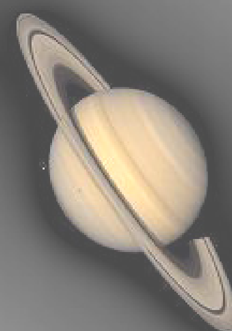
PLANETARY FORMATION
1) PROTO-PLANETARY DISCS

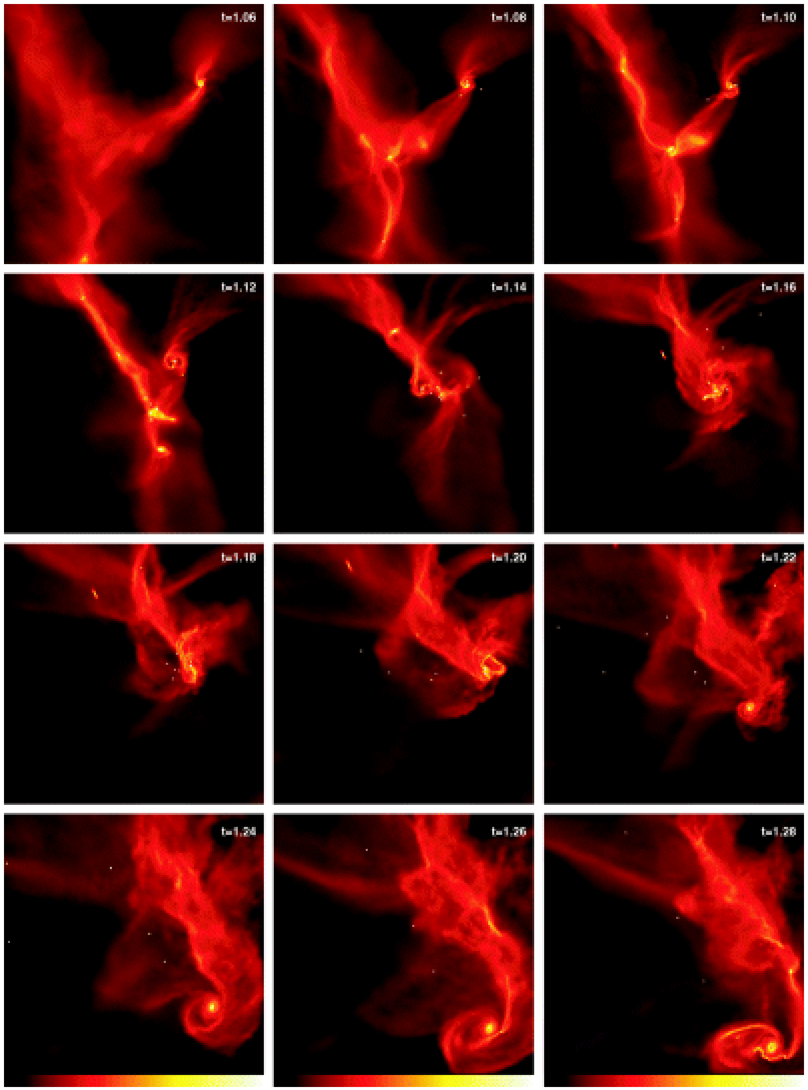


Aurélien CRIDA

Introduction

Stars form by the collapse of a molecular gaseous cloud.

Ex: numerical simulations
by Bate et al. (2003).



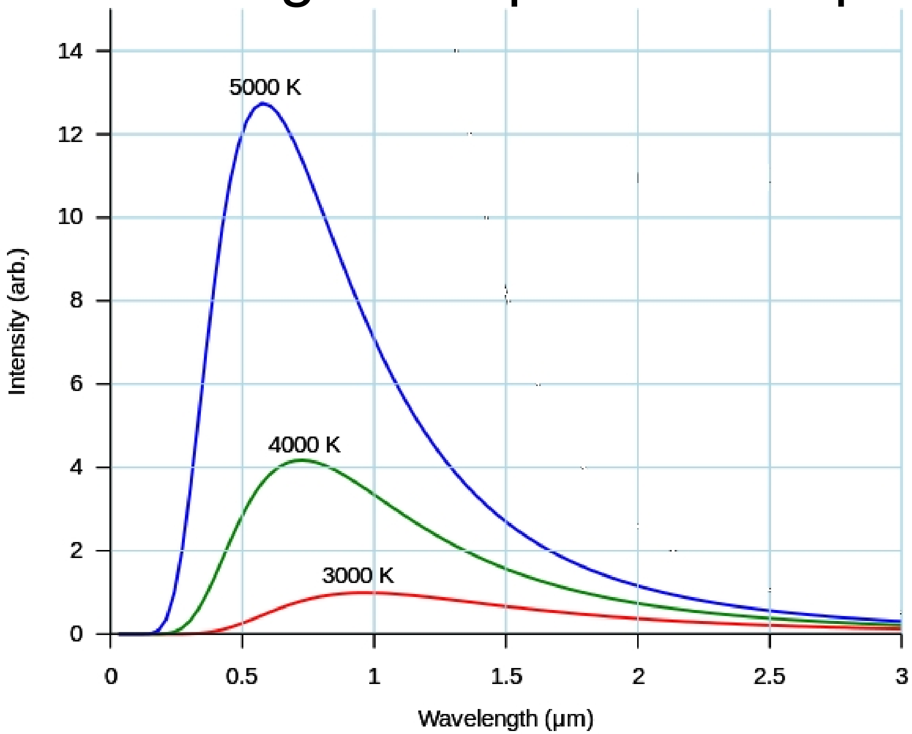
SED

<u>def: S.E.D.</u> = Spectral Energy Distribution = broad spectrum.

