

PLANETARY MIGRATION

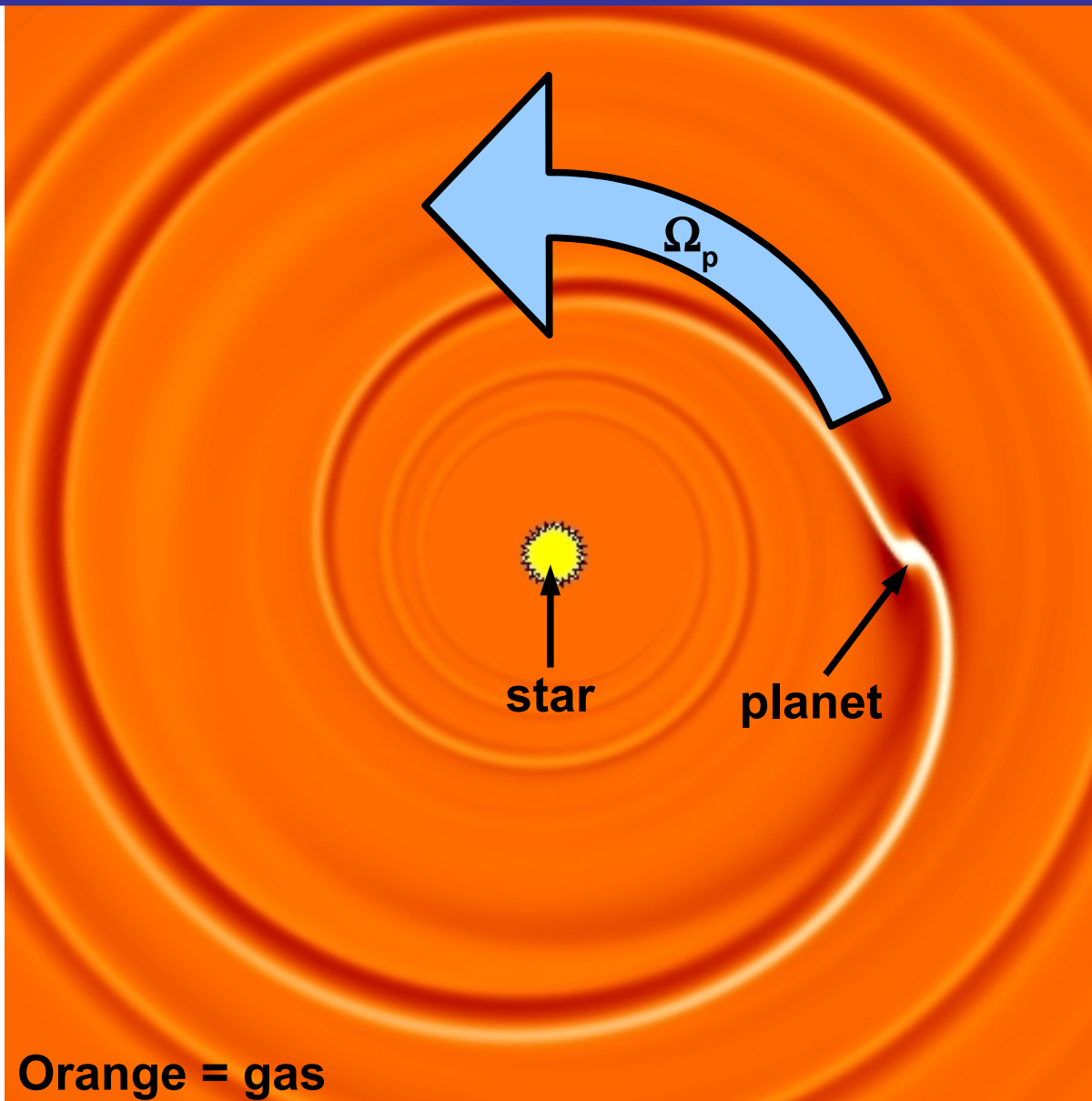
A. CRIDA



(the migration of Jupiter
carrying its satellites.
Alegory.)



INTRODUCTION



A planet orbiting in a protoplanetary disk launches a one-armed, spiral wake.

This is a pressure-supported wave, caused by the planet's gravity perturbing the trajectories of the gas particles.

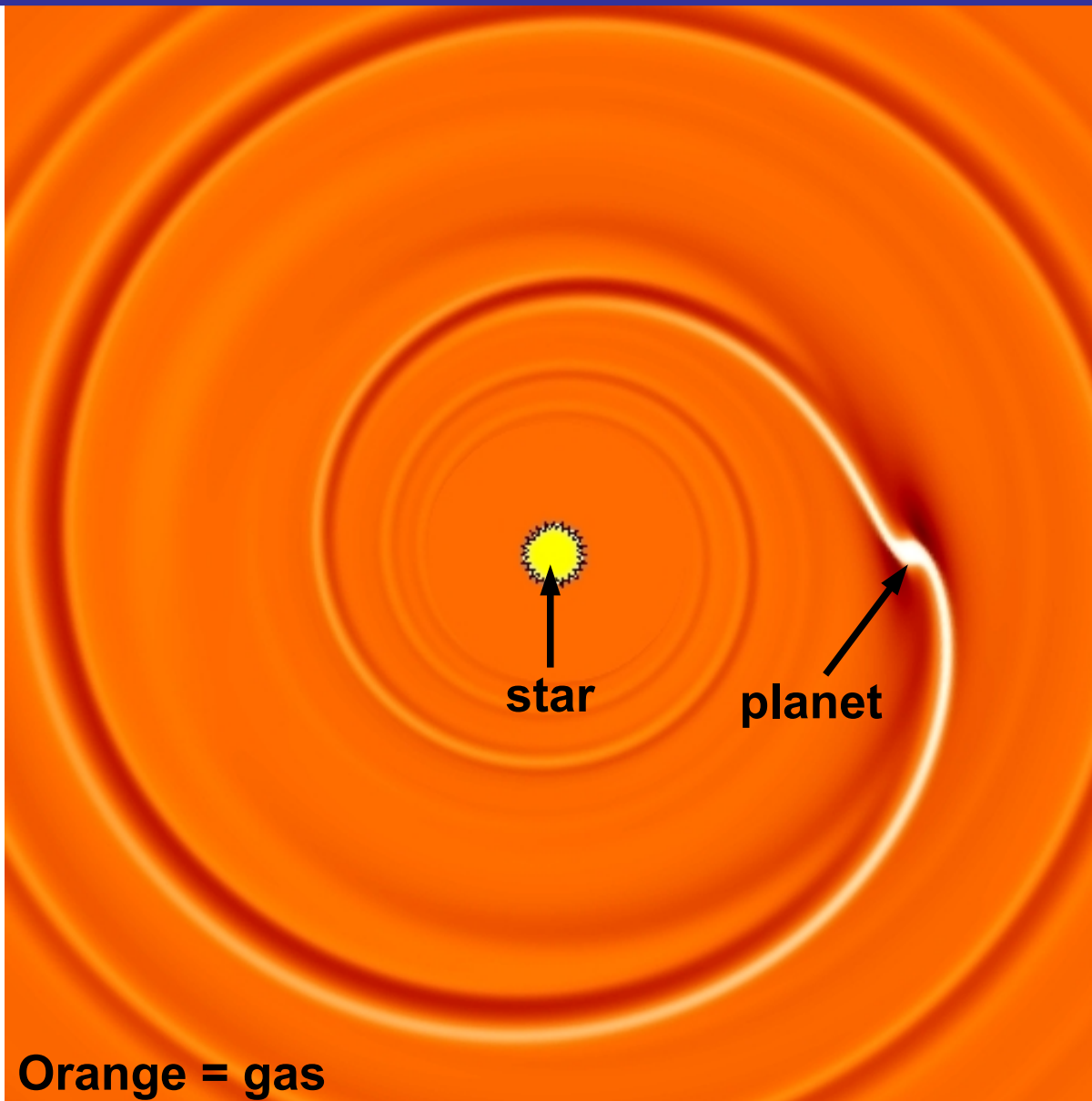
The wave corotates with the planet at the same angular velocity

$$\Omega_p = (GM_*/a_p^3)^{1/2}.$$

Orange = gas

Clear/white = over-density , Dark = underdensity.

INTRODUCTION



Wake = over-density.
This mass in turn
attracts the planet
gravitationally.

These forces exerted
on the planet have a
momentum with respect
to the central star.

It leads to a torque, and
thus a change in the
planet's orbital angular
momentum

$$L_p = M_p (GM_* a_p)^{1/2},$$

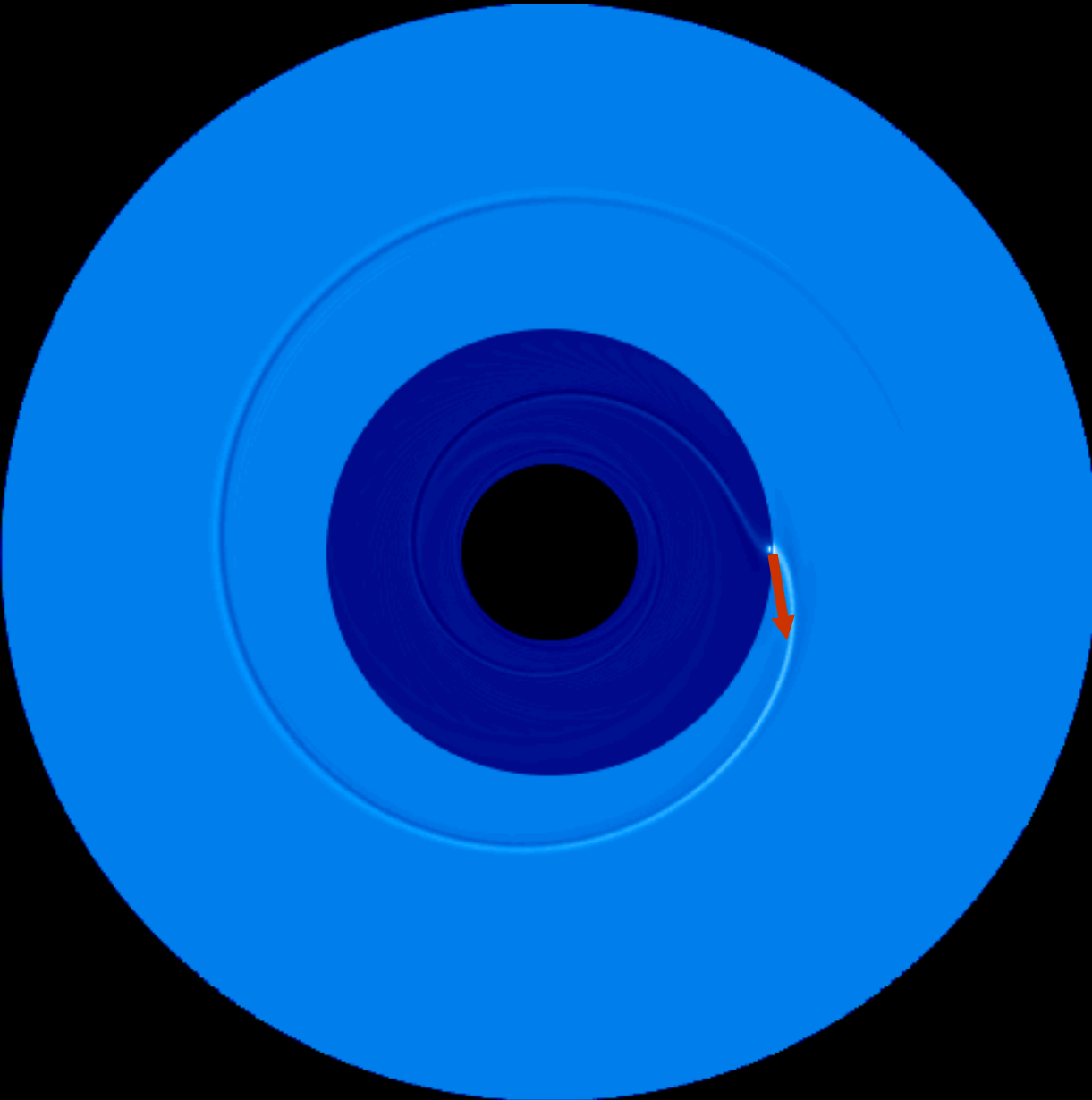
thus a change in a_p :

planetary migration.

Orange = gas

Clear/white = over-density , Dark = underdensity.

