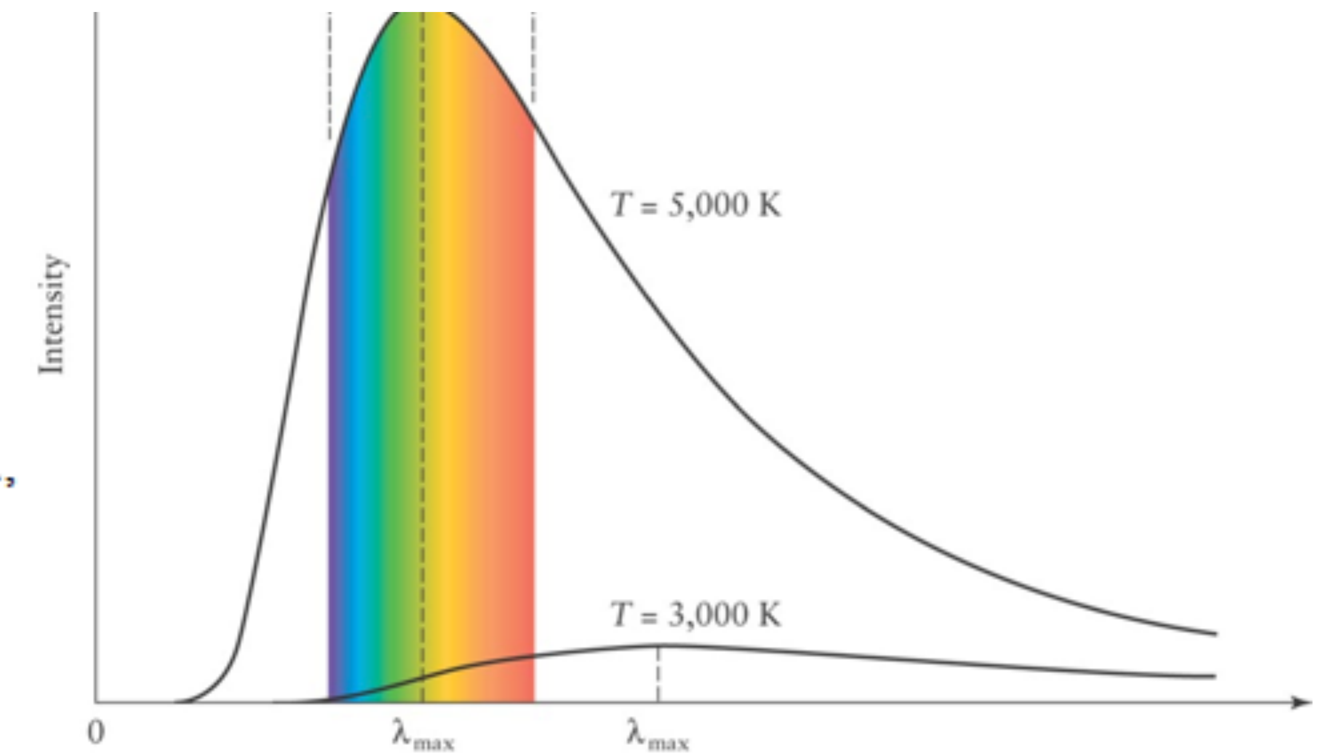
Require

$$I_\lambda \, d\lambda = -I_\nu \, d\nu$$
$$I_\lambda = -I_\nu \frac{d\nu}{d\lambda}$$

where $\nu = c/\lambda$ and $d\nu = -(c/\lambda^2)\, d\lambda$.
Substitute (s1) into (s2) while expressing everything in terms of λ,

$$I_\lambda = -\frac{2h(c/\lambda)^3}{c^2} \frac{1}{e^{hc/\lambda kT} - 1} \left(\frac{-c}{\lambda^2}\right)$$

➡ $$= \frac{2hc^2}{\lambda^5} \frac{1}{e^{hc/\lambda kT} - 1}$$



Planck radiation law as a function of wavelength

$$I_\lambda(\lambda, T) = \frac{2hc^2}{\lambda^5} \frac{1}{e^{hc/(\lambda kT)} - 1} \qquad (\mathrm{W\ m^{-3}\ sr^{-1}})$$

Blackbody spectrum
Peak wavelength

Define a new variable $x = hc/\lambda kT$ so $\lambda = hc/xkT$, and rewrite (s3),

$$I_\lambda = 2\,hc^2\left(\frac{kT}{hc}\right)^5 x^5\,\frac{1}{e^x - 1}$$

Take the derivative and set it to zero to find

$$\frac{5x^4}{e^x - 1} = \frac{x^5 e^x}{(e^x - 1)^2}$$

which reduces to

$$5 = \frac{x}{1 - e^{-x}}$$

We encounter again the transcendental equation (75), this time with $n = 5$ and solution $x = 4.9651$, from Table 2,

$$\frac{hc}{\lambda_{\text{peak}} kT} = 4.965$$

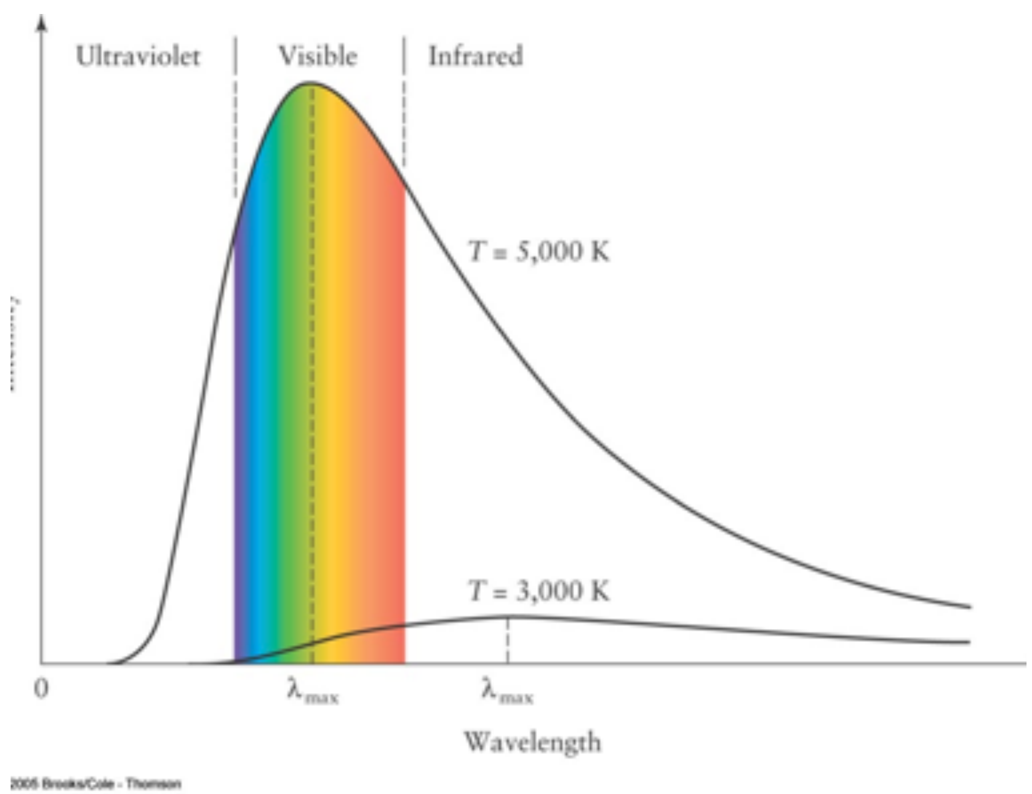
Blackbody spectrum
Peak wavelength vs. Frequency

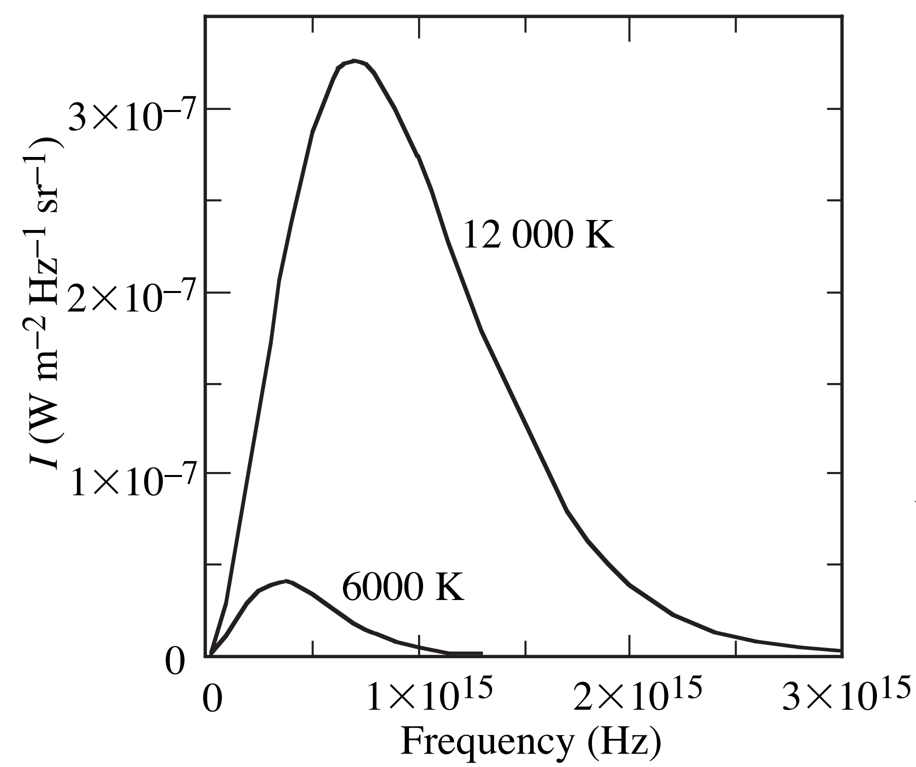
$$I_\lambda(\lambda, T) = \frac{2hc^2}{\lambda^5}\frac{1}{e^{hc/(\lambda kT)}-1} \quad (\mathrm{W\ m^{-3}\ sr^{-1}})$$