Planck's law for a black body :

$$I(\nu,T) = \frac{2h\nu^3}{c^2}\frac{1}{e^{\frac{h\nu}{kT}}-1}.$$

I is the energy radiated per unit time by a unit surface, per unit solid angle and per unit frequency,



c = speed of light (3×10^8 m.s^{-1}),

h = Planck's constant (6.6×10^{-34} J.s),

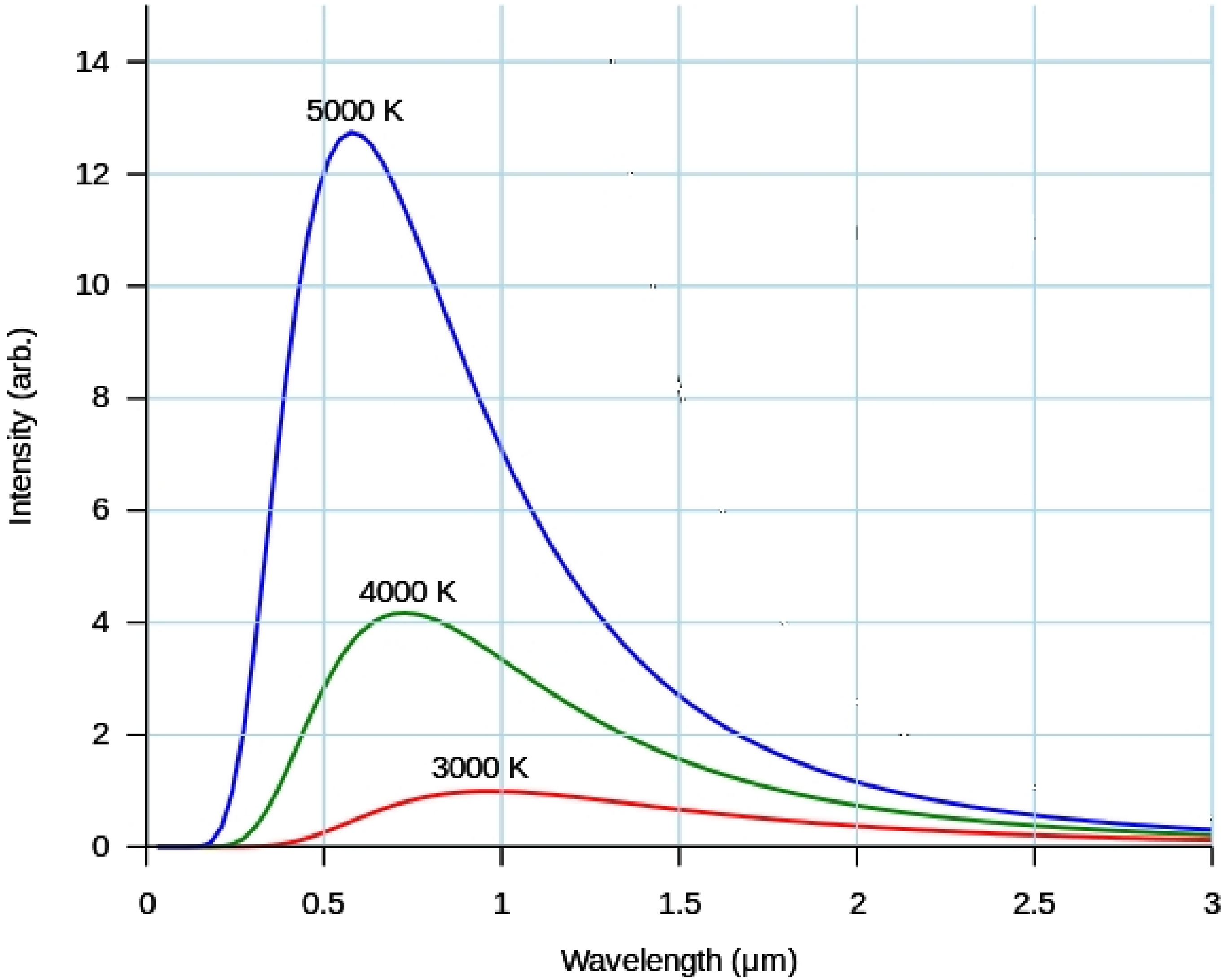
k = Boltzmann's constant (1.38×10^{-23} J.K^{-1}),

ν = frequency (in Hz=s^{-1}),

T = temperature (in K).