WAVE TORQUE

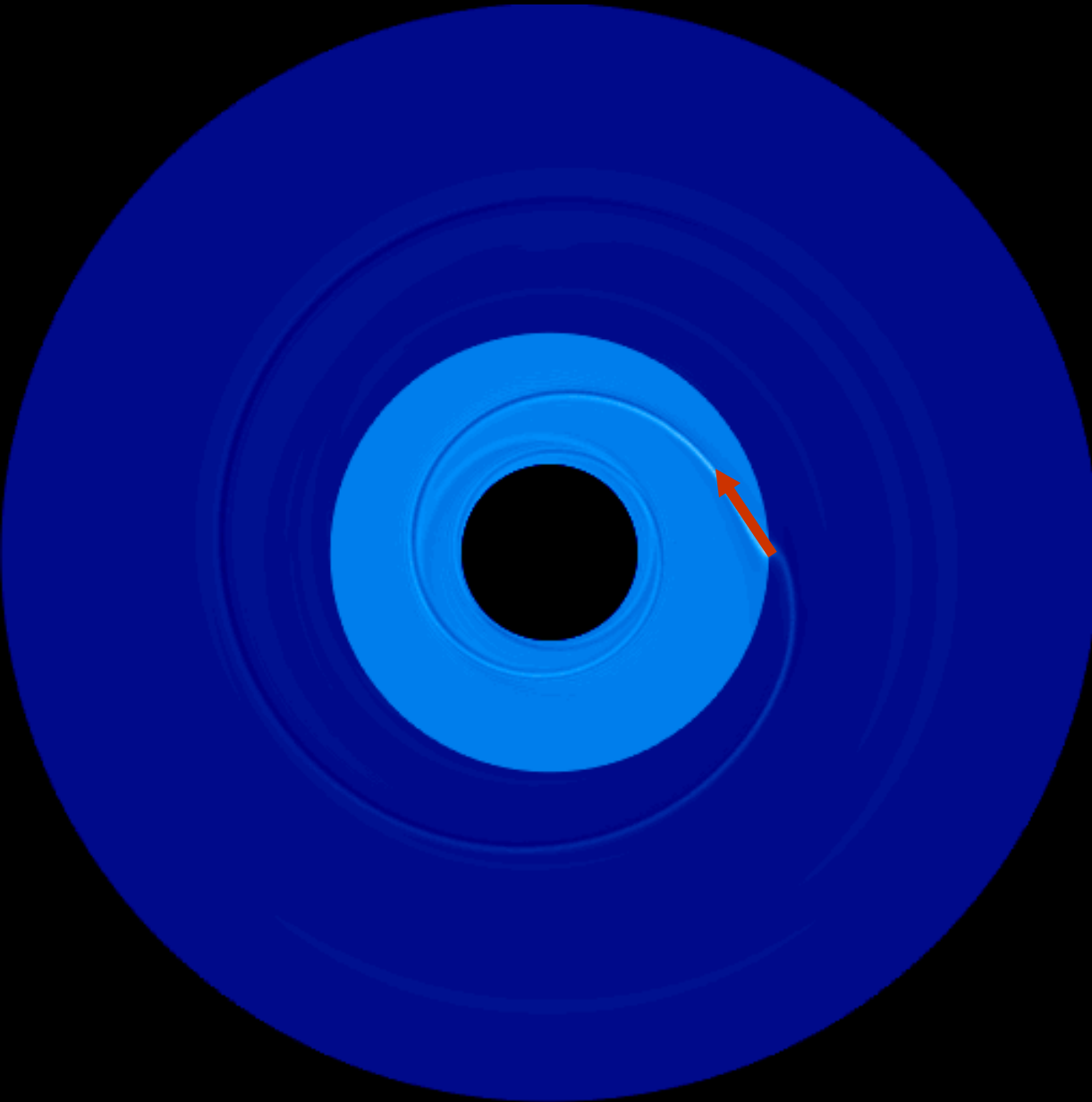


In the outer disk, the keplerian rotation is slower, so the wake trails behind the planet.

Thus, it exerts on the planet a negative torque.

It slows the planet down, and pushes it towards the central star.

WAVE TORQUE



In the inner disk, the keplerian rotation is faster, so the wake leads the planet.

Thus, it exerts on the planet a positive torque.

It accelerates the planet, and pulls it outwards.

TYPE I MIGRATION

In general, the (negative) outer torque is larger (in magnitude) than the (positive) inner torque (Ward, 1997). The planet loses orbital angular momentum, and its orbital radius a_p decreases.

The inner / outer torque is called *one sided Lindblad torque*. The total is called the *differential Lindblad torque* and is :

$$\gamma \Gamma_L / \Gamma_0 = -2.5 - 1.7\beta_T + 0.1\alpha_\Sigma$$

where $\Gamma_0 = q^2 \Sigma a_p^4 \Omega_p^2 (H/r)^{-2}$,

$q = M_p / M_*$, Σ = surface density of the gas, $\Sigma \sim r^{-\alpha_\Sigma}$, $T \sim r^{-\beta_T}$,

H/r = aspect ratio of the disk, γ = adiabatic index.

(Paardekooper et al. 2010)

Note: in the linear regime, the amplitude of the wake is proportionnal to the planetary mass, and the force is proportionnal to the product of the planetary mass and that of the wake. Therefore, Γ_0 is proportionnal to q^2 .

TYPE I MIGRATION

Migration in the linear regime is often called *type I migration*. It concerns small mass proto-planets (~ a few Earth masses).

The migration speed is proportionnal to the planet's mass.

Exer : Express dr_p / dt .

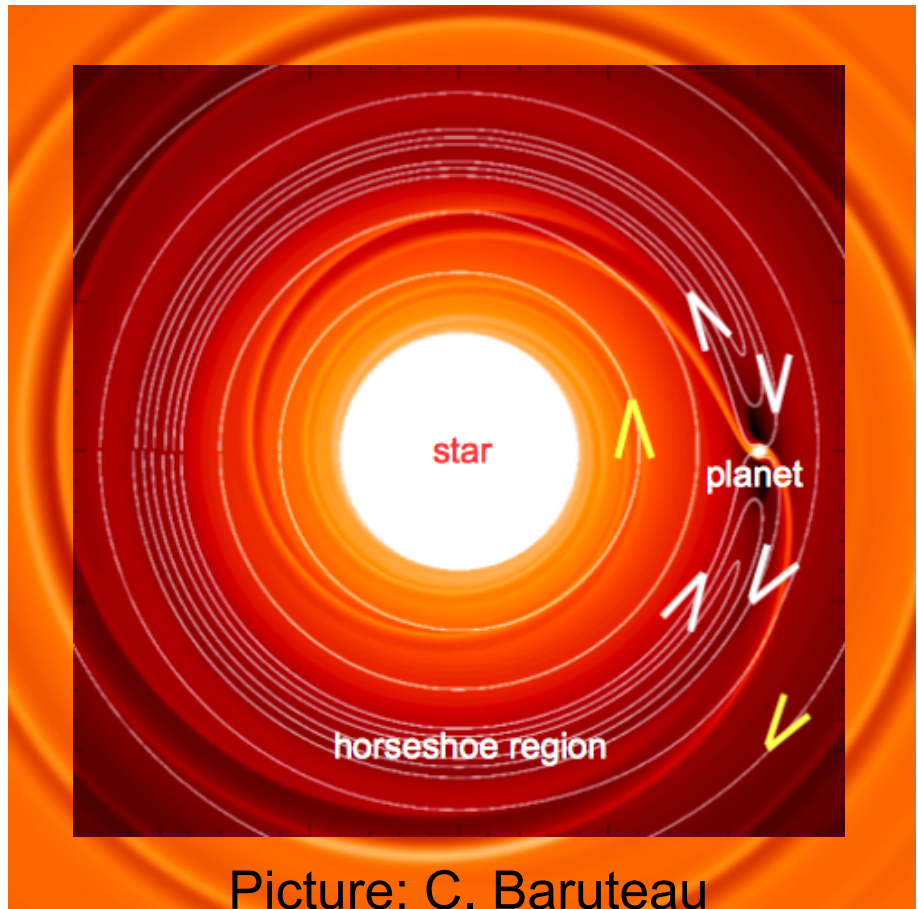
A.N. : Calculer le temps de migration $t_{\text{migr}} = r_p / (dr_p/dt)$ associé au couple différentiel de Lindblad si :

$$M_p = 1M_{\text{Earth}}, \Sigma = 1700 \text{ g/cm}^2, H/r = 0.05$$

$$M_p = 5M_{\text{Earth}}, \Sigma = 5100 \text{ g/cm}^2, H/r = 0.1$$

COROTATION TORQUE

Around the planetary orbit, the gas corotates with the planet. The streamlines of the velocity field have horseshoe shapes.

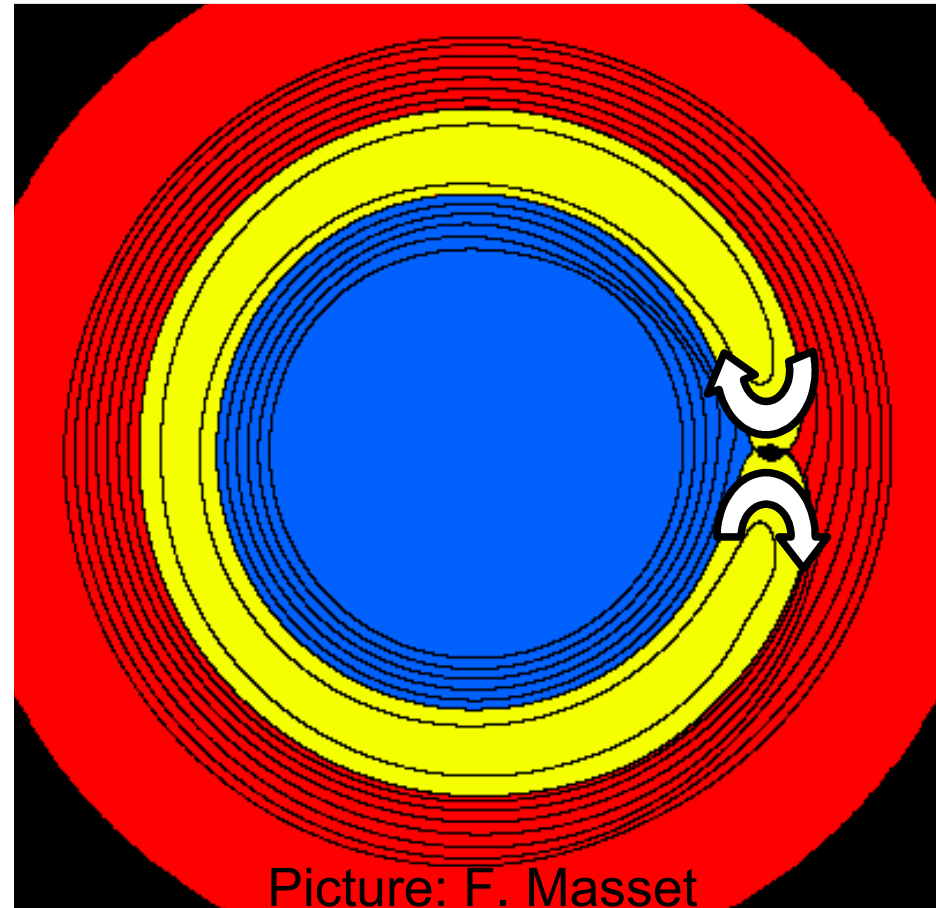


Picture: C. Baruteau

COROTATION TORQUE

Around the planetary orbit, the gas corotates with the planet. The streamlines of the velocity field have horseshoe shapes.

The torque arising from this «horseshoe region», the *corotation torque* Γ_c , has been widely studied in the last ~7 years.