$$I(\nu, T) = \frac{2h\nu^3}{c^2}\frac{1}{e^{h\nu/kT}-1} \quad (\mathrm{Wm^{-2}Hz^{-1}sr^{-1}})$$

$$T\lambda_{\rm peak} = 2.898 \times 10^{-3} \qquad (\mathrm{Km})$$

$$\nu_{\rm peak}(\mathrm{Hz}) = 5.88 \times 10^{10} \times T(\mathrm{K})$$





The formulas do not give the same result !

$$\nu_{\rm peak}\ \lambda_{\rm peak} \simeq 1.7 \times 10^8\ \mathrm{m/s}$$

Energy flux density through a fixed surface

$$I(\nu,T) = \frac{2h\nu^3}{c^2}\frac{1}{e^{h\nu/kT}-1} \qquad (\mathrm{Wm^{-2}Hz^{-1}sr^{-1}})$$

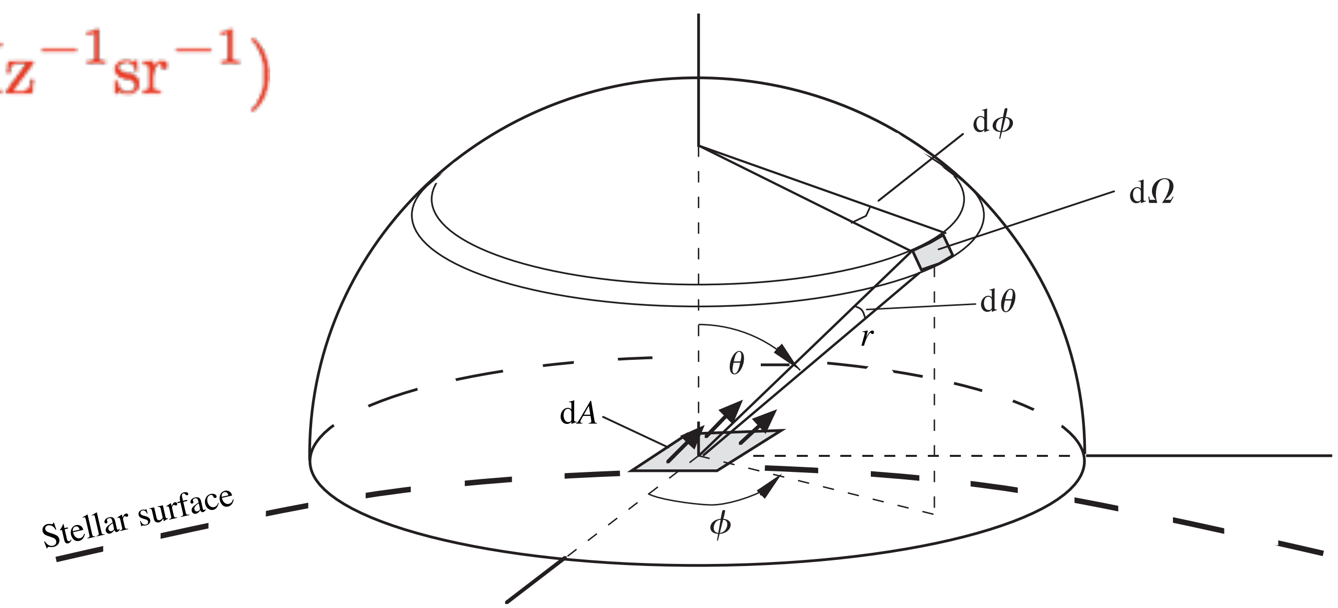


Fig. 6.4: Astrophysics Processes (CUP), © H Bradt 2008

Total power leaving 1 m^2 of the stellar (BB) surface

$$S = \int_{\nu=0}^{\infty}\int_{\theta=0}^{\pi/2}\int_{\Phi=0}^{2\pi} B(\nu,T)\frac{\mathrm{d}A\ \cos\theta}{\mathrm{d}A}\sin\theta\ \mathrm{d}\theta\ \mathrm{d}\Phi\ \mathrm{d}\nu \quad (\mathrm{W/m^2})$$

$$\int_{\theta=0}^{\pi/2} \int_{\phi=0}^{2\pi} \cos\theta \ \sin\theta \, d\theta \, d\phi$$

Note that $\sin\theta \, d\theta = -\, d(\cos\theta)$, so,

$$\Rightarrow \quad 2\pi \int_{1}^{0} \cos\theta \ (-)\, d(\cos\theta) \ = -\, 2\pi \, \frac{\cos^2\theta}{2} \, \Big|_1^0 \ = \ \pi$$

which is the indicated result in (16).

The second integral (17) is

$$\mathscr{F} = \pi \int_0^\infty \frac{2h\,\nu^3}{c^2} \ \frac{d\nu}{e^{h\nu/kT} - 1}$$

Change variable to $x = h\nu/kT$. Hence $\nu = kTx/h$ and

$$\mathscr{F} = \frac{2\pi h}{c^2} \left(\frac{kT}{h}\right)^4 \int_0^\infty \frac{x^3 \, dx}{e^x - 1} \tag{6.23.s4}$$

The integral is proportional to the Riemann zeta function (74) of order $z = 4$, which is tabulated (Table 1, Section 4) to be $\zeta(4) = \pi^4/90$, so

$$\int_0^\infty \frac{x^3 \, dx}{e^x - 1} = \zeta(4)\,\Gamma(4) \ = \frac{\pi^4}{90} \cdot 6 \ = \frac{\pi^4}{15}$$

where the gamma function is $\Gamma(z) \equiv (z-1)!$ Substitute this into (s4) to find the result (18) and (19),

$$\mathscr{F} = \frac{2\pi h}{c^2} \left(\frac{kT}{h}\right)^4 \frac{\pi^4}{15}$$

$$\Rightarrow \quad = \frac{2\,\pi^5\,k^4}{15\,c^2\,h^3} \ T^4 \ \equiv \sigma T^4$$