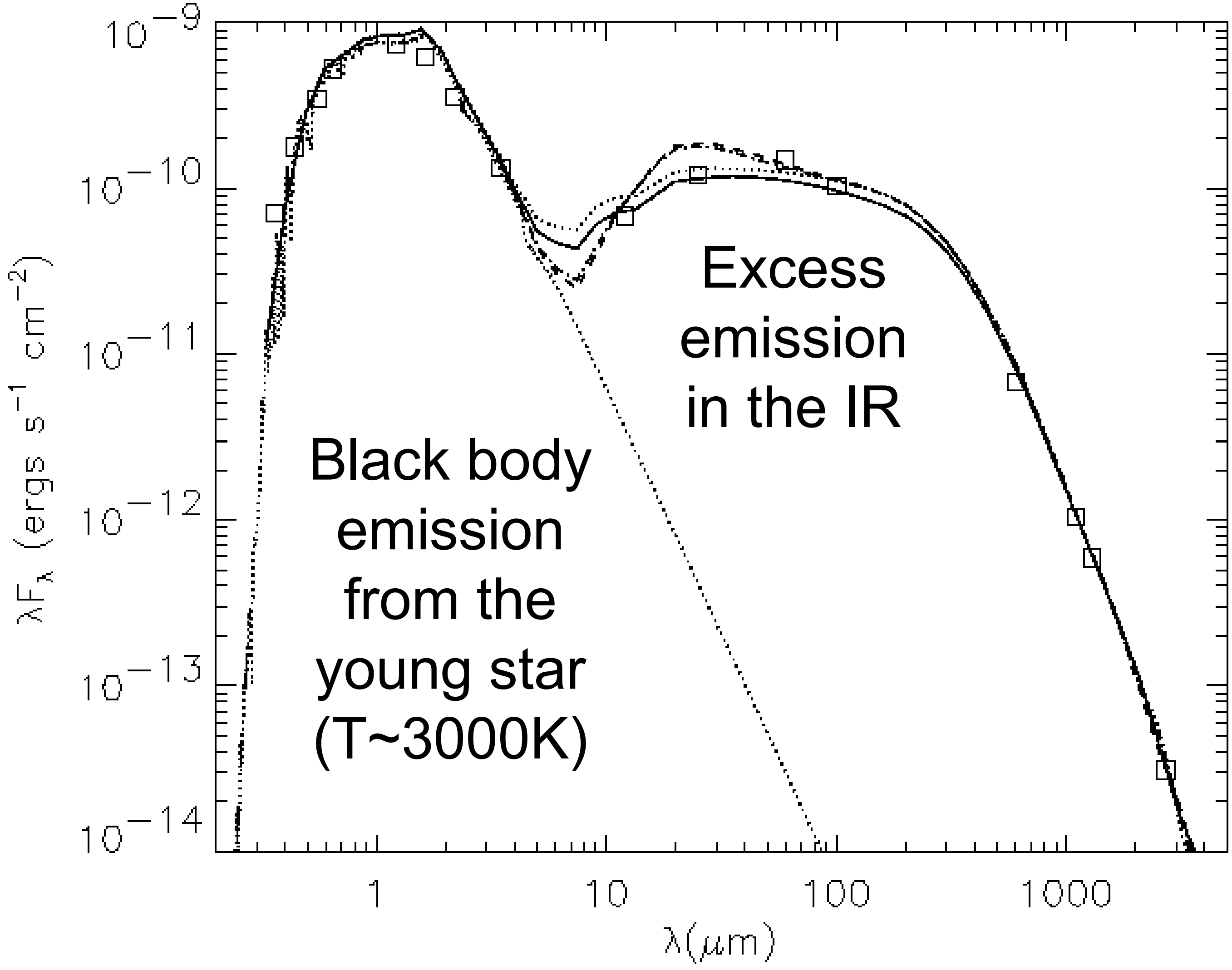
SED

Wien's law : λ_{max} = 2,9 m / T[K]