Picture: F. Masset

COROTATION TORQUE

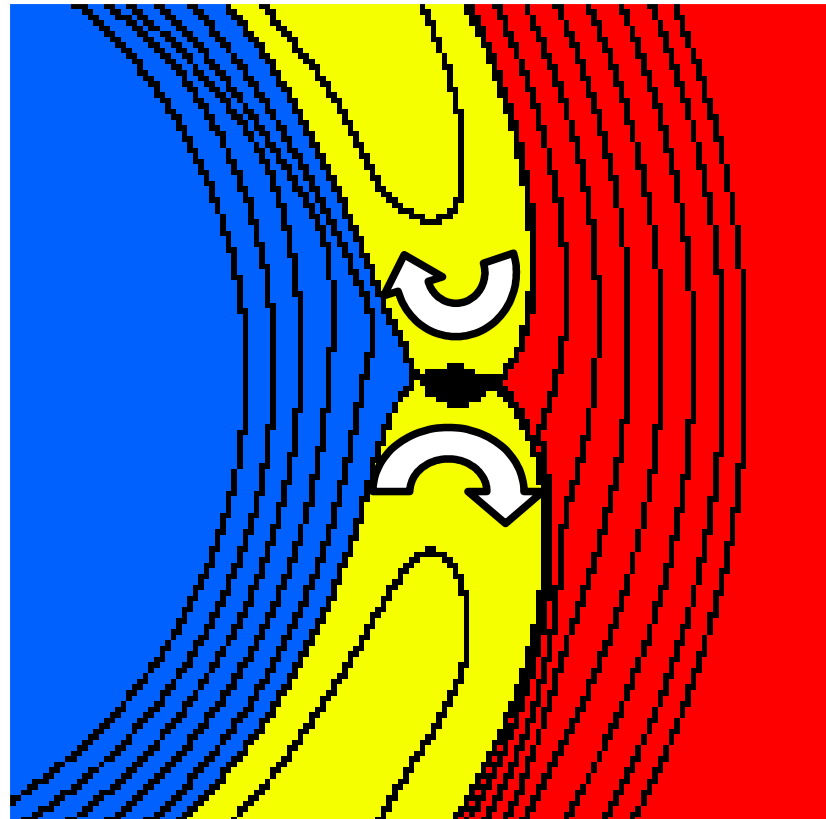
$$\gamma \Gamma_c / \Gamma_0 = 1.1 (3/2 - \alpha_\Sigma) + 7.9 \xi / \gamma$$

$$\xi = \beta_T - (\gamma - 1) \alpha_\Sigma$$

(Paardekooper et al. 2010)

1st term : barotropic part

(e.g.: Ward 1991, Masset 2001,
Paardekooper & Papaloizou 2009)



COROTATION TORQUE

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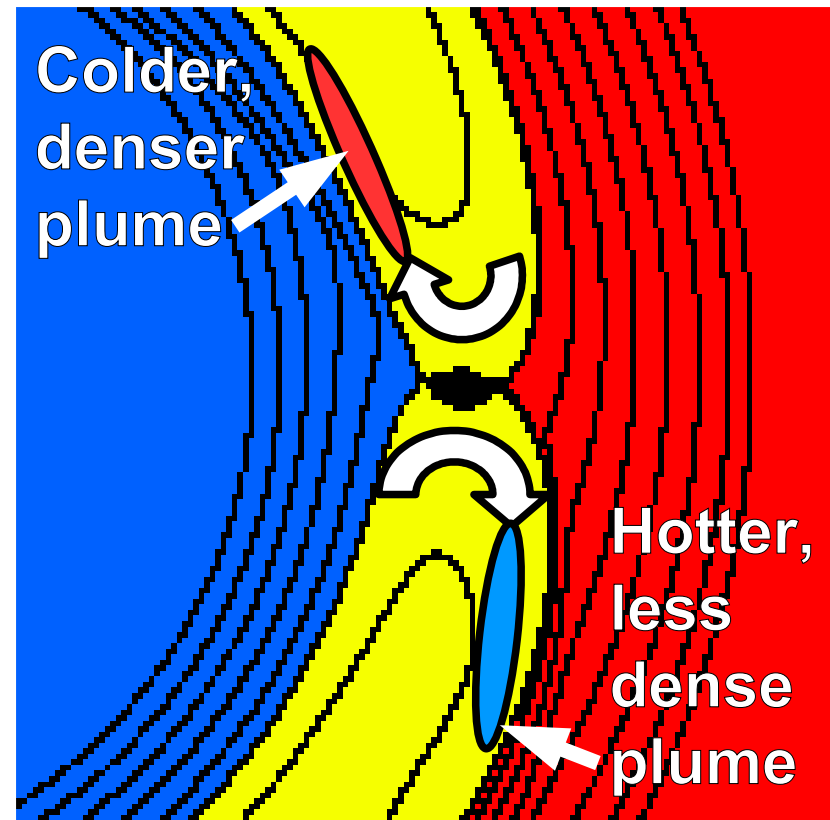
1st term : barotropic part

(e.g.: Ward 1991, Masset 2001,
Paardekooper & Papaloizou 2009)

2nd term : thermal part, due to
the advection of the entropy :

$$\xi = - \text{dlog}(\text{entropy}) / \text{dlog}(r)$$

(Paardekooper & Mellema 2008,
Baruteau & Masset 2008)



TYPE I MIGRATION

$$\gamma \Gamma_c / \Gamma_0 = 1.1 (3/2 - \alpha_\Sigma) + 7.9 \xi / \gamma$$

(Paardekooper et al. 2010)

As $\alpha_\Sigma < 3/2$ and $\xi > 0$, this torque is generally positive, and can overcome the negative Γ_L ,

→ outward migration !

Total torque (assuming $\gamma=1.4$) :

$$(\Gamma_L + \Gamma_c) / \Gamma_0 = -0.64 - 2.3 \alpha_\Sigma + 2.8 \beta_T$$

TYPE I MIGRATION

$$\gamma \Gamma_{\text{c,circ,unsaturated}} / \Gamma_0 = 1.1 (3/2 - \alpha_\Sigma) + 7.9 \xi / \gamma$$



This is only true on circular orbits.

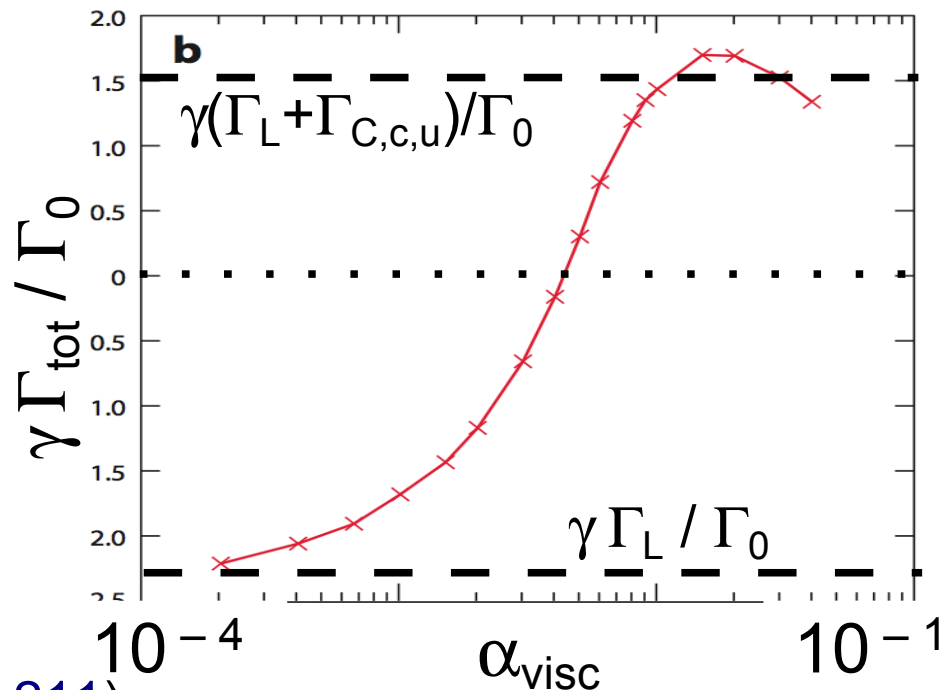
$\Gamma_c \rightarrow 0$ for large e . (Bitsch & Kley 2010, Fendyke & Nelson 2013)



The corotation torque is prone to *saturation*.

The horseshoe region only has a limited a.m. to exchange.
Needs to be refreshed, through **viscosity**, otherwise $\Gamma_c \rightarrow 0$.

(see Masset & Casoli 2010,
Paardekooper et al. 2011,
Fig: Kley & Nelson 2012, ARAA, 52, 211).



TYPE I MIGRATION

The total torque depends on α_Σ and β_T ,
thus on the disc structure.

Viscous heating $><$ radiative cooling :
large β_T , non flared disks, easy outward migration.

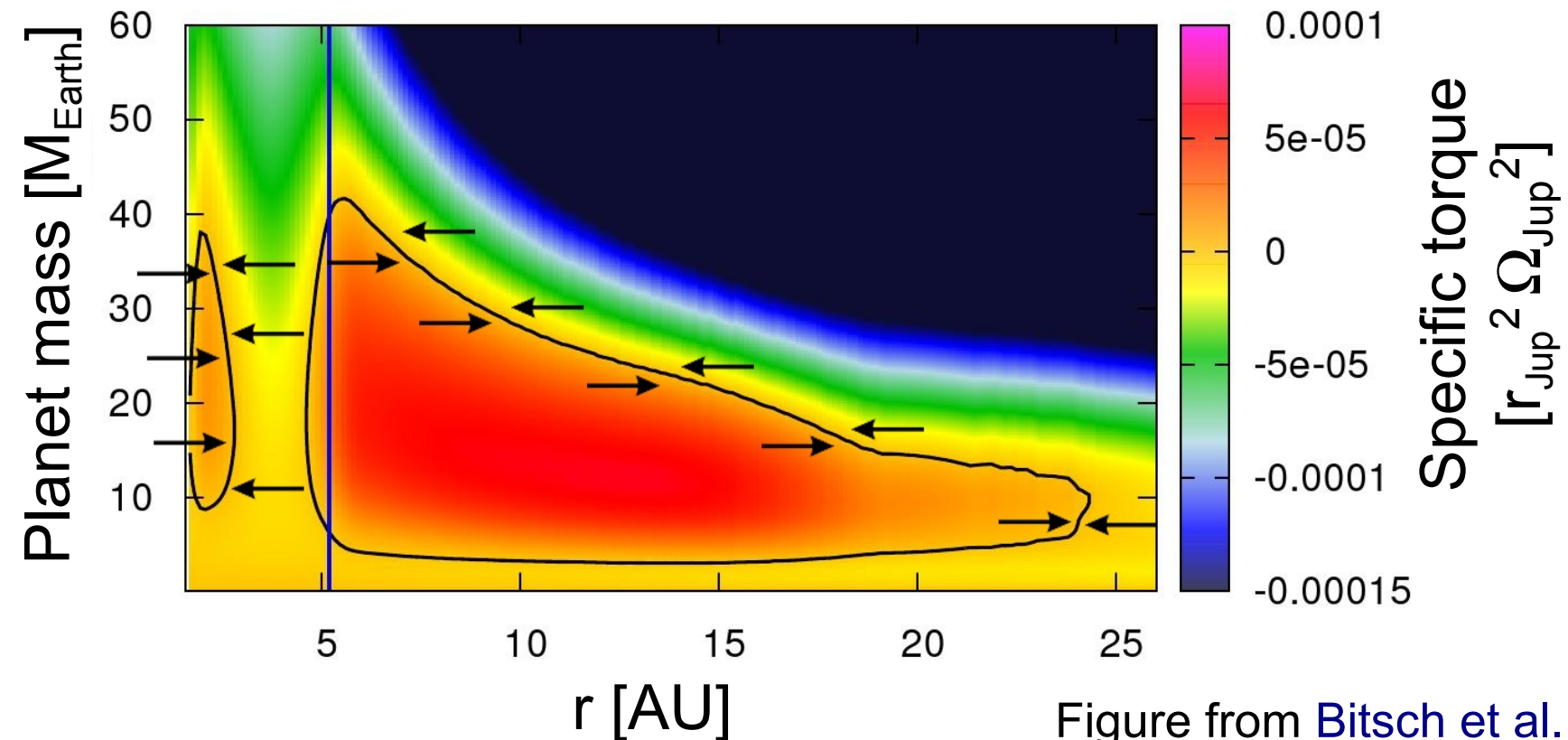


Figure from Bitsch et al. 2013

TYPE I MIGRATION

The total torque depends on α_Σ and β_T ,
thus on the disc structure.

Stellar irradiation + Viscous heating $><$ radiative cooling :
large β_T , flared disks, smaller outward migration zone.

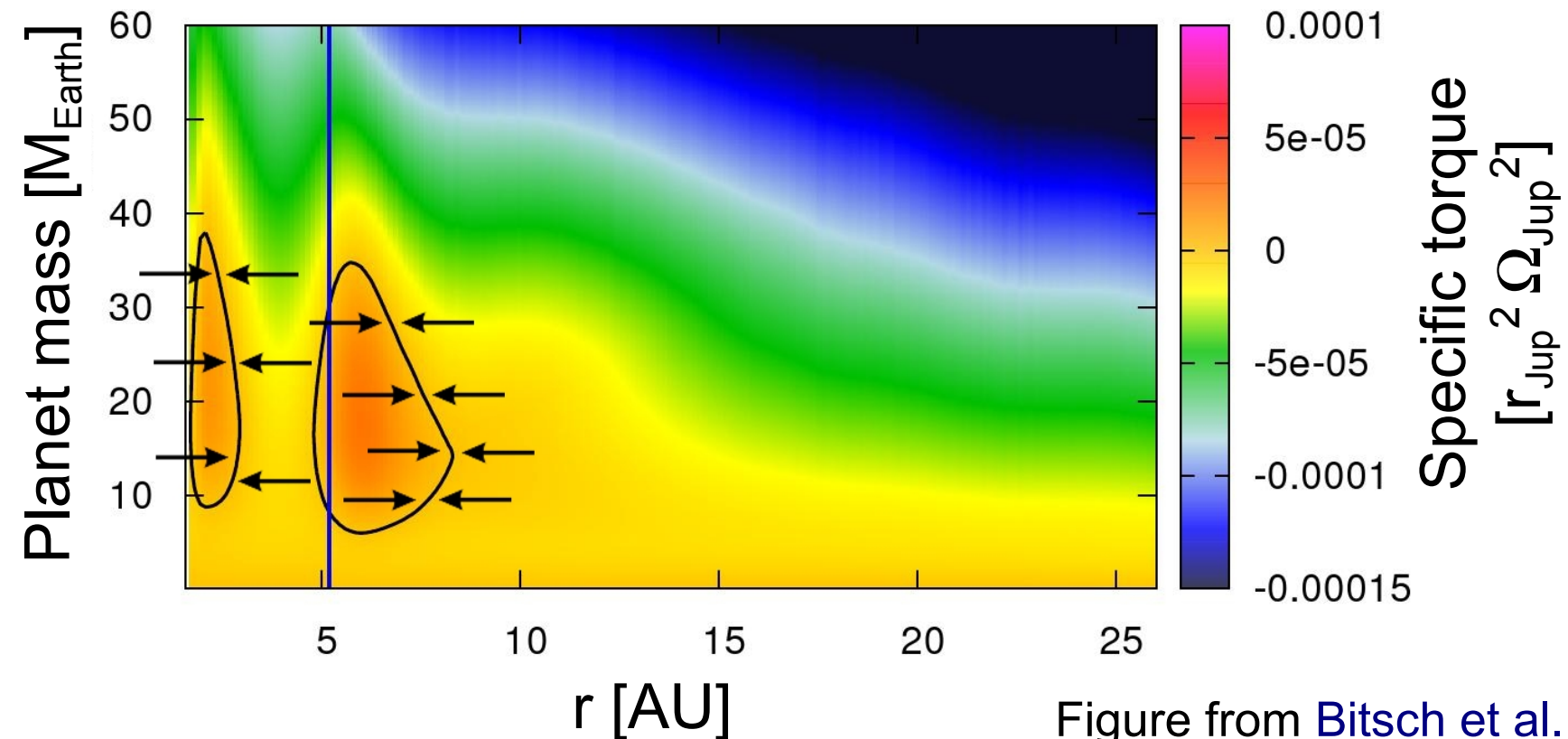


Figure from Bitsch et al. 2013

TYPE I MIGRATION

Summary :

Small mass planets launch a spiral wake (*sillage*).