Energy flux of a blackbody

Stefan-Boltzmann radiation law

$$S = \sigma T^4 \quad (\mathrm{W/m^2})$$

= flux which passes in one direction through a surface immersed in BB radiation

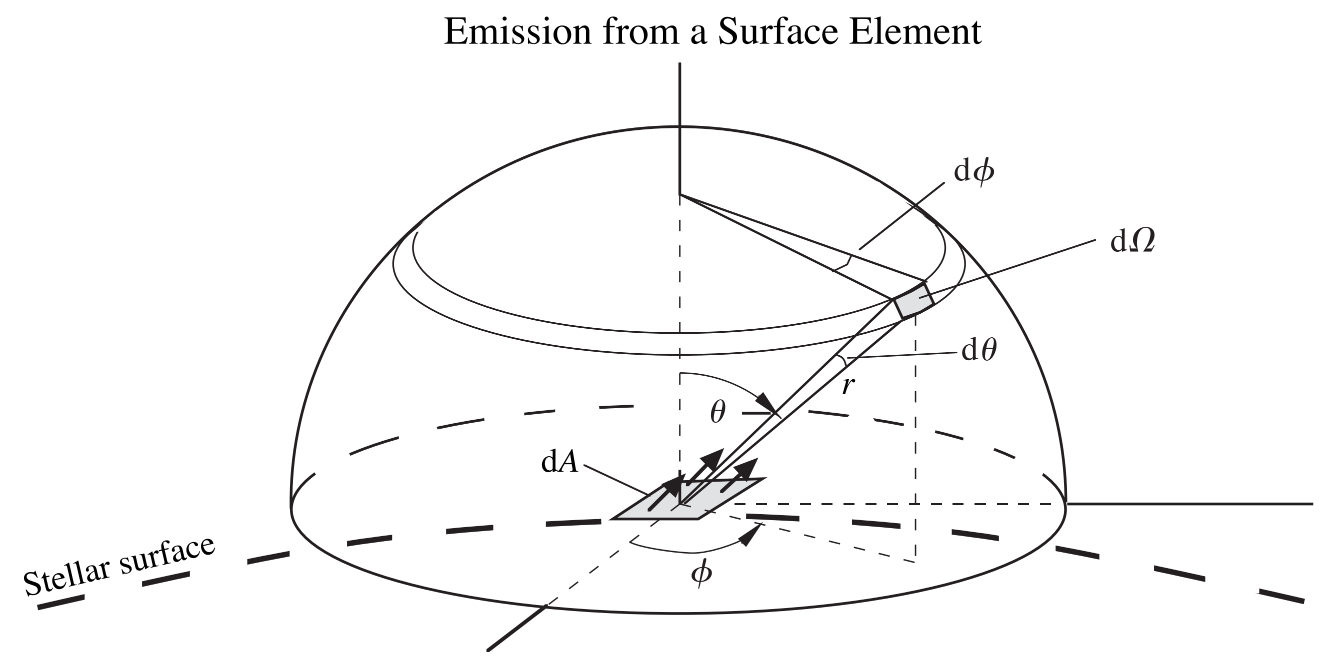
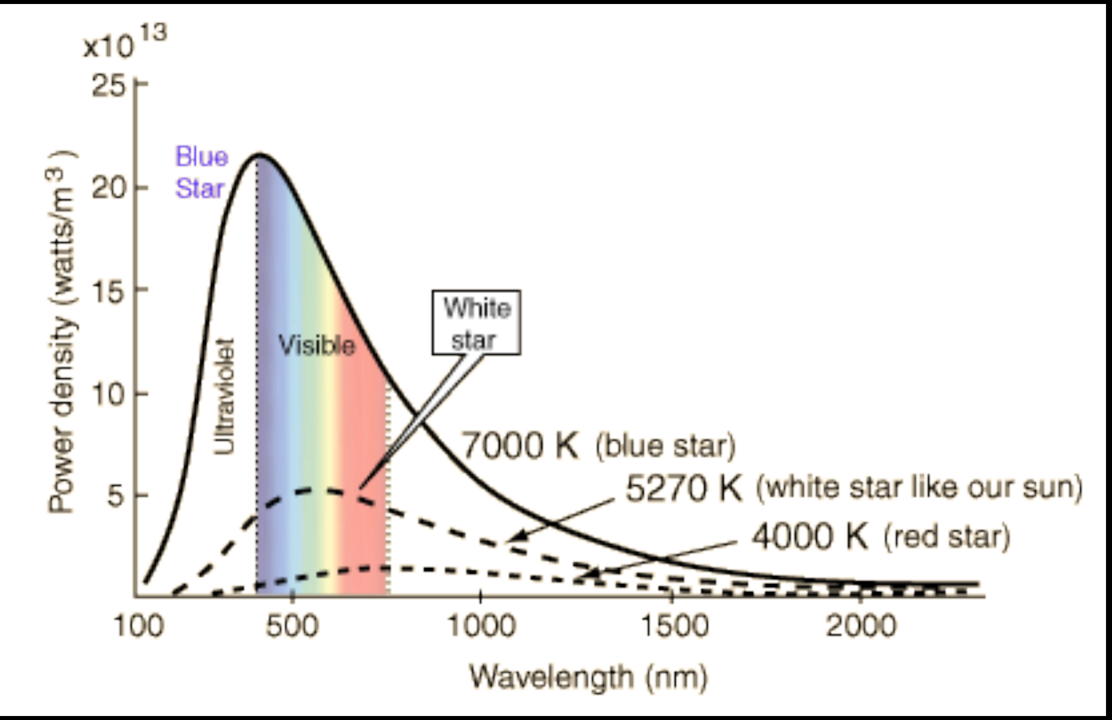


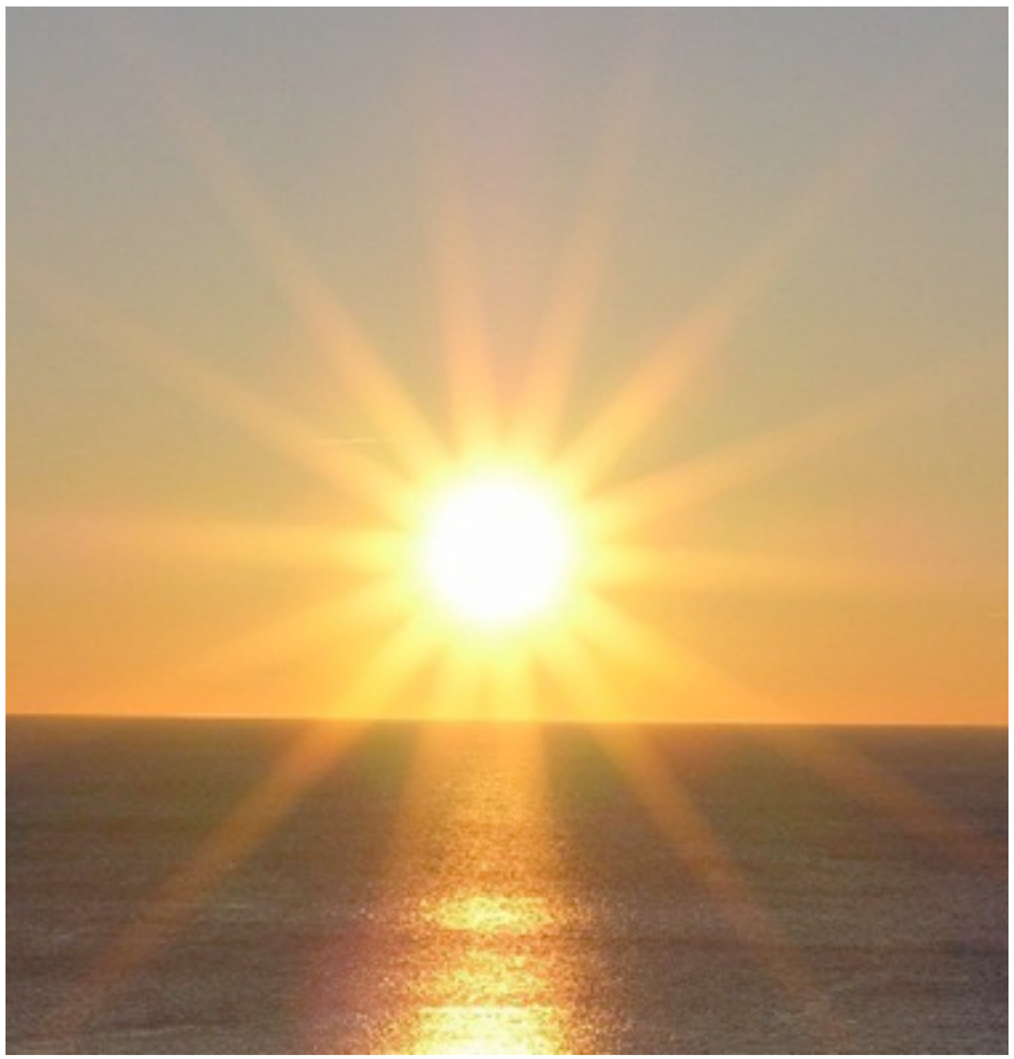
Fig. 6.4: Astrophysics Processes (CUP), © H Bradt 2008

$$\sigma = \frac{2\pi^5 k^4}{15 c^2 h^3} = 5.670 \times 10^{-8} \quad \mathrm{W m^{-2} K^{-4}}$$