SED

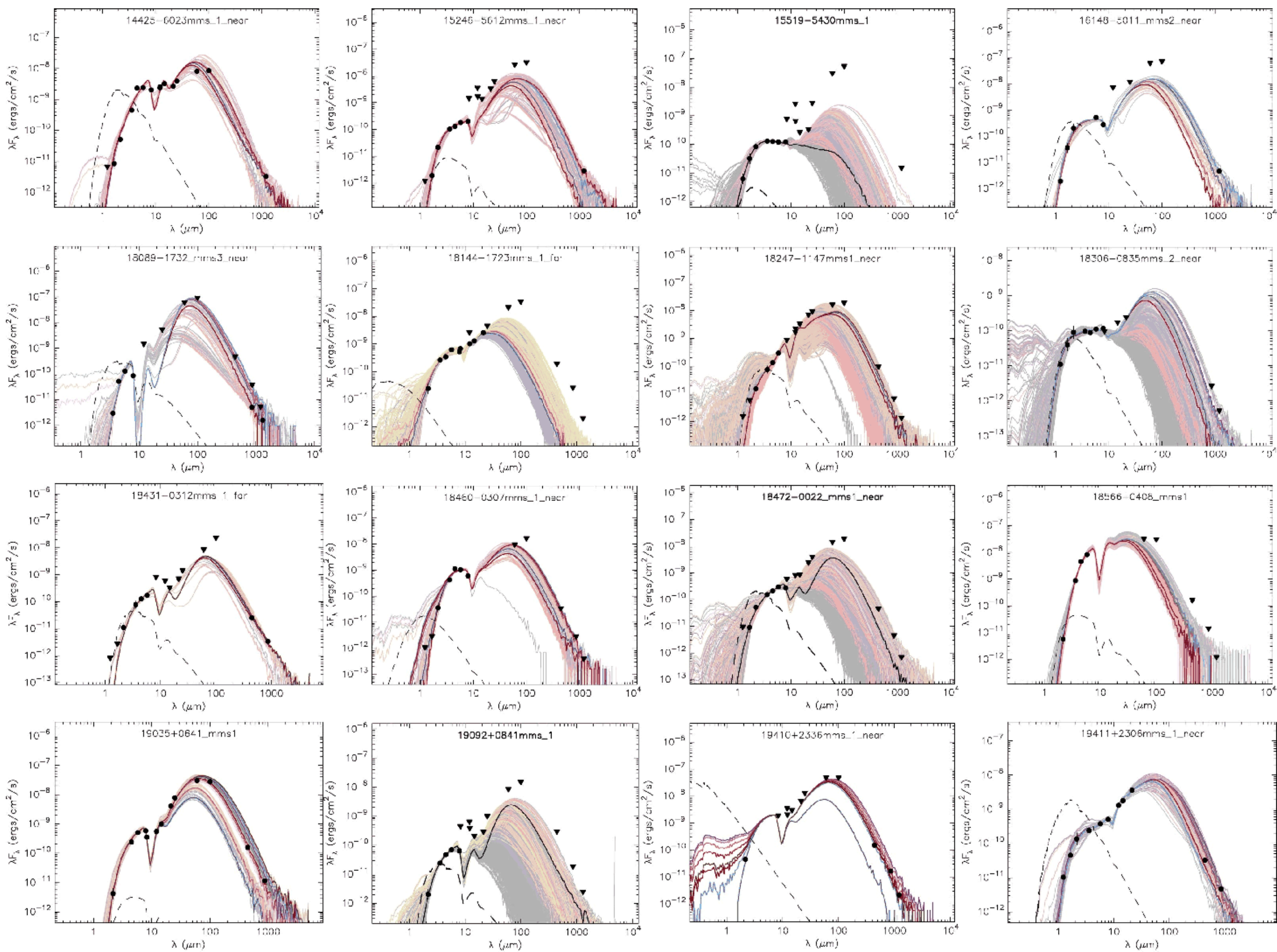
<u>IR excess</u> ex: case of GM Aurigae



SED

IR excess: around high mass protostellar objects

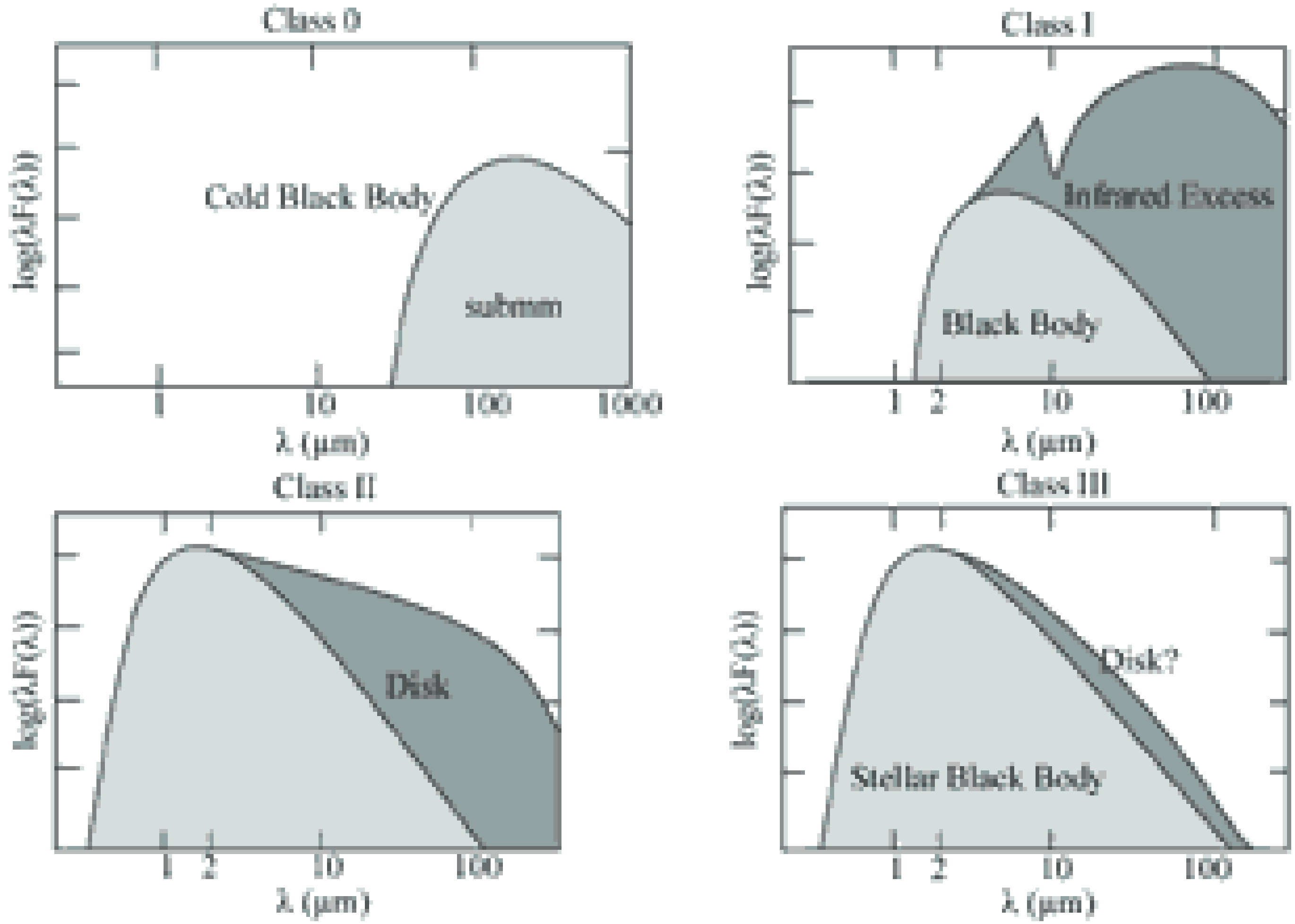
(Grave & Kumar 2009)



What can cause an IR excess ????????

SED

Classification: around high mass protostellar objects



SED

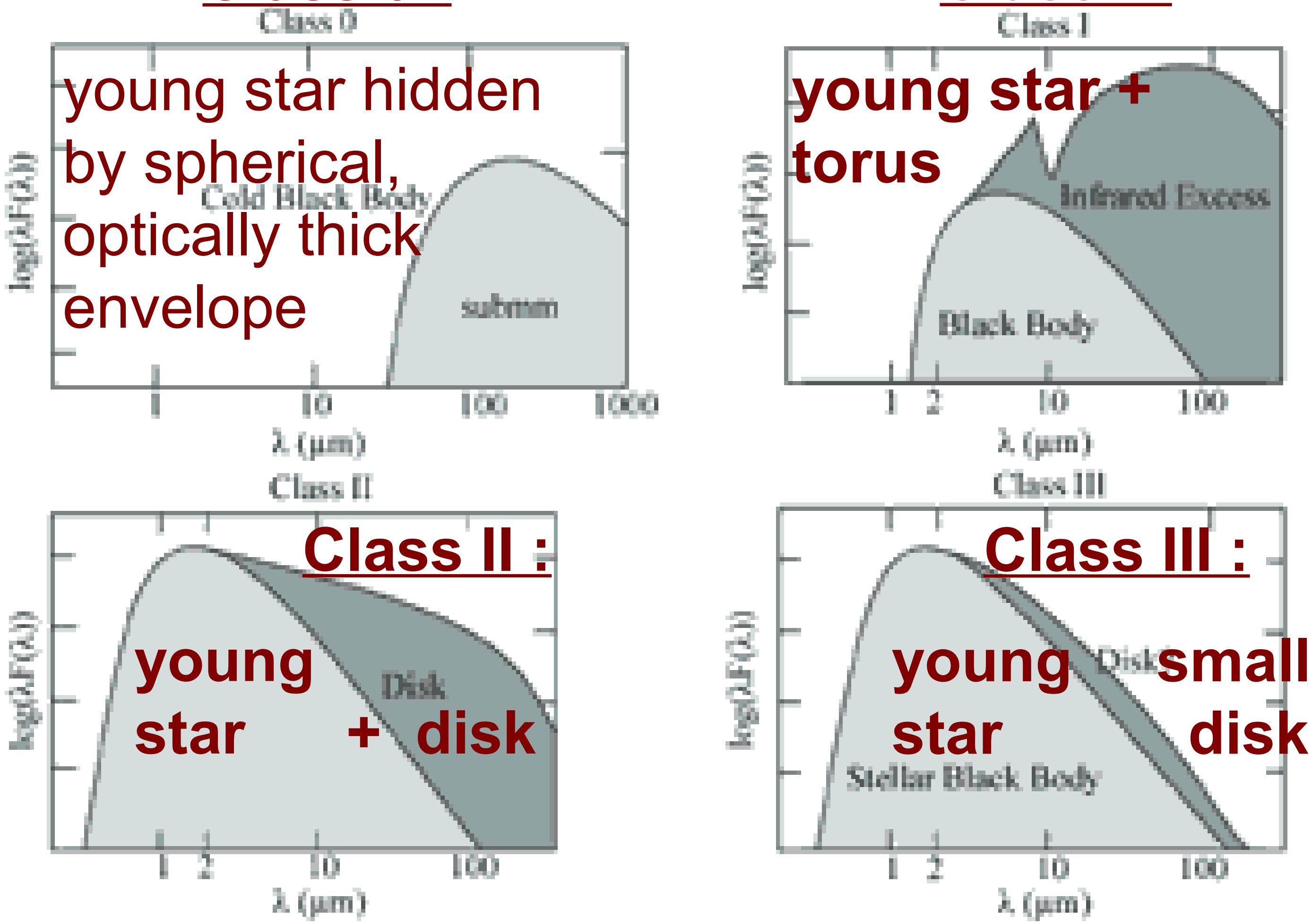
Classification: around high mass protostellar objects