They feel a torque from the disk, prop to $\Gamma_0 = (q/h)^2 \Sigma a_p^4 \Omega_p^2$, which changes their orbital angular momentum, hence radius: migration !

- the Differential Lindblad Torque promotes (too fast) inwards migration
 - the Corotation Torque can promote outward migration
 - the total torque depends on the planet mass, and on disk local properties.
- zero torque radius, where embryos converge ?

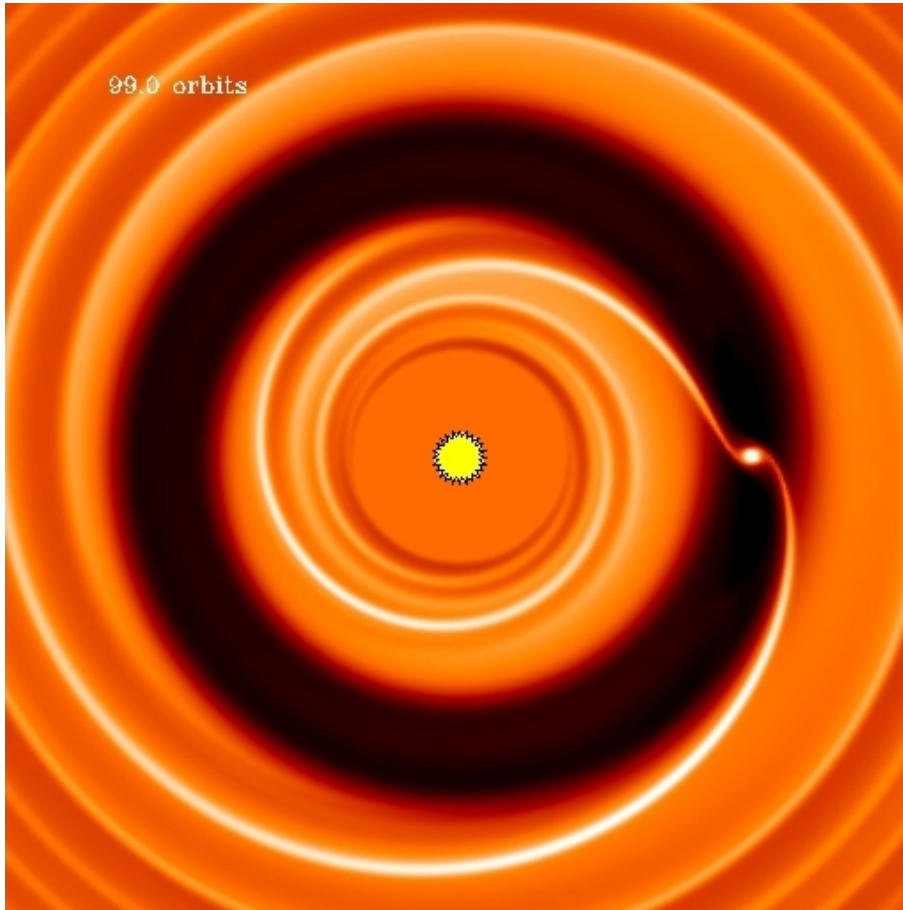
GIANT PLANETS

In the case of giant planets, the linear regime is not valid anymore. The wake shocks, and deposits angular momentum. Thus, giant planets perturb the density profile of the disk.

The outer disk is taking angular momentum from the planet, and is accelerated by the planet. Therefore, it shifts outwards.

The inner disk is giving angular momentum to the planet, and it is slowed down by the planet, because the wake is leading in front of the planet. Therefore, the inner disk loses orbital angular momentum, and shifts towards the star.

GAP OPENING



The outer wake has a larger angular velocity than the local gas ($\Omega_p > \Omega_{\text{gas}}$).

It accelerates the gas, gives angular momentum to the gas.

In contrast, the inner wake carries a negative angular momentum flux, given to the gas.

So, the gas moves inwards.

In the end, the planet tends to open a gap around its orbit, and to split the disk into an inner disk and an outer disk, separated by an empty region (Lin & Papaloizou, 1986).

GAP OPENING

The amount of gas remaining in the gap depends on the competition between the torques from the planet, which repel the gap, and the effects of the viscosity and pressure in the gap, which tend to make the profile smooth.

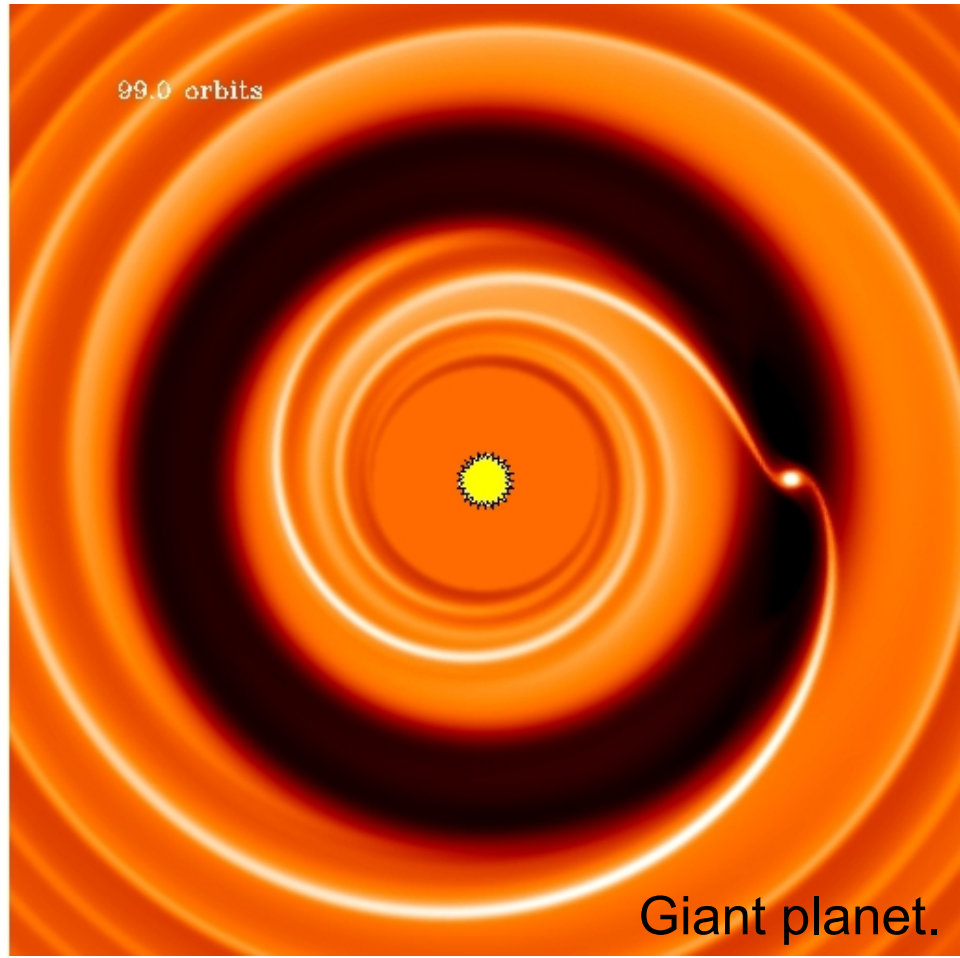
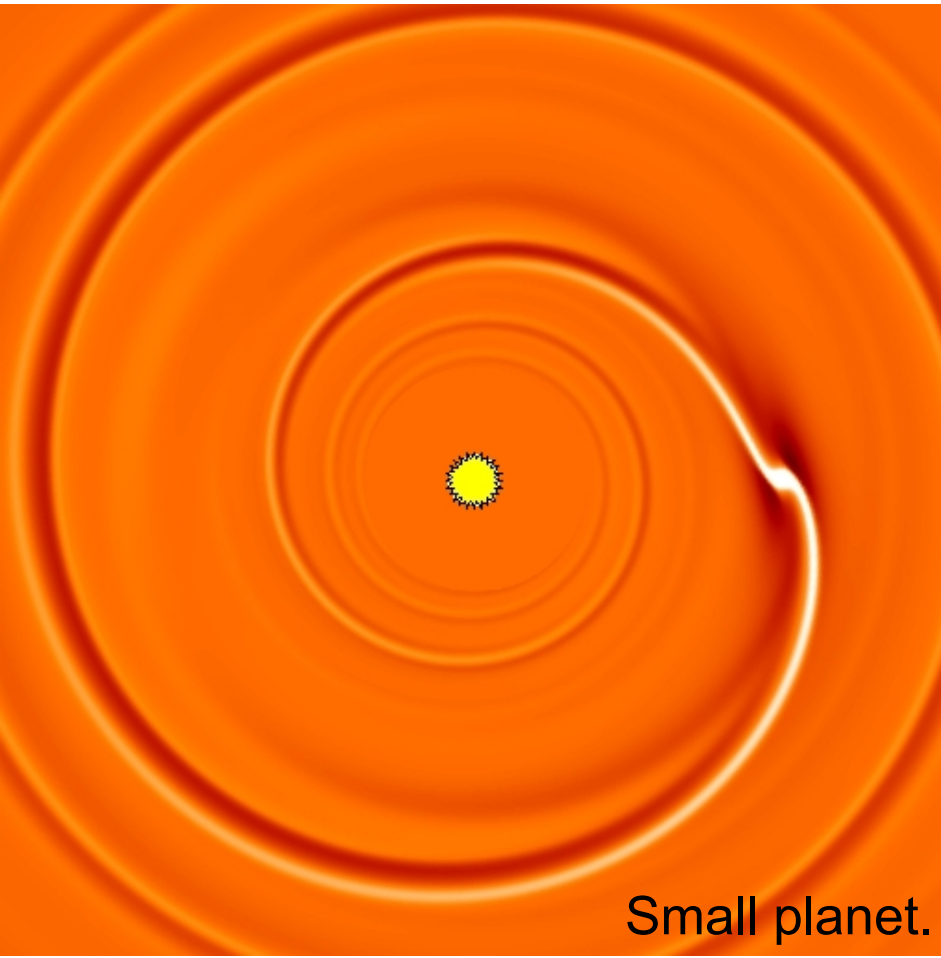
The more viscous or thick the disk is, the higher the planetary mass must be. Therefore, there is an opening criterion ([Crida et al. 2006](#)) for the density in the gap to be less than 10% of the unperturbed density :

$$3H / [4r_p(M_p/3)^{1/3}] + 50\nu / [r_p^2\Omega_p M_p] < 1$$

Application : In a standard disk, $H = \sim 0.05 r_p$, $\nu = \sim 10^{-5} r_p^2 \Omega_p$, and $M_p > M_{\text{Saturn}}$ is enough.

GAP OPENING

In ~ 100 orbits, a giant planet of a Jupiter mass repels the disk around its orbit, and opens a gap.



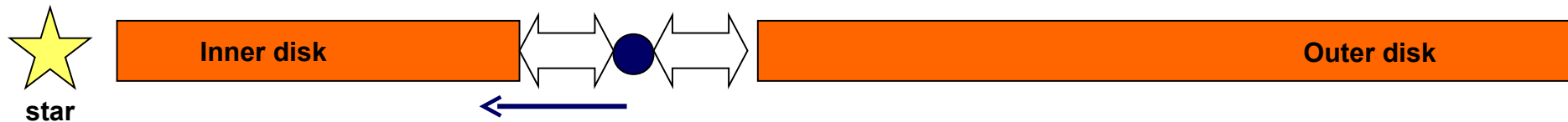
TYPE II MIGRATION

The planet is repelled

→ outwards by the inner disk

→ inwards by the outer disk.