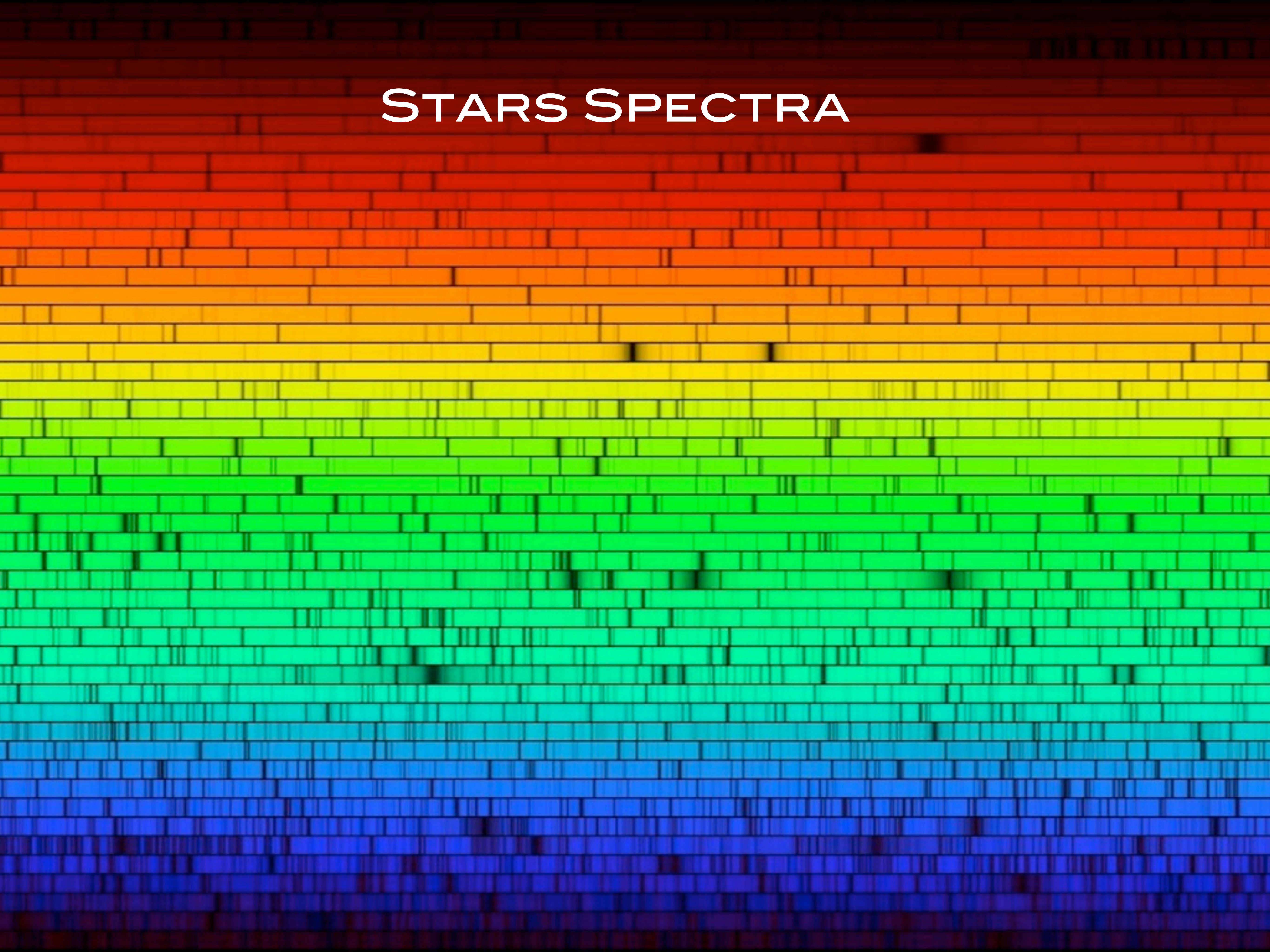
x10^13
25
20
15
10
5
Power density (watts/m^3)
Blue
Star
Ultraviolet
Visible
White
star
7000 K (blue star)
5270 K (white star like our sun)
4000 K (red star)
100
500
1000
1500
2000
Wavelength (nm)

Effective Temperature of a Star

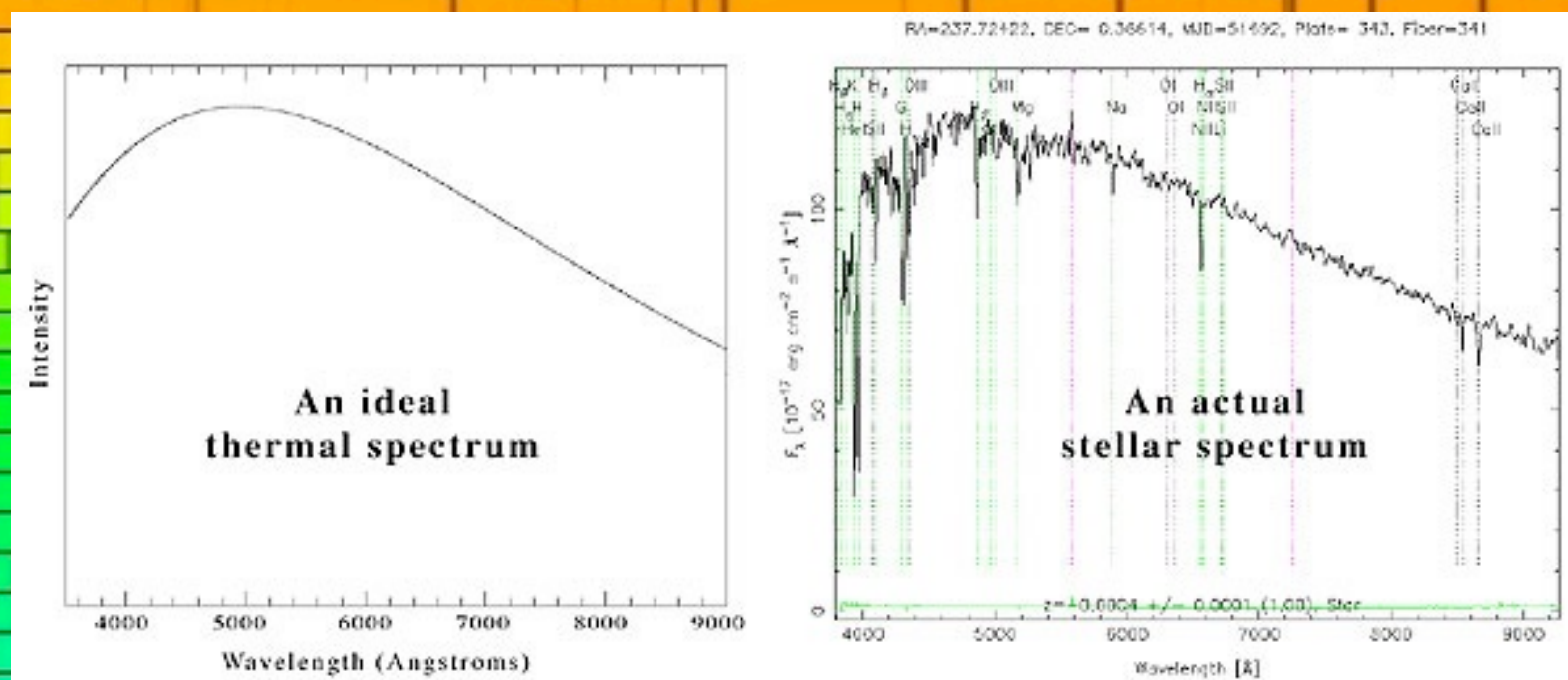


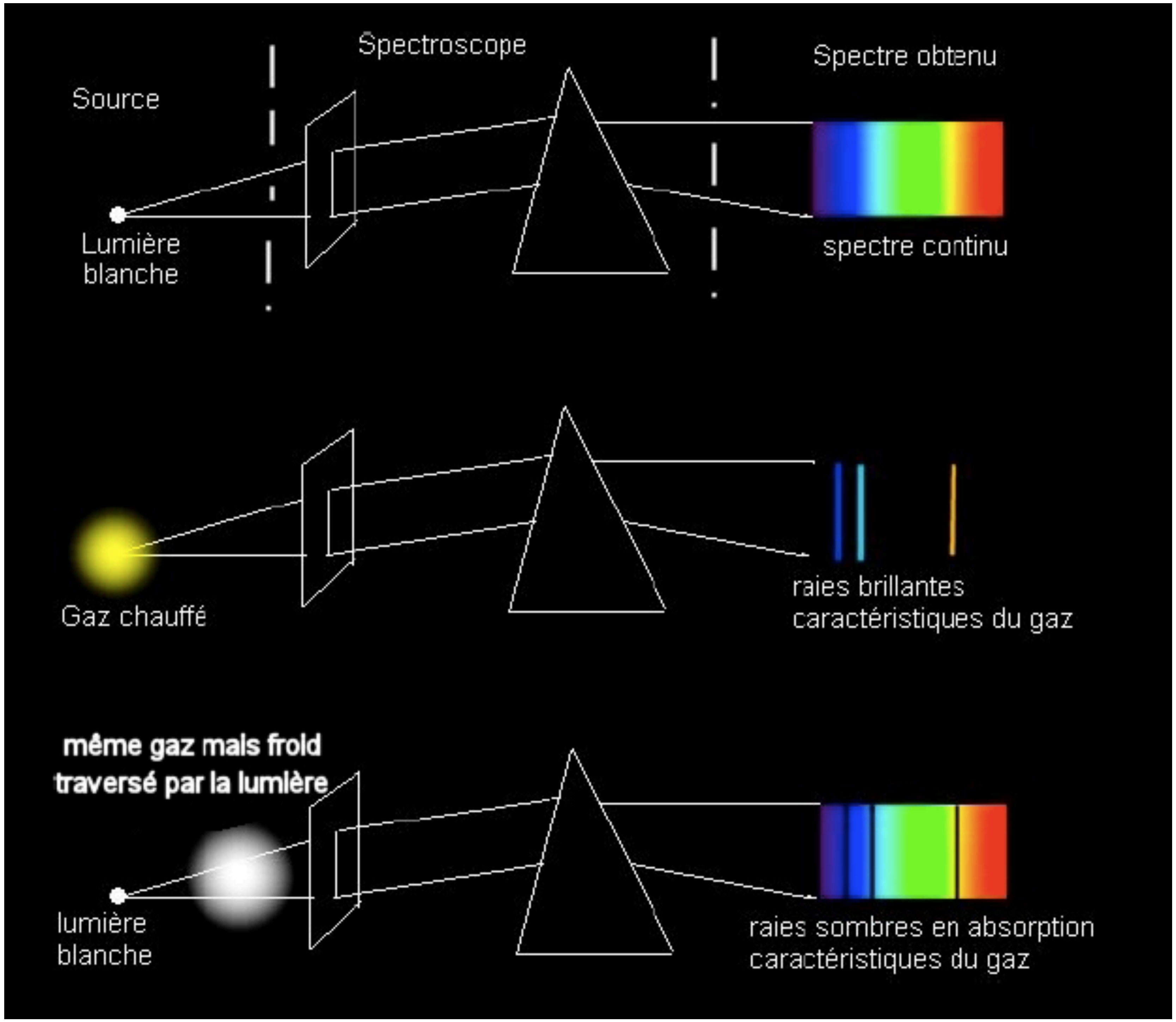
$$L = 4\pi R^2 \sigma T_{\text{eff}}^4 \quad W$$