Class 0 : young star hidden by spherical, optically thick envelope

Class I : young star + torus

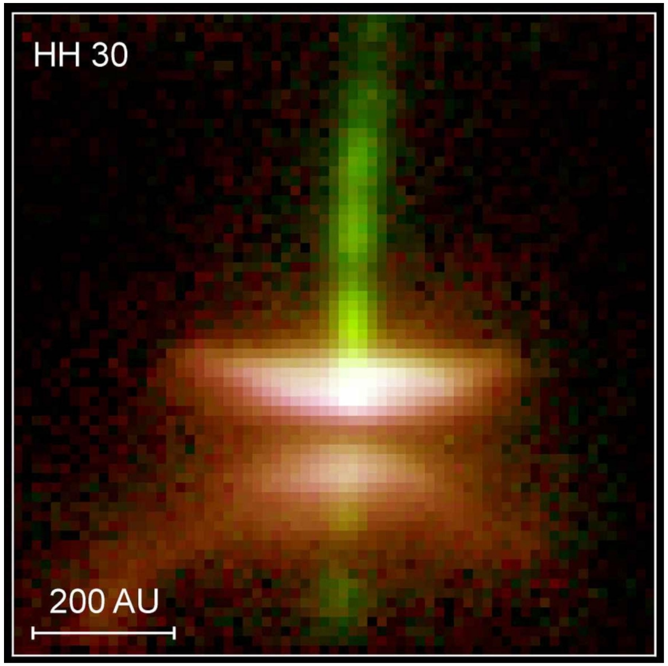
Class II : young star + disk

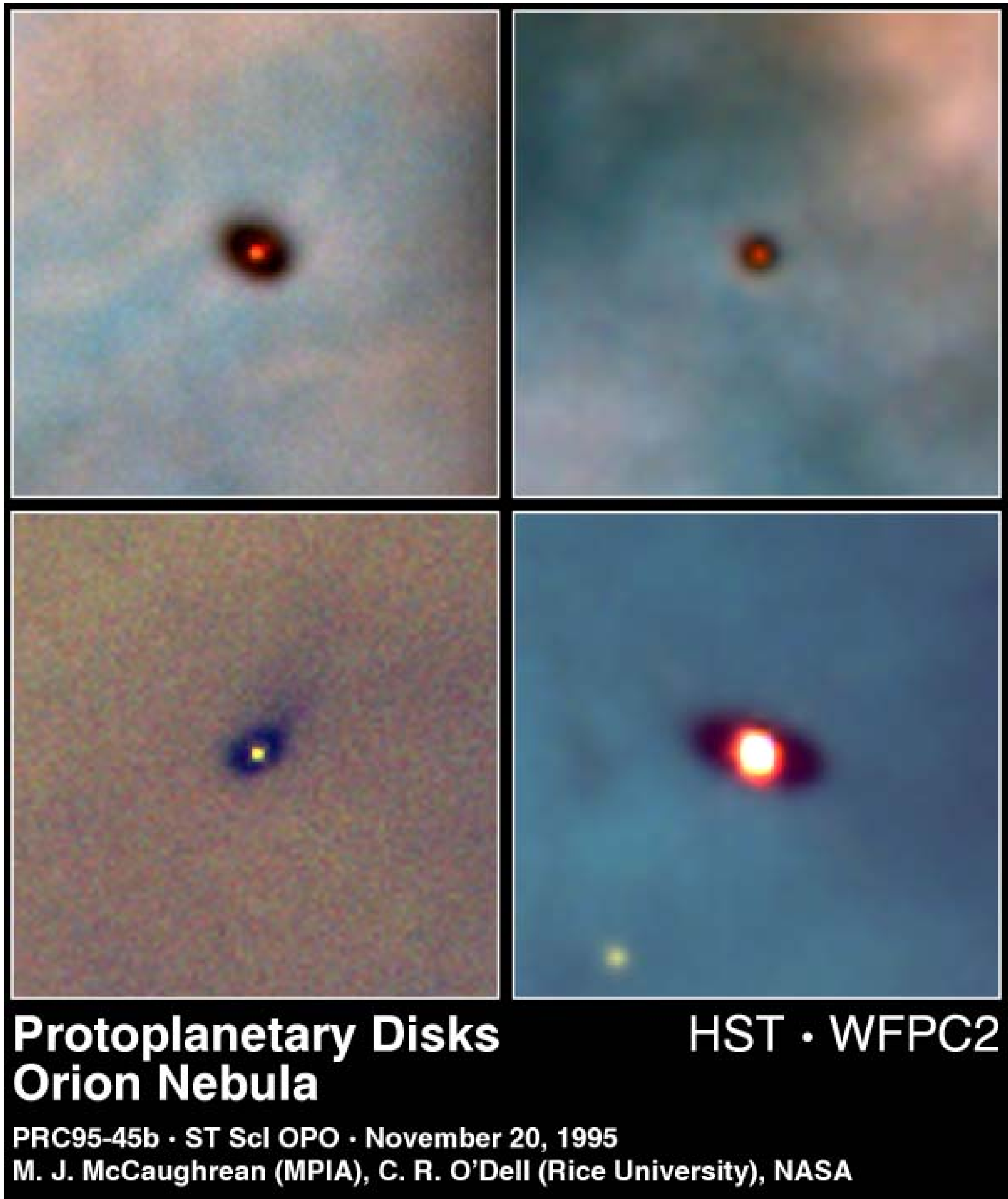
Class III : young star small disk



OBSERVATIONS

Discs, discs, disc...