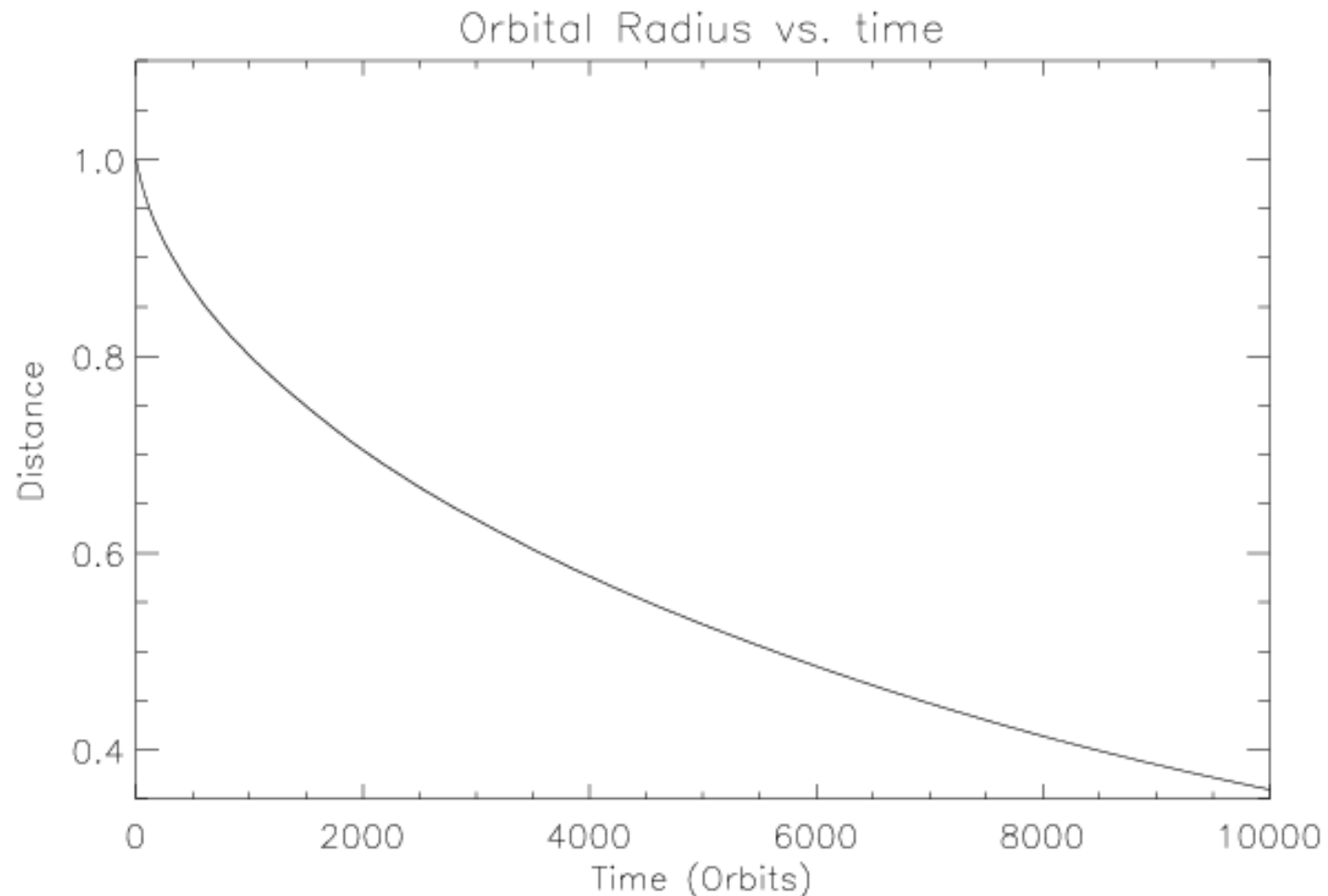
It is locked in the middle of the gap, and can not migrate **with respect to** the gas of the disk anymore.



But the disk falls onto the star (accretion), driving the planet inwards.

TYPE II MIGRATION

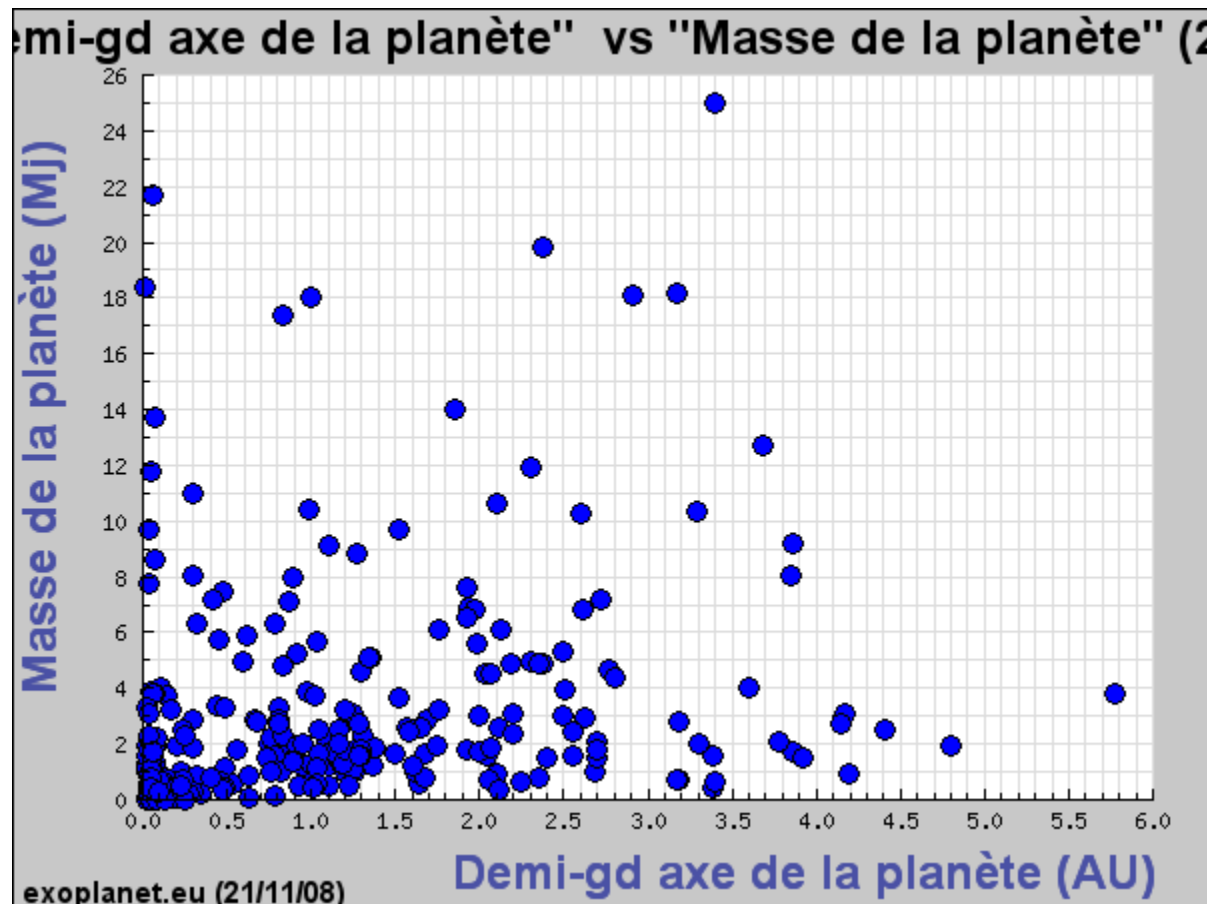
The planet follows the viscous accretion of the disk towards the star



TYPE II MIGRATION

Many exoplanets are giant planets, close to their host star, where they couldn't form : the *hot Jupiters*.

This is a strong indication of type II migration....



SUMMARY

Planet – disk interactions => wake (sillage)

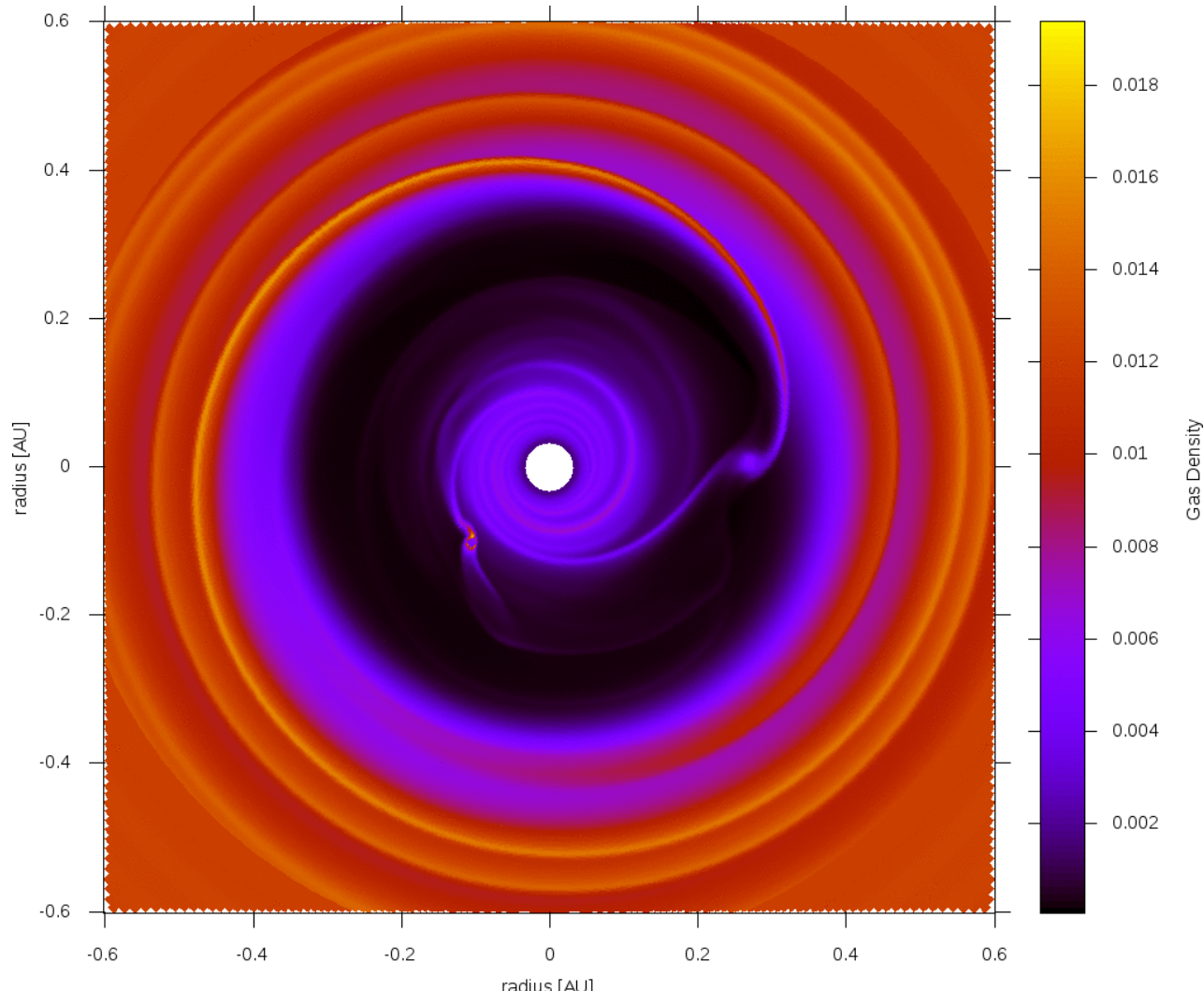
→ type I migration of small mass planets

→ gap opening and slow inwards migration of giant planets

What happened in the Solar System ?

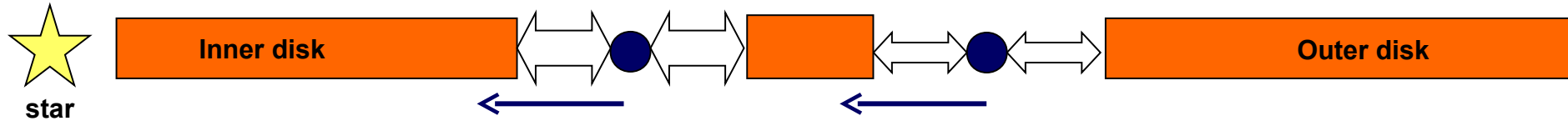
MIGRATION in RESONANCE

If two planets in the disk + convergent migration
then resonance capture is possible.