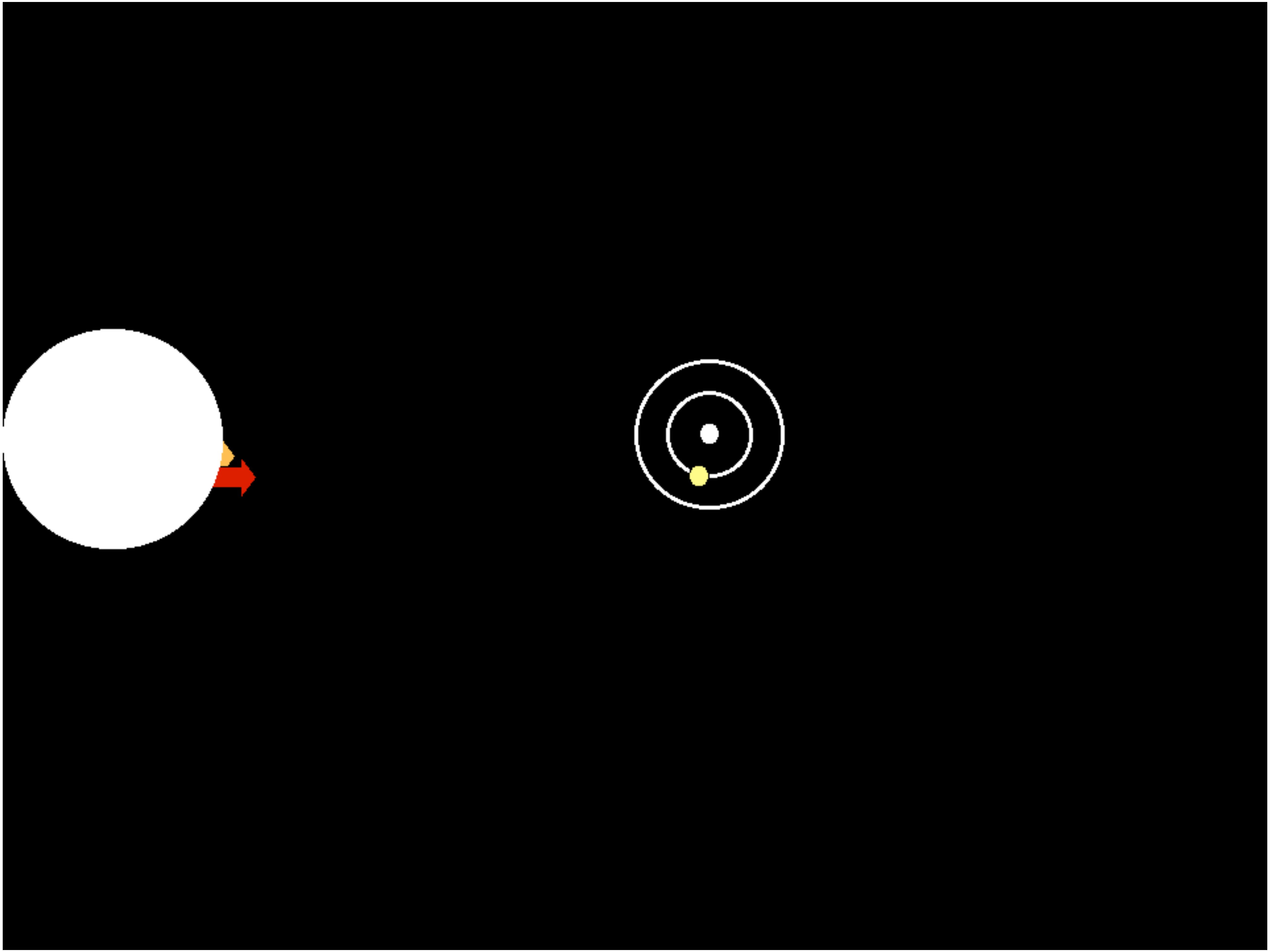
Stars Spectra

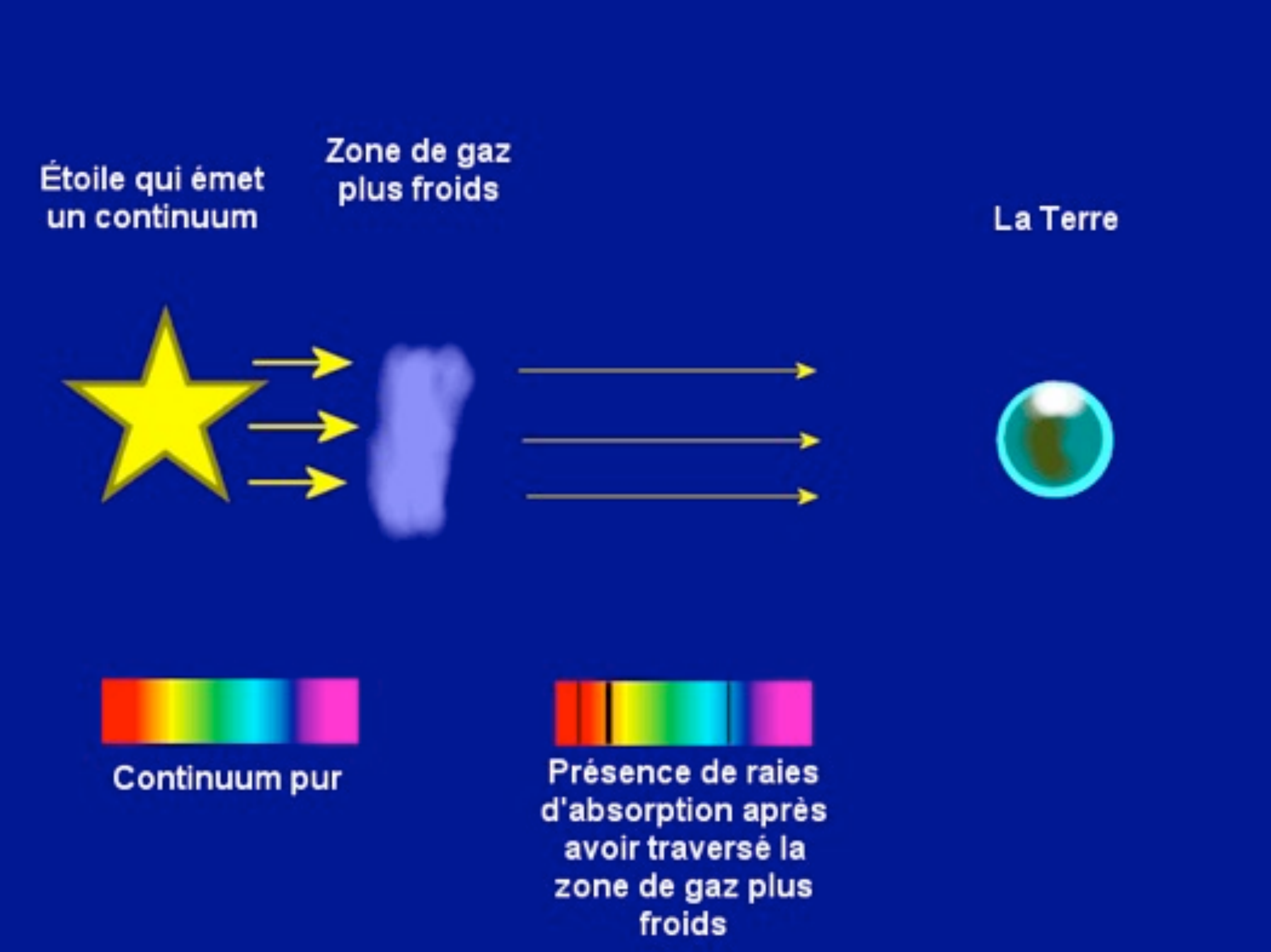


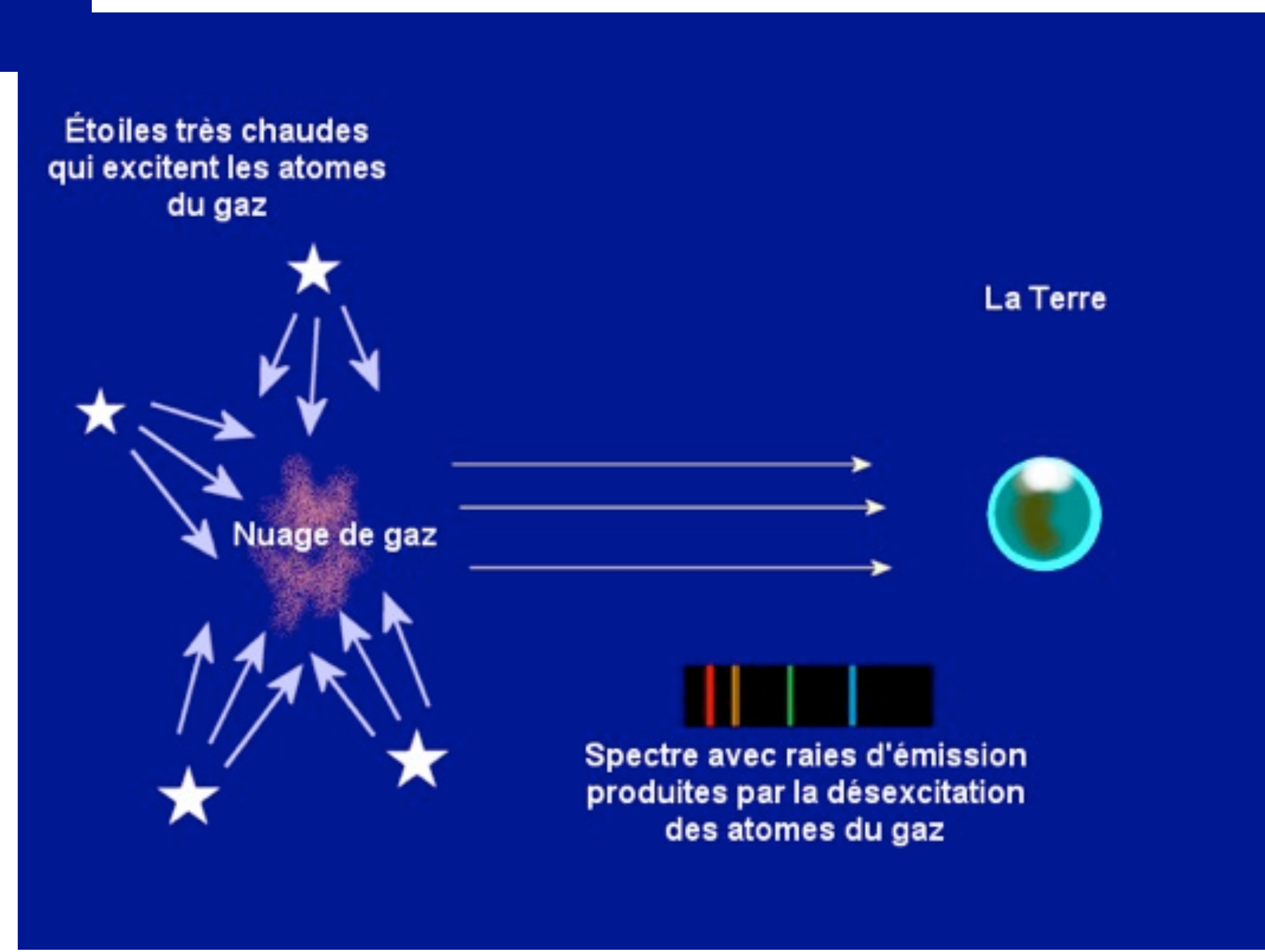
Stars Spectra




Spectroscope
Spectre obtenu
Source
Lumière
blanche
spectre continu
Gaz chauffé
raies brillantes
caractéristiques du gaz
même gaz mais froid
traversé par la lumière
lumière
blanche
raies sombres en absorption
caractéristiques du gaz




Étoile qui émet un continuum
Zone de gaz plus froids
La Terre
Continuum pur
Présence de raies d'absorption après avoir traversé la zone de gaz plus froids


Étoiles très chaudes qui excitent les atomes du gaz
La Terre
Nuage de gaz