DISC STRUCTURE

<u>Vertical structure :</u>

Vertical hydrostatic equilibrium.

Perfect gas : $P = R\rho T/\mu$

Equation Of State : $P = c_s^2 \rho / \gamma$

c_s : sound speed

γ : adiabatic index

Exercise : Find $\rho(z)$.

Solution : $\rho(z) = \rho_0 \exp(-z^2/2H^2)$, where $H = c_s / \gamma^{1/2}\Omega_K$.

<u>Nota Bene :</u>

Perfect gas : $P = R\rho T/\mu$

μ : molecular weight
$= 2{,}35 \times 10^{-3}$ kg/mol
R : perfect gases constant
$= 8.314$ J/mol/K

$T = (H/r)^2 \, (GM_* \mu / Rr)$

DISC STRUCTURE

Vertical structure :

$H = c_s \, \Omega$ (assuming $\gamma=1$)

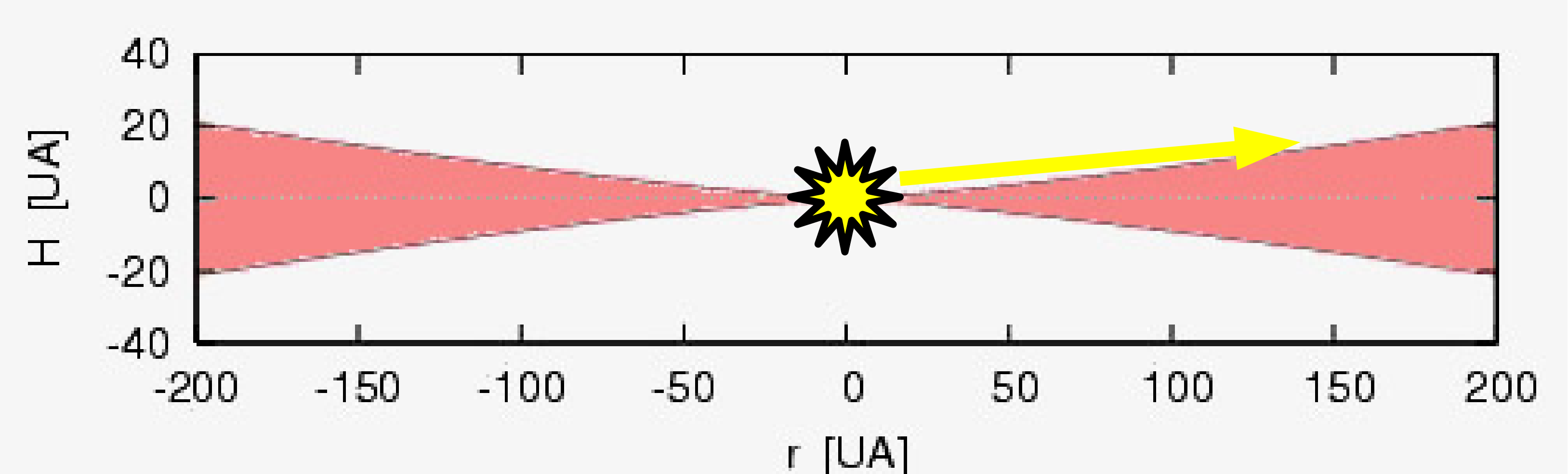
Exercise : What is H(r) ?

Assume disk = black body of height H(r).

Heating from the star >< Cooling from the surface.

Consider an elementary ring of width δr .

Solution : $H/r = h_0 \, (r/r_0)^{2/7}$.



DISC STRUCTURE

Horizontal Structure :

Gravity + Centrifugal force + Pressure gradient

Perfect gas : $P=R\rho T/\mu$

μ : molecular weight = 2,35 x 10^{-3} kg/mol
R : perfect gases constant = 8.314 J/mol/K

Equation Of State : $P = c_s^2 \rho / \gamma$

c_s : sound speed = $H\,\Omega_K$ (where $\Omega_K^2 = GM_*/r^3$)
γ : adiabatic index

Exercise: Find $\Omega(r)$, if $\rho=\rho_0\,(r/r_0)^{-a}$.

Solution: $\Omega_{gas} = \Omega_K\,[\,1 - (1+a-2\beta)(H/r)^2\,]^{1/2}$, where $H/r = h_0\,(r/r_0)^{\beta}$.

The gas is **subkeplerian** (in general, because $a>0$ and $\beta<0.5$).

DISC EVOLUTION

<u>Tenseur des taux de déformation, D :</u>

2 dimensions, coordonnées polaires (r, θ) :

$D_{rr} = \partial v_r / \partial r = 0$; $D_{\theta\theta} = 1/r\ \partial v_\theta / \partial \theta + \partial v_r / \partial r = 0$.

$D_{r\theta} = D_{\theta r} = 1/2\ (\partial v_r / r \partial \theta + r\ \partial (v_\theta / r) / \partial r) = -3/4\ \Omega$.

<u>Tenseur de contraintes :</u> $\sigma = 2 \Sigma \nu D$

Où Σ = densité surfacique du disque, et ν = viscosité cinématique.