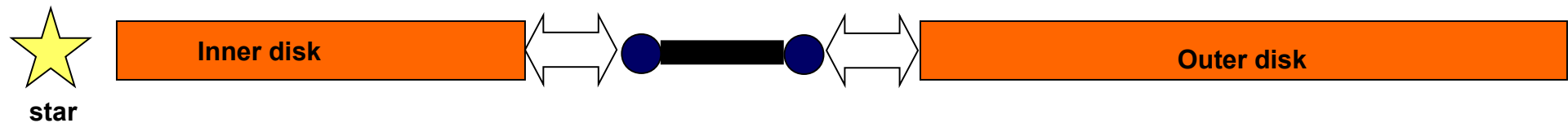


MIGRATION in RESONANCE

Two planets in their own gaps migrate in parallel.

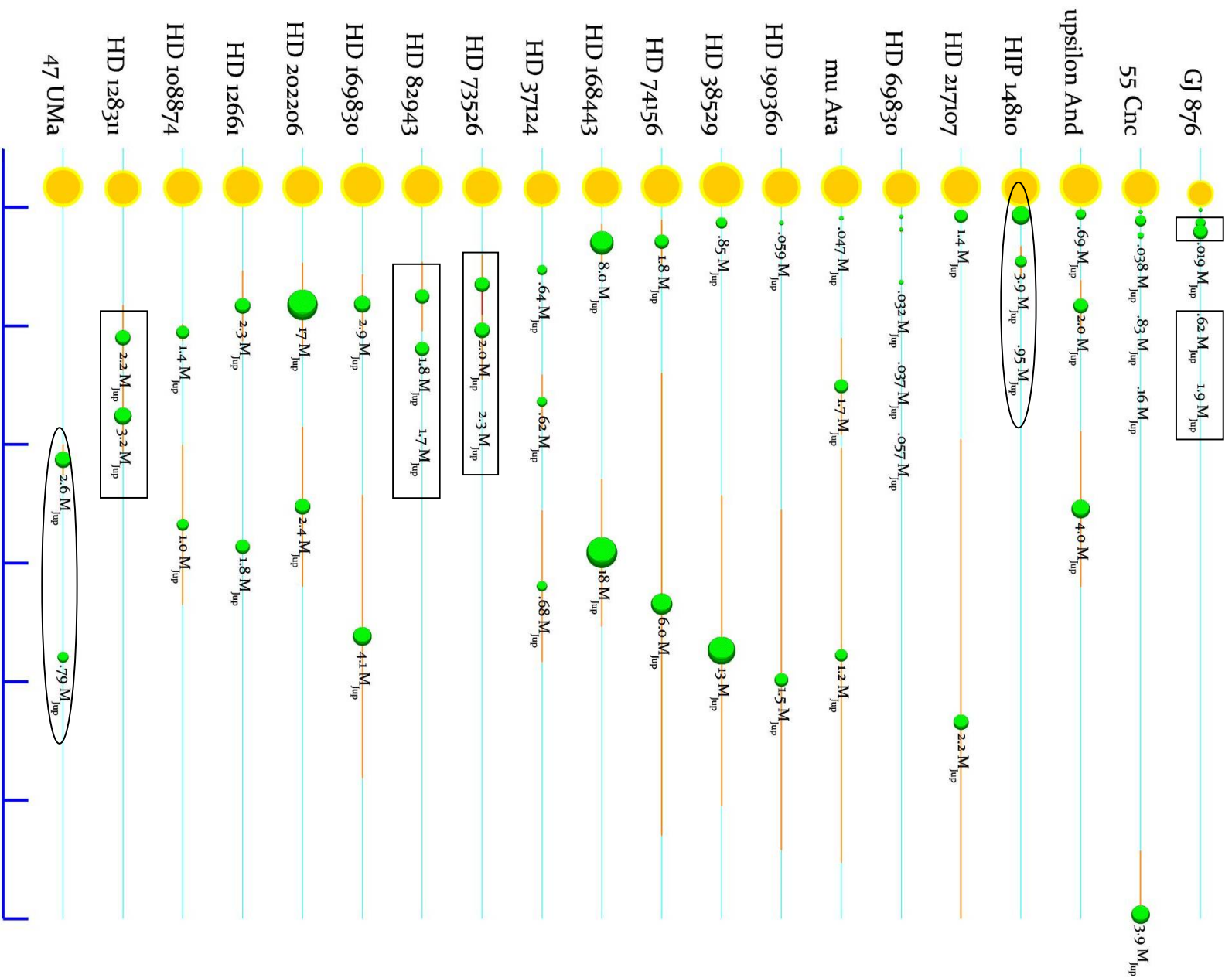


Two planets in a same gap approach each other → résonance.



Once locked in resonance, the pair of planet migrates in this configuration.

The outer disk pushes the outer planet inwards ;
in turn, the outer planet pushes the inner planet inward,
due to the resonance.



MIGRATION in RESONANCE

MIGRATION in RESONANCE

Eccentricities are excited by the resonance forcing,
and damped by the gas disc.
→ non zero equilibrium value.

Ex: GJ876

(Lee & Peale 2002, Crida et al 2008)

Possibility of close encounters,
in particular if 3 planets.

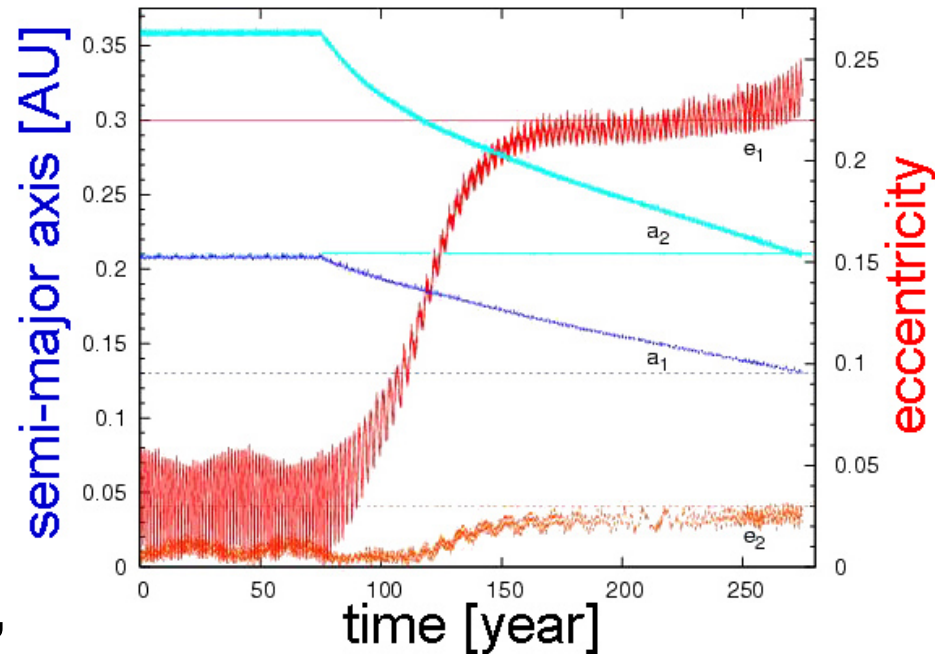
Then, scattering and e and i increase.

(Marzari et al. 2010, Juric & Tremaine 2008, Chatterjee et al. 2008)

But the disc damps e and i of an isolated planet.

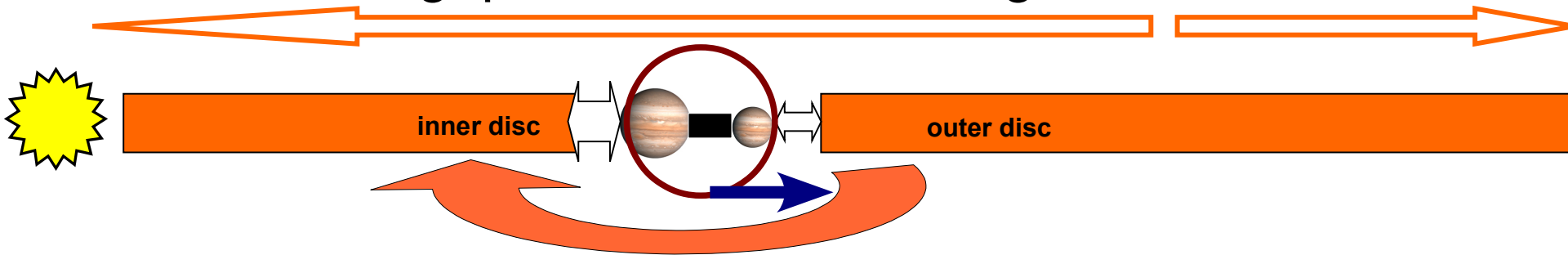
(Xiang-Gruess & Papaloizou 2013, Bitsch et al. 2013)

→ Scattering should take place as the disc dissipates in order to explain the observed high e of exoplanets. Unlikely fine tuning of the timing (Lega et al. 2013).



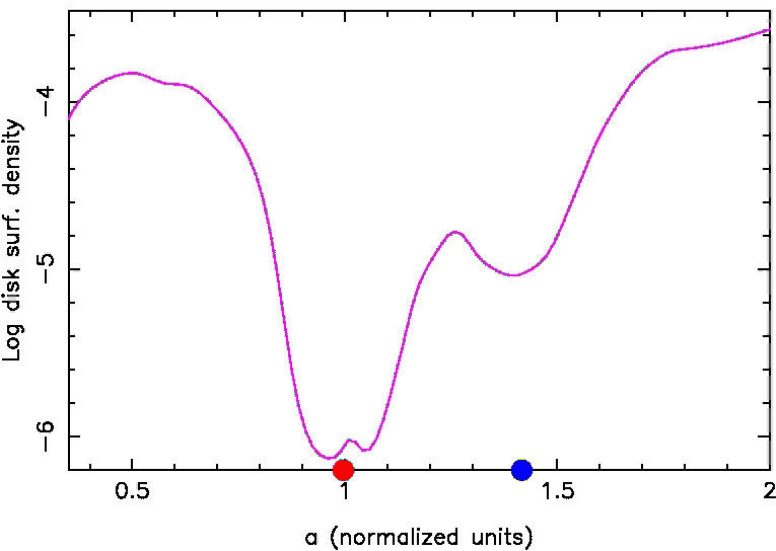
MIGRATION in RESONANCE

Common gap + resonance locking case :

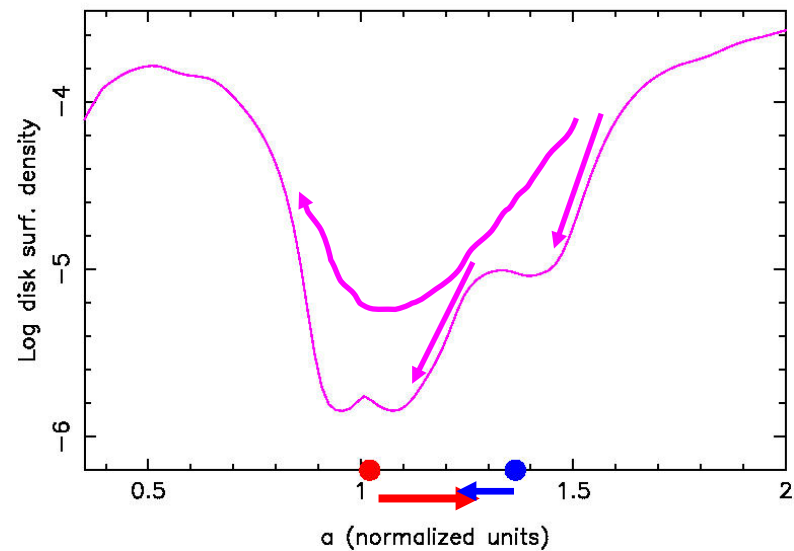


$M_2 < M_1 \Rightarrow$ smaller negative torque from outer disc than positive torque from inner disc (Masset & Snellgrove 2001).
The pair goes outwards, even if the disc goes inwards.