Spectre avec raies d'émission produites par la désexcitation des atomes du gaz

Radiation densities
Energy density

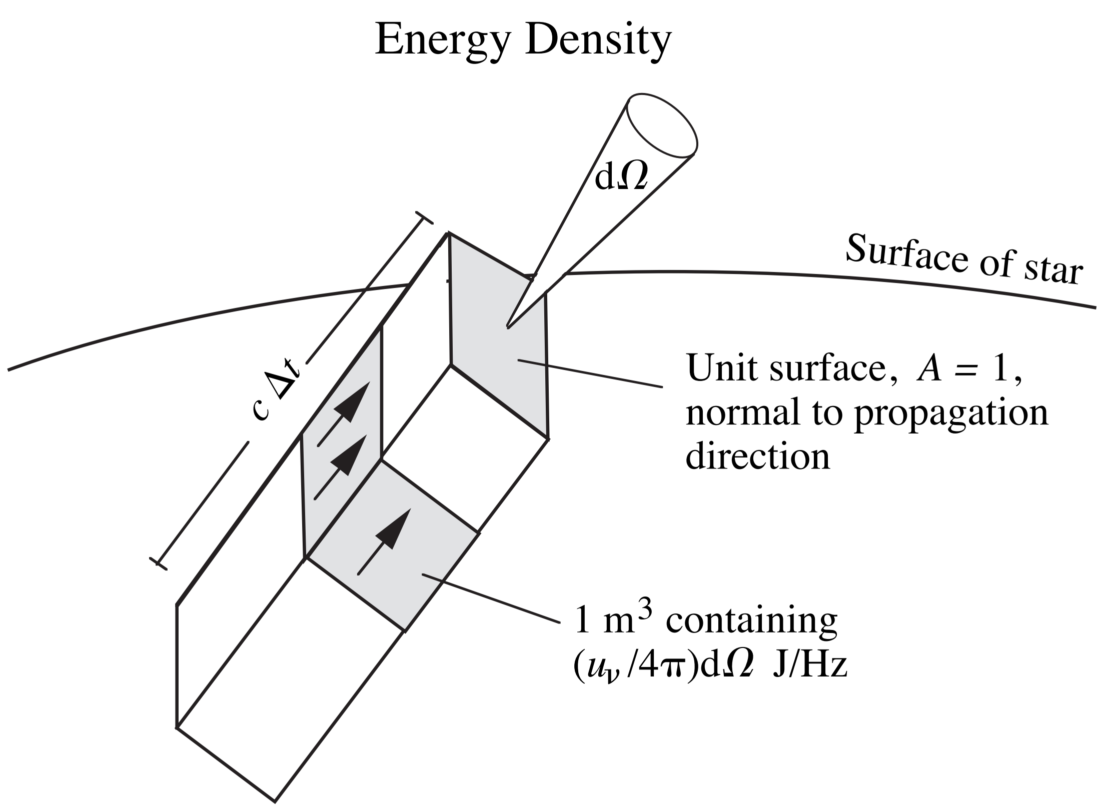


Fig. 6.5: Astrophysics Processes (CUP), © H Bradt 2008

Blackbody energy density:
sum of the energies hν of photons contained at one instant in 1 m^3

$u_\nu(\nu, T)$: Spectral energy density (J m^{-3} Hz^{-1})

$u(T)$: Energy density (J m^{-3})

Radiation densities
Energy density

Energy Density

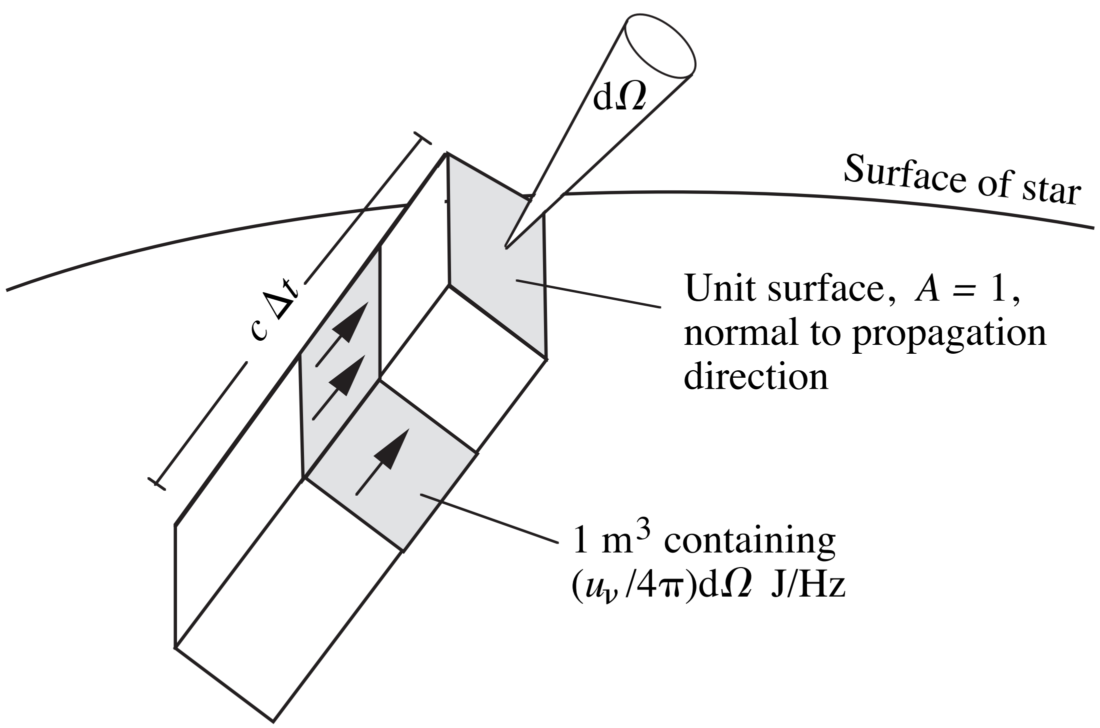


Fig. 6.5: Astrophysics Processes (CUP), © H Bradt 2008

Blackbody energy density:
sum of the energies hν of photons contained at one instant in 1 m³