<u>Force interne volumique :</u> $\mathbf{F} = \mathrm{div}\ \sigma = (3/2)(\Sigma \nu \Omega / r)\ \mathbf{u}_\theta$

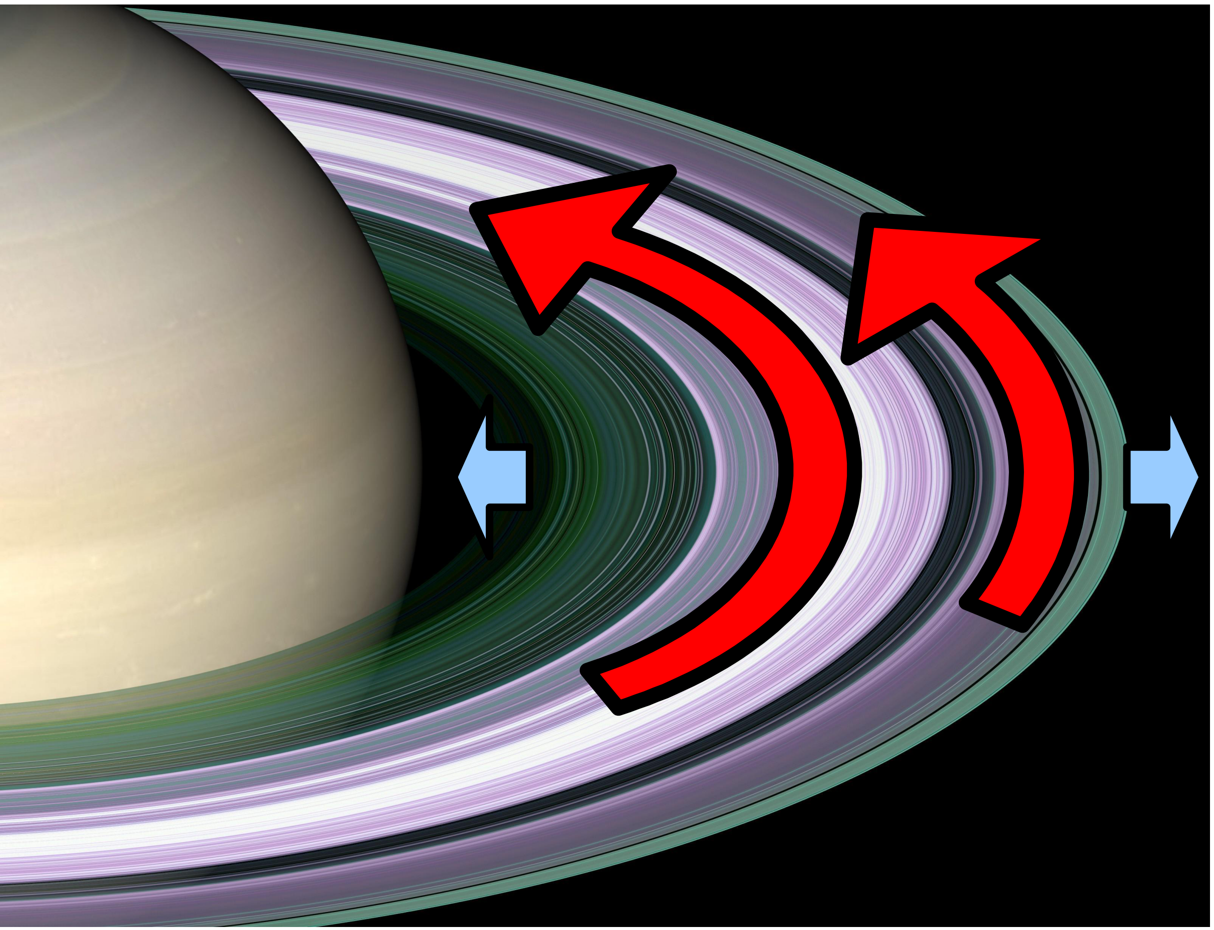
<u>Couple exercé par $\{r < r_0\}$ sur $\{r > r_0\}$:</u>

$T_\nu(r_0) = \int_{\{r < r0\}} \mathbf{r} \times \mathbf{F} = +3 \pi r_0^2 \Omega_0 \Sigma \nu$.

DISC EVOLUTION

T[illegible]$(r_0) = +3$ [illegible]r_0^2 [illegible]$_0$ [illegible]

Transfert de moment cinétique de l'intérieur vers l'extérieur.



Perte de moment cinétique pour disque interne
→ r diminue.

Gain de moment cinétique pour disque externe
→ r augmente.

Bilan: étalement !

Etat final : énergie minimale, toute la masse au centre, tout le moment cinétique porté par une particule infinitésimale située à l'infini.

DISC EVOLUTION

Mass conservation :

$$r\frac{\partial \Sigma}{\partial t}+\frac{\partial}{\partial r}\left(r\,\Sigma\, v_r\right)=0$$

Angular momentum conservation :

$$\frac{\partial}{\partial t}\left(\Sigma\, r^2\,\Omega\right)+\frac{1}{r}\frac{\partial}{\partial r}\left(\nu\,\Sigma\, r^3\frac{\partial \Omega}{\partial r}\right)=0$$

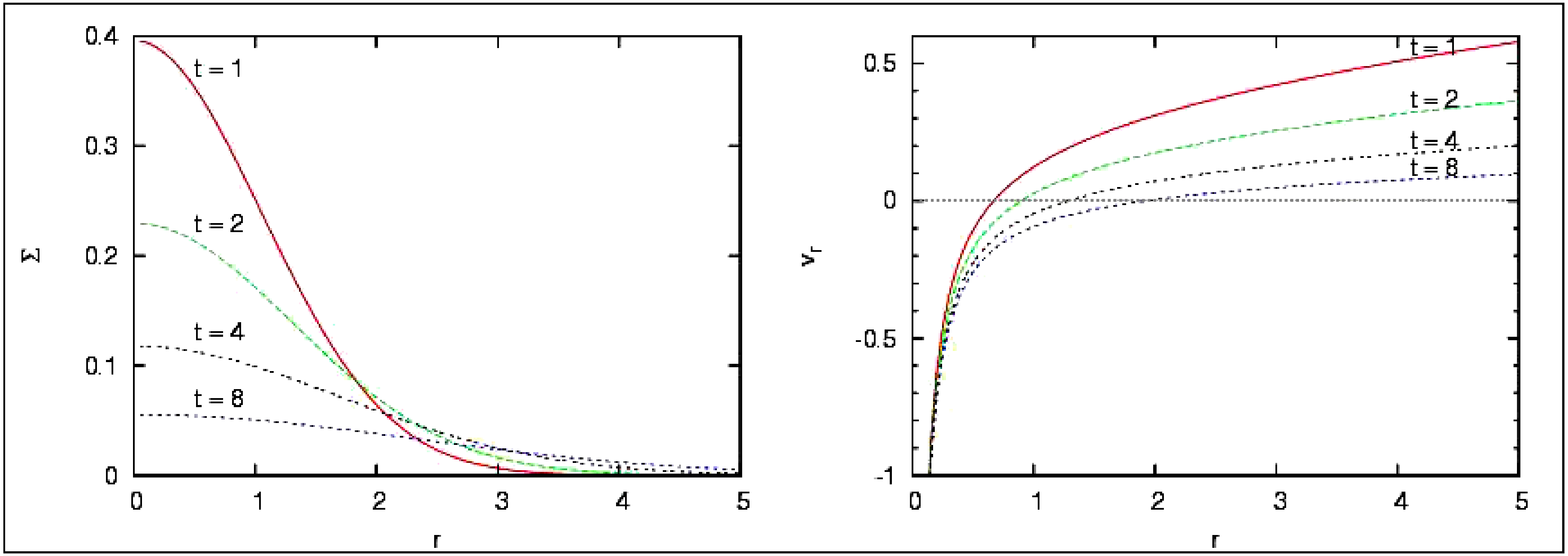
Thus density evolution :