T= 0



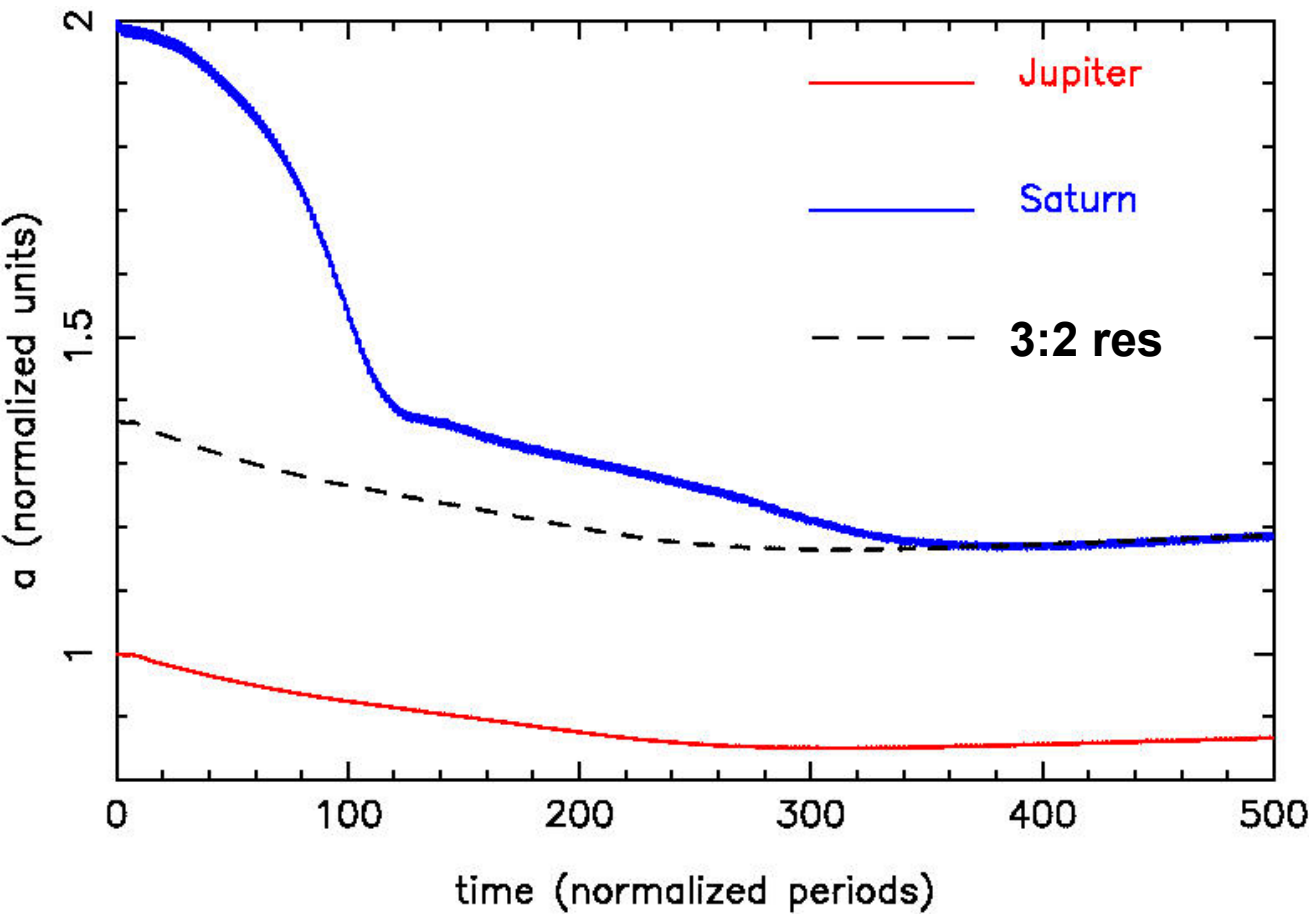
T= 500



SOLAR SYSTEM

How to prevent Jupiter from becoming « hot » ?

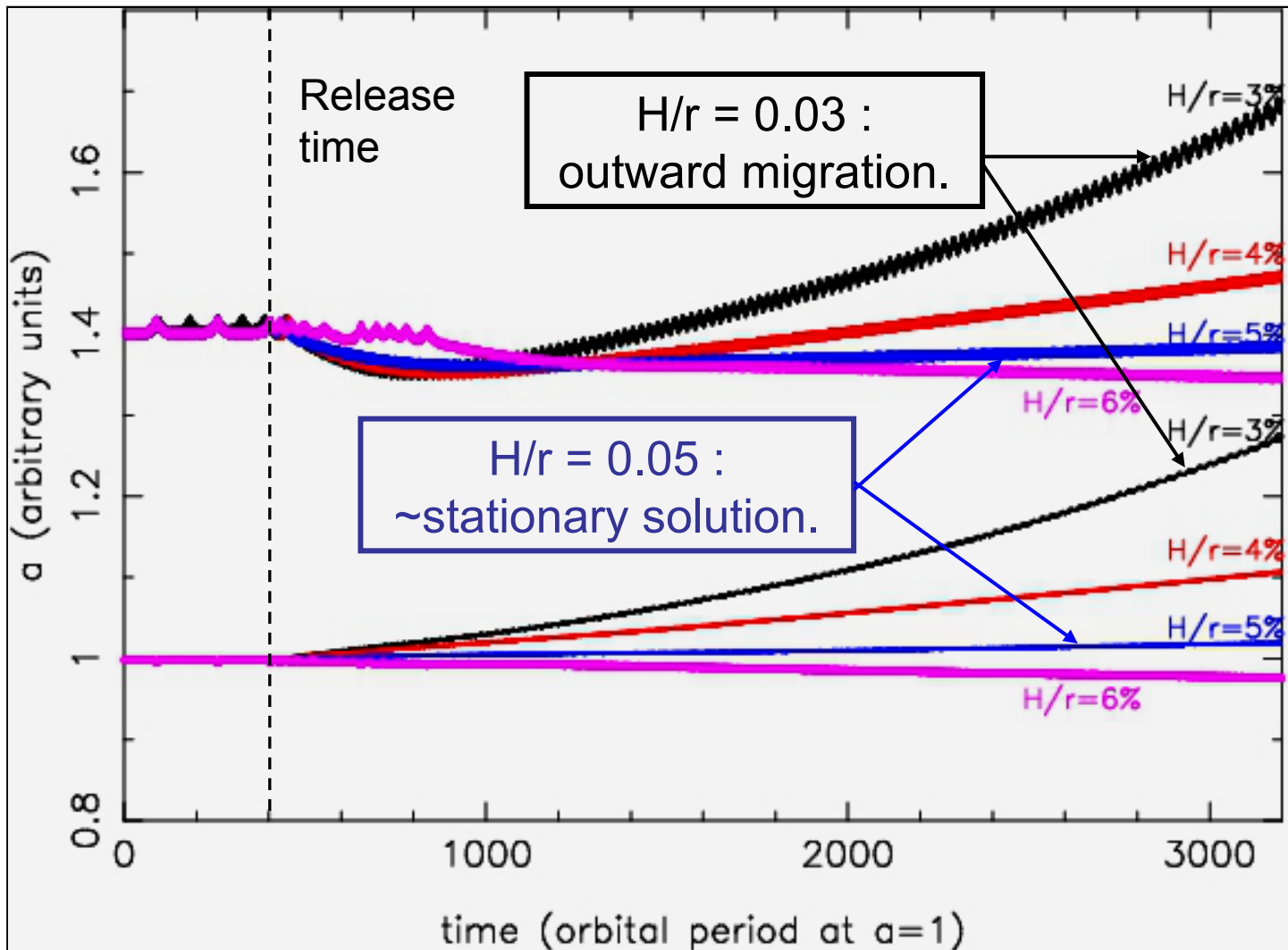
$M_{\text{Saturn}} \sim M_{\text{Jupiter}} / 3 \Rightarrow$ they can decouple from the disk !



Pierens & Nelson
2008:
most likely
outcome is a
capture in 3:2
MMR.

SOLAR SYSTEM

Morbidelli & Crida 2007: once in 3:2 MMR, migration speed and direction depends on disk parameters (in particular H/r).

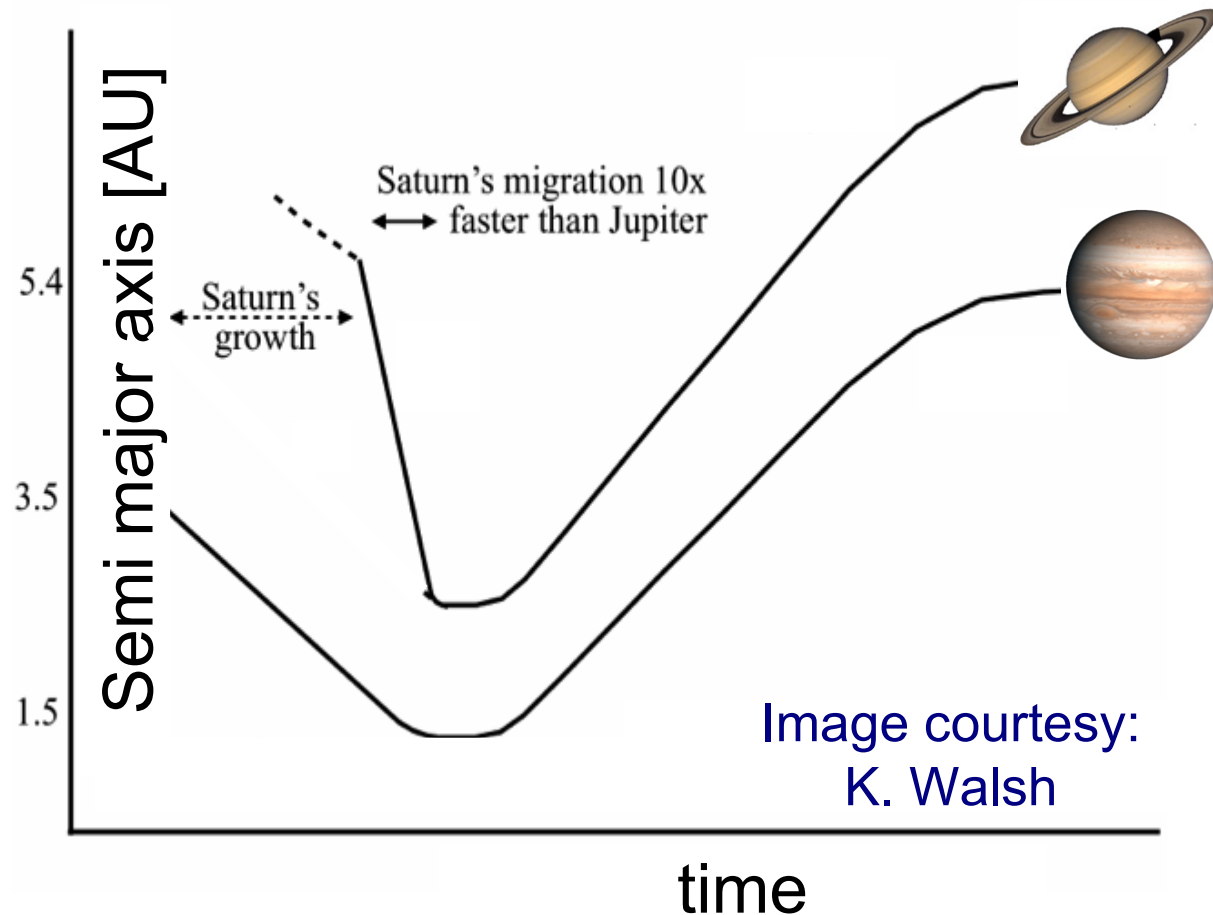


GRAND TACK SCENARIO

1) Jupiter's core grows at the zero-torque migration radius. Jupiter becomes giant, opens a gap, migrates inwards in type II, from $\sim 4\text{-}6$ AU down to 1.5 AU.

2) Saturn's core forms, grows, migrates faster than Jupiter (the migration map has changed), catches up with it in MMR.

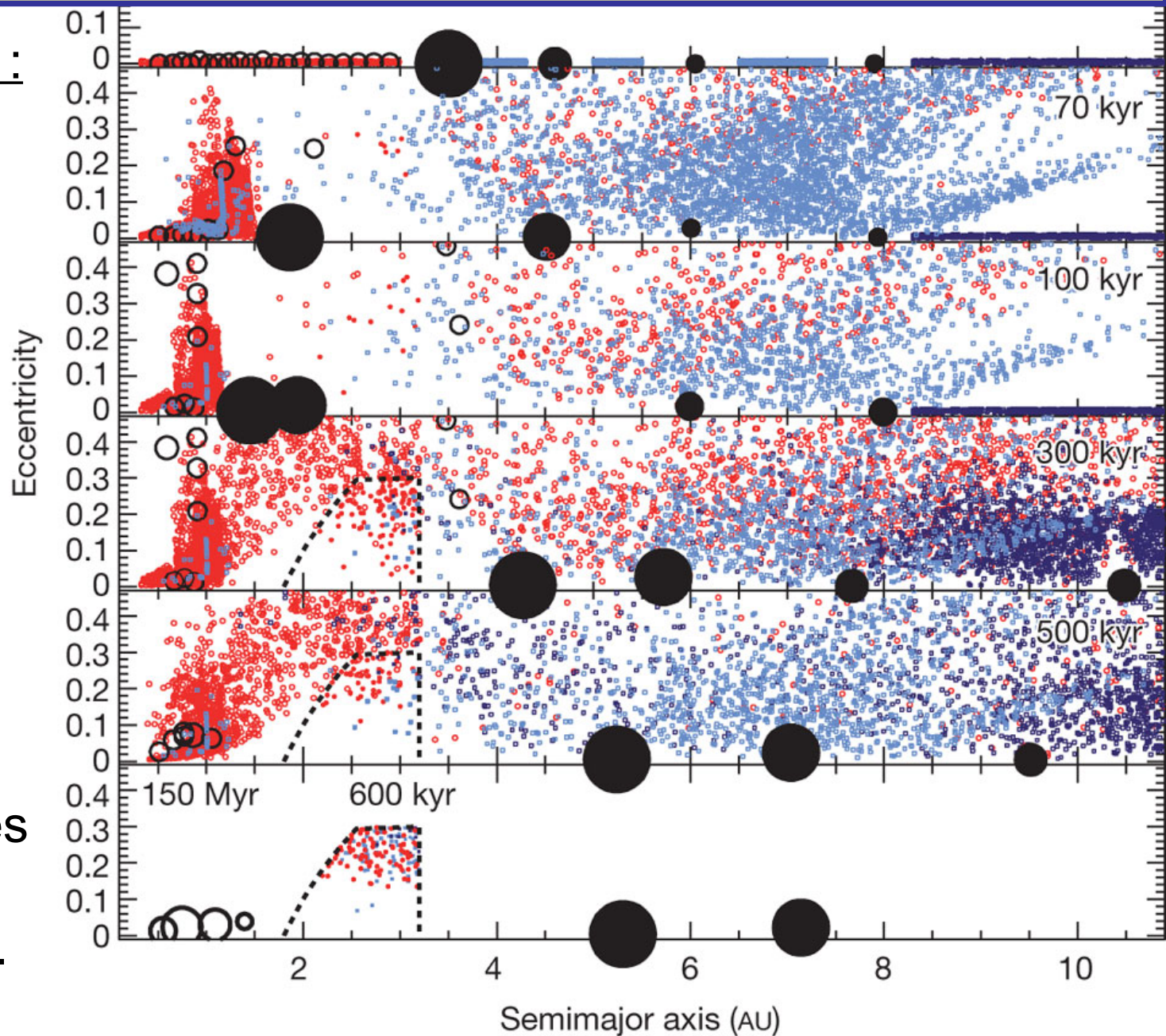
3) Jupiter tacks, and the pair of planet migrates outwards, until $H/r = 0.05$ (hopefully around 5 and 8 AU respectively).