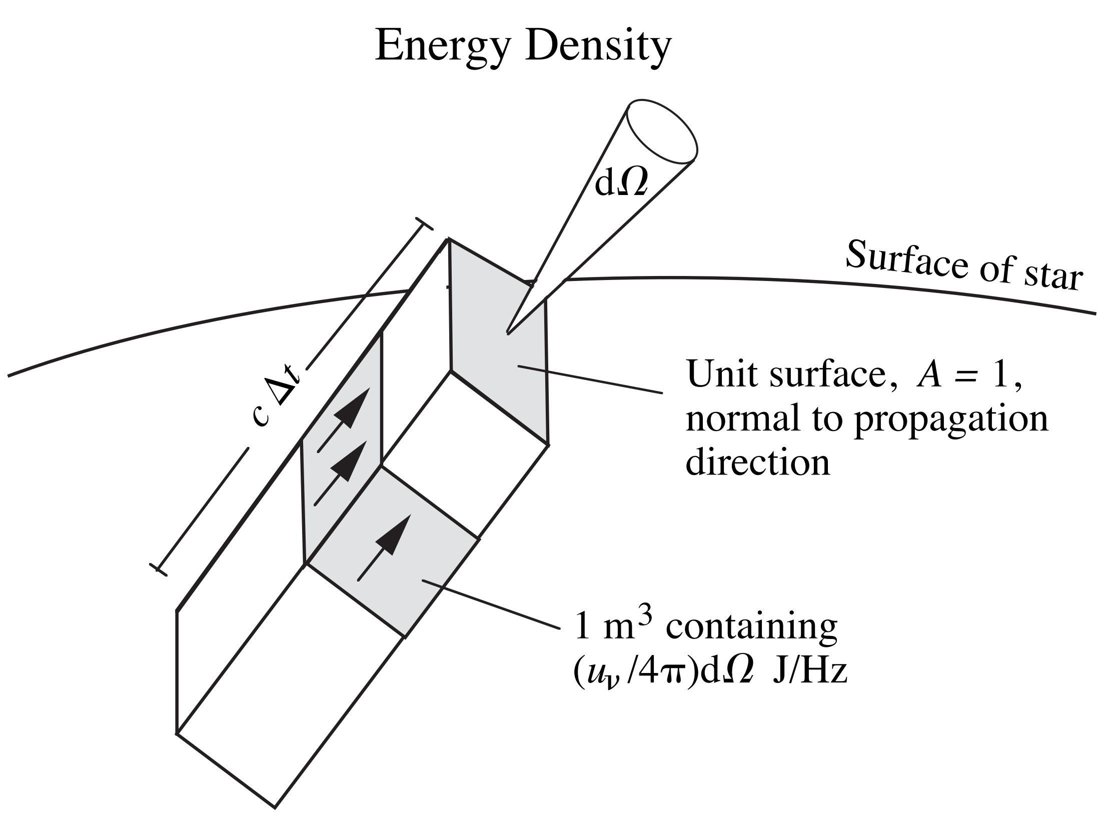
$$u_\nu(\nu, T): \text{ Spectral energy density} \quad (\mathrm{J\ m^{-3}\ Hz^{-1}})$$

$$u(T): \text{ Energy density} \quad (\mathrm{J\ m^{-3}})$$

$$I(\nu, T) = u_\nu(\nu, T)\underbrace{\frac{c}{4\pi}} \quad [\mathrm{W\ s\ m^{-3}\ Hz^{-1} \times m\ s^{-1} \times sr^{-1}}]$$

Energy flux per steradian passing through 1 m² in 1 s
≡ specific intensity

Radiation densities
Energy density



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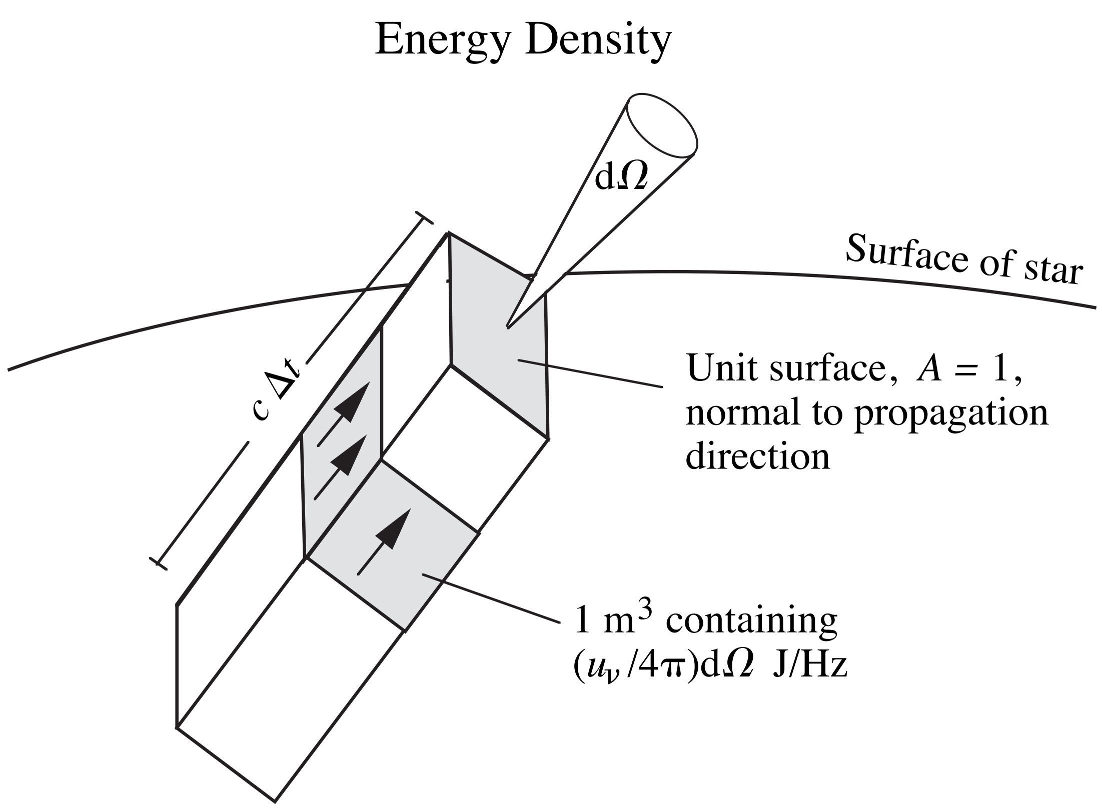
$$I(\nu, T) = \frac{2h\nu^3}{c^2} \frac{1}{e^{h\nu/kT} - 1} \qquad (\mathrm{Wm^{-2}Hz^{-1}sr^{-1}})$$

$$\underbrace{I(\nu, T) = u_\nu(\nu, T)\frac{c}{4\pi}}_{\text{Energy flux per steradian passing through 1 m}^2\text{ in 1 s} \equiv \text{specific intensity}} \qquad [\mathrm{W\ s\ m^{-3}\ Hz^{-1} \times m\ s^{-1} \times sr^{-1}}]$$

Energy flux per steradian passing through 1 m² in 1 s
≡ specific intensity

Radiation densities
Energy density



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$$I(\nu, T) = u_\nu(\nu, T)\frac{c}{4\pi} \qquad [\mathrm{W\ s\ m^{-3}\ Hz^{-1}} \times \mathrm{m\ s^{-1}} \times \mathrm{sr^{-1}}]$$

$$u(\nu, T) = \frac{8\pi h\nu^3}{c^3}\frac{1}{e^{h\nu/kT}-1} \qquad (\mathrm{J\ m^{-3}\ Hz^{-1}})$$