GRAND TACK SCENARIO

Consequences :

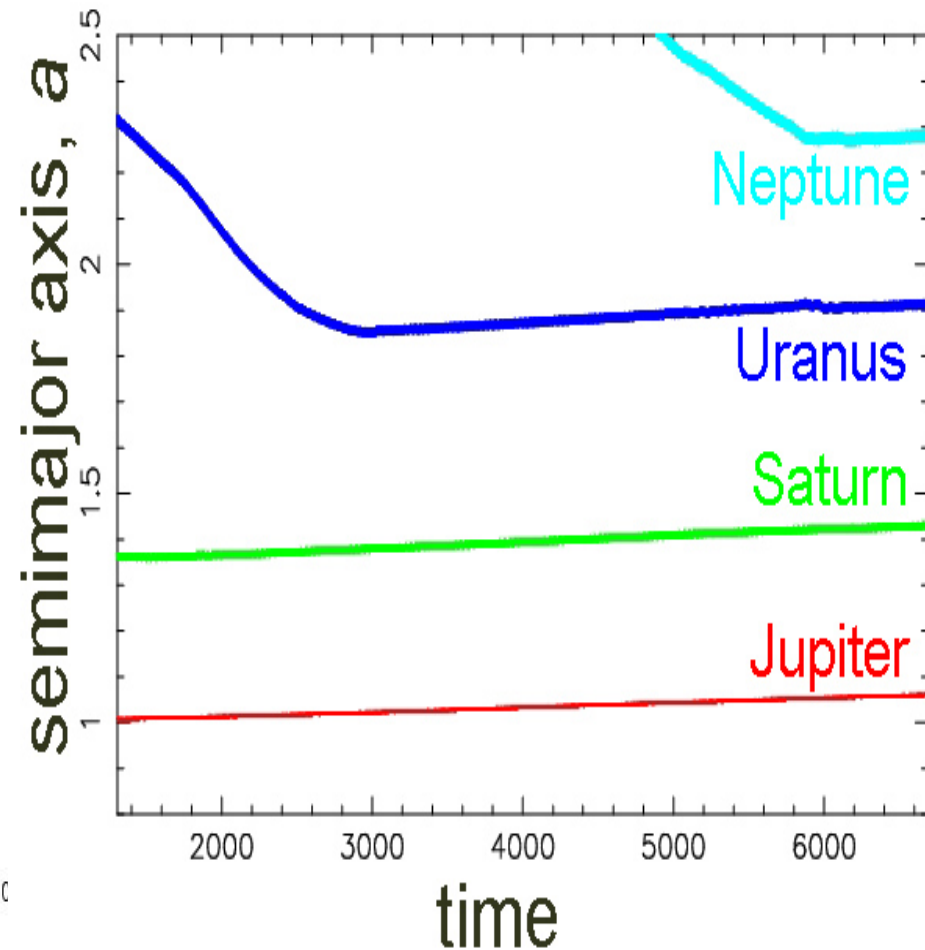
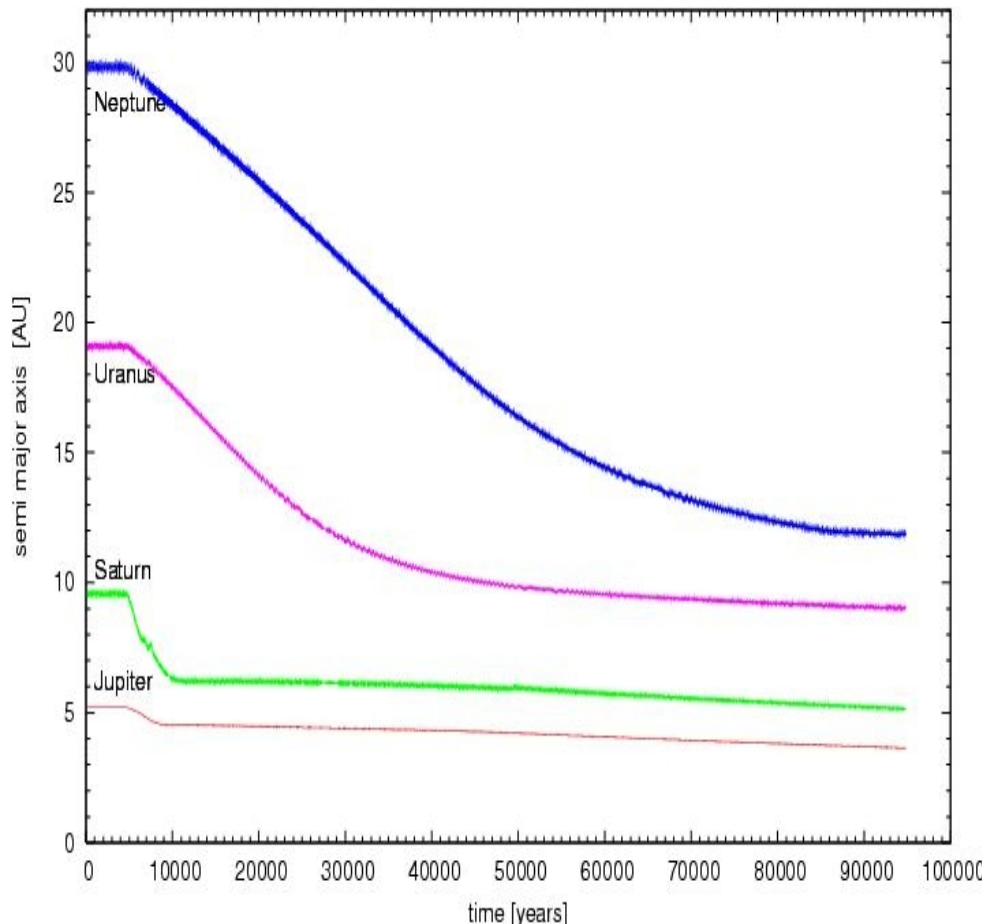
- a) The disk of planetesimals and embryos in the terrestrial planets region is truncated at 1 AU.
- b) The MAB region is populated with scattered bodies from in and out of the snowline.



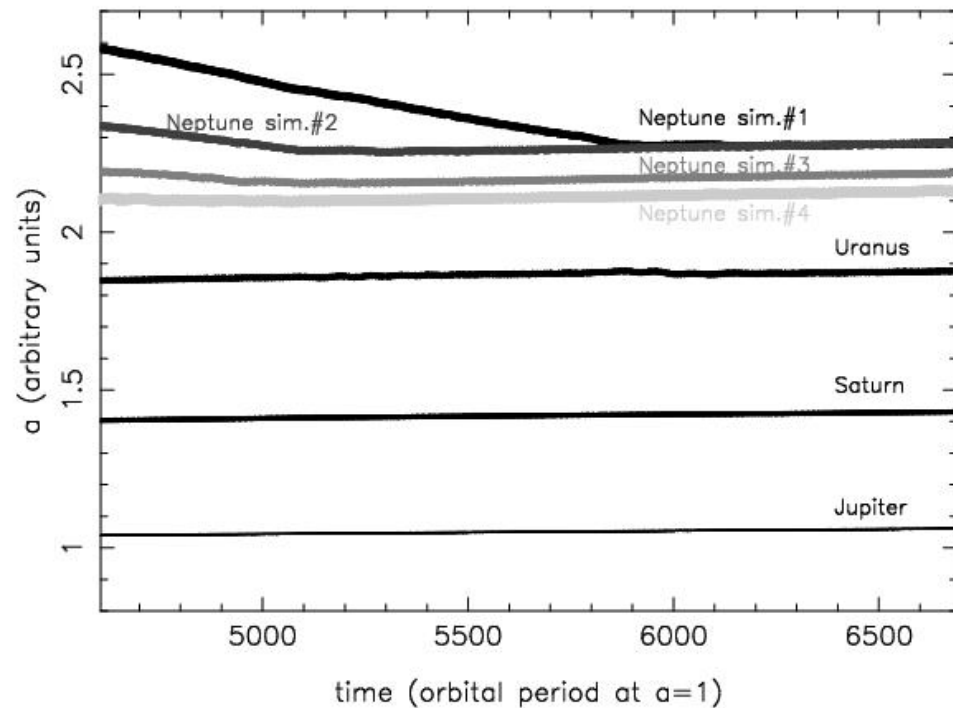
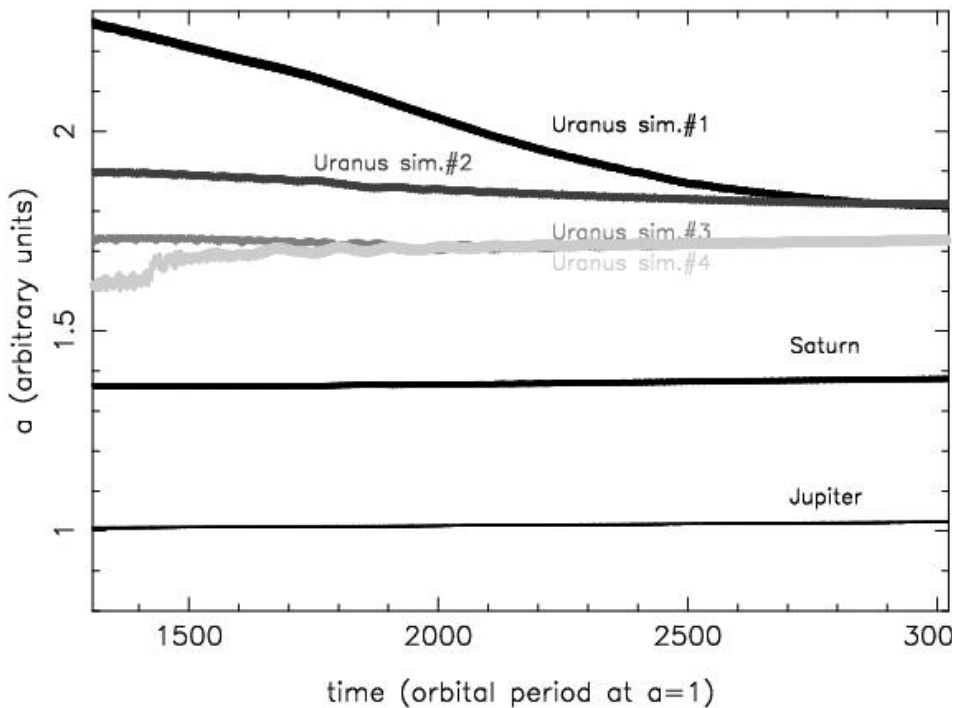
GRAND TACK SCENARIO

Then, the ice giants (Uranus and Neptune) can be trapped in mean motion resonance as well...

Migration in the Hayashi (1981) nebula ; $H/r=0.05$; $\alpha=0.004$



GRAND TACK SCENARIO



Possible configurations :

J:S in 3:2, S-U in 3:2 or 4:3, U-N in 4:3, 5:4 or 6:5.

→ 6 possible configurations. (ex : J:S:U:N in 12:8:6:5)

4 unstable in a few Myrs, 2 stable for more than 100 Myrs.

(Morbidelli, Tsiganis, Crida, Levison, Gomes, 2007)

GRAND TACK SCENARIO

We are left with a favourable situation
for the formation of the terrestrial planets,
but the giant planets are in a strange, compact configuration.

What next ?