Radiation densities
Energy density

$u_\nu(\nu, T):$ Spectral energy density $\quad (\mathrm{J\ m^{-3}\ Hz^{-1}})$

$u(T):$ Energy density $\quad (\mathrm{J\ m^{-3}})$

$$u(\nu, T) = \frac{8\pi h\nu^3}{c^3}\frac{1}{e^{h\nu/kT} - 1} \quad (\mathrm{J\ m^{-3}\ Hz^{-1}})$$

$$u(T) = \int_0^\infty u_\nu(\nu, T)\mathrm{d}\nu = \frac{4\pi}{c}\int_0^\infty u_\nu I(\nu, T)\mathrm{d}\nu \quad (\mathrm{J\ m^{-3}})$$

Radiation densities
Energy density

$u_\nu(\nu, T):$ Spectral energy density $\quad (\mathrm{J\ m^{-3}\ Hz^{-1}})$

$u(T):$ Energy density $\quad (\mathrm{J\ m^{-3}})$

$$u(\nu, T) = \frac{8\pi h\nu^3}{c^3}\frac{1}{e^{h\nu/kT} - 1} \quad (\mathrm{J\ m^{-3}\ Hz^{-1}})$$

$$u(T) = \int_0^\infty u_\nu(\nu, T)\mathrm{d}\nu = \frac{4\pi}{c}\int_0^\infty u_\nu I(\nu, T)\mathrm{d}\nu \quad (\mathrm{J\ m^{-3}})$$

$$u(T) = \frac{4}{c}\sigma T^4 = aT^4 \quad (\mathrm{J\ m^{-3}})$$

$$a = \frac{4\sigma}{c} = 7.566 \times 10^{-16} (\mathrm{J\ m^{-3}\ K^{-4}})$$

Radiation densities
Spectral number density

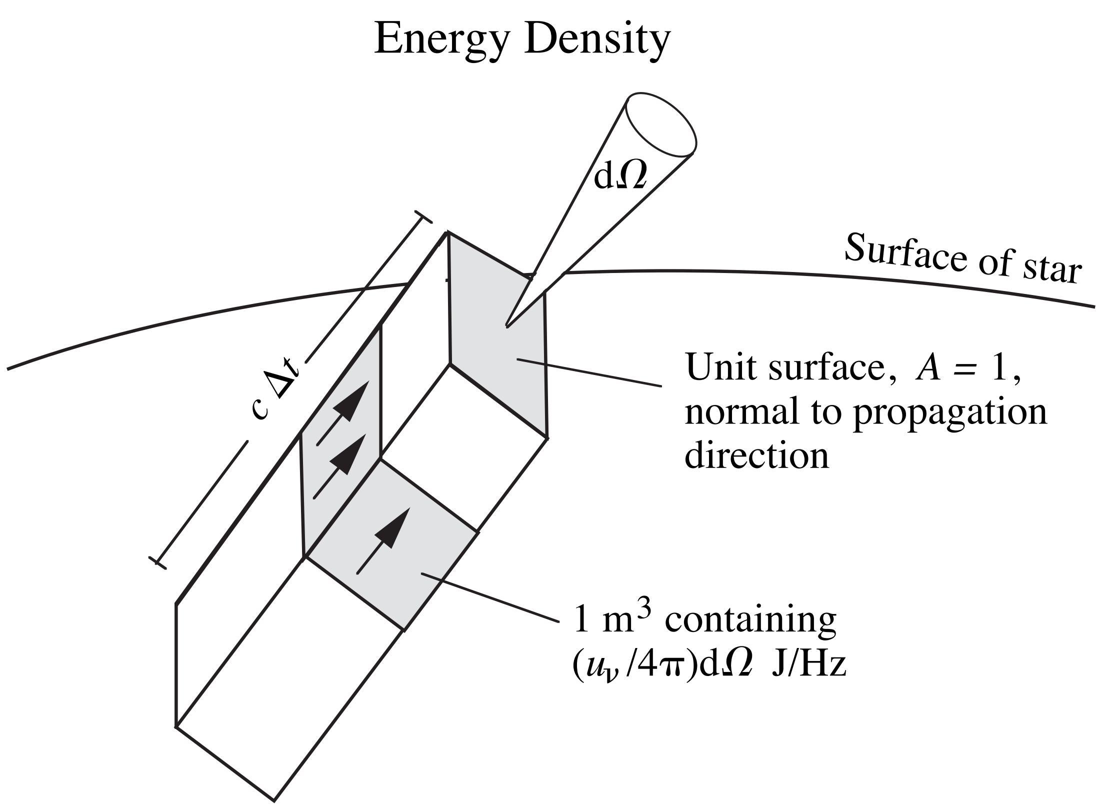

Fig. 6.5: Astrophysics Processes (CUP), © H Bradt 2008

Blackbody energy density:
sum of the energies hν of photons contained at one instant in 1 m^3

$u_\nu(\nu, T)$: Spectral energy density $\quad (\mathrm{J\ m^{-3}\ Hz^{-1}})$

$u(T)$: Energy density $\quad (\mathrm{J\ m^{-3}})$

$$u(\nu, T) = \frac{8\pi h\nu^3}{c^3}\frac{1}{e^{h\nu/kT}-1} \quad (\mathrm{J\ m^{-3}\ Hz^{-1}})$$

$$n_\nu(\nu, T) = \frac{u_\nu(\nu, T)}{h\nu} = \frac{8\pi\nu^2}{c^3}\frac{1}{e^{h\nu/kT}-1} \quad (\mathrm{m^{-3}\ Hz^{-1}})$$

Spectral number density = Spectral energy density / Energy of a single photon

Radiation densities
Total number density

$$n_\nu(\nu, T) = \frac{u_\nu(\nu, T)}{h\nu} = \frac{8\pi\nu^2}{c^3}\frac{1}{e^{h\nu/kT} - 1} \quad (\mathrm{m}^{-3}\ \mathrm{Hz}^{-1})$$

$$n(T) = \int_0^\infty n_\nu(\nu, T)\mathrm{d}\nu = \frac{8\pi}{c^3}\int_0^\infty \frac{\nu^2}{e^{h\nu/kT} - 1} \quad (\mathrm{m}^{-3})$$

$$n(T) = 16\pi\left(\frac{kT}{hc}\right)^3 \times 1.202 = 2.029 \times 10^7\ T^3 \quad (\mathrm{m}^{-3})$$

Radiation densities
Total number density

$$n_\nu(\nu, T) = \frac{u_\nu(\nu, T)}{h\nu} = \frac{8\pi\nu^2}{c^3}\frac{1}{e^{h\nu/kT} - 1} \quad (\mathrm{m^{-3}\ Hz^{-1}})$$

$$n(T) = \int_0^\infty n_\nu(\nu, T)\mathrm{d}\nu = \frac{8\pi}{c^3}\int_0^\infty \frac{\nu^2}{e^{h\nu/kT} - 1} \quad (\mathrm{m^{-3}})$$

$$n(T) = 16\pi\left(\frac{kT}{hc}\right)^3 \times 1.202 = 2.029 \times 10^7\ T^3 \quad (\mathrm{m^{-3}})$$

$$T \sim 3\ \mathrm{K} \ \rightarrow \ \approx 0.55\ \mathrm{photons\ mm^{-3}}$$

$$T \sim 3\ \mathrm{K} \ \rightarrow \ \lambda_{\mathrm{peak}} \approx 1.0\ \mathrm{mm}$$

At 3 K, each cubic millimeter contains ~ 1 photons of wavelength ~ 1 mm

Radiation densities
Total number density

$$n_\nu(\nu,T) = \frac{u_\nu(\nu,T)}{h\nu} = \frac{8\pi\nu^2}{c^3}\frac{1}{e^{h\nu/kT}-1} \quad (\mathrm{m^{-3}\ Hz^{-1}})$$

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$$T\sim 3\ \mathrm{K} \ \rightarrow \ \approx 0.55\ \text{photons mm}^{-3}$$

$$T\sim 3\ \mathrm{K} \ \rightarrow \ \lambda_{\mathrm{peak}} \approx 1.0\ \mathrm{mm}$$

At 3 K, each cubic millimeter contains ~ 1 photons of wavelength ~ 1 mm

$$T\lambda_{\mathrm{peak}} = 2.898\times10^{-3} \quad (\mathrm{Km}) \quad \rightarrow \quad \lambda_{\mathrm{peak}} \propto T^{-1}$$

$$+ \quad n(T) \propto T^3$$

A cube of size about equal to the peak wavelength will contain about one photons

Radiation densities
Average photon energy

$$n(T) = 16\pi \left(\frac{kT}{hc}\right)^3 \times 1.202 = 2.029 \times 10^7\ T^3 \quad (\mathrm{m}^{-3})$$

$$u(T) = \frac{4}{c}\sigma T^4 = aT^4 \quad (\mathrm{J\ m}^{-3})$$

$$h\nu_{av} = \frac{u(T)}{n(T)} = \frac{\pi^4}{30 \times 1.202} kT = 2.70\ kT \quad (\mathrm{J})$$

$$h\nu_{\mathrm{peak}} = 2.82\ k\ T$$

Image credits for this lecture

- H. Bradt, 2008, “Astrophysical processes”, Cambridge University Press
- ESA web page