$$\frac{\partial \Sigma}{\partial t}=\frac{3}{r}\frac{\partial}{\partial r}\left[\sqrt{r}\,\frac{\partial}{\partial r}\left(\nu\,\Sigma\,\sqrt{r}\right)\right]$$

DISC EVOLUTION

Evolution (Lynden-Bell & Pringle, 1974) :

Accrétion et étalement. $v_r(r)$ croissante. r_c tel que $v_r(r_c)=0$ augmente.



Quand la densité est suffisamment faible, photo-évaporation du gaz par l'étoile centrale, et le disque disparaît rapidement.

DISC EVOLUTION

Viscous stress :

Final state = minimal energy :

all the mass in the center, all the angular momentum carried by an infinitesimal particle at infinity...

Stellar accretion : ~ 10^{-8} M_{sun}/year

Disk life time : ~ 10^{6} years.

Viscous time : $t_{\nu} = r^2 / \nu$

Gas viscosity : too low.

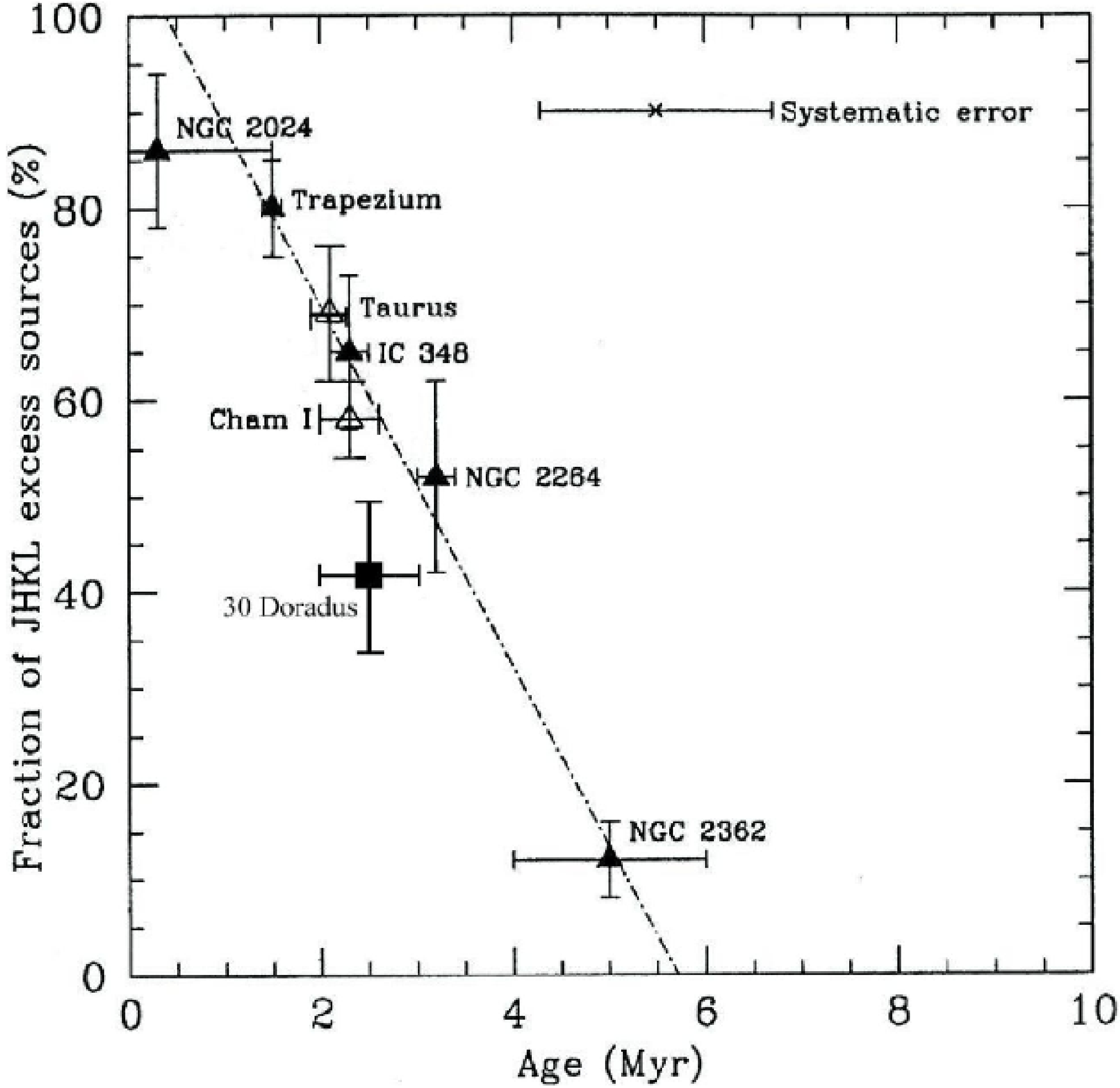
It is considered that ***turbulence*** is responsible for this angular momentum transfer, mimicking viscosity.

Shakura & Sunyaev (1973) : $\nu = \alpha\, c_s\, H$ ($-4 < \log(\alpha) < -2$)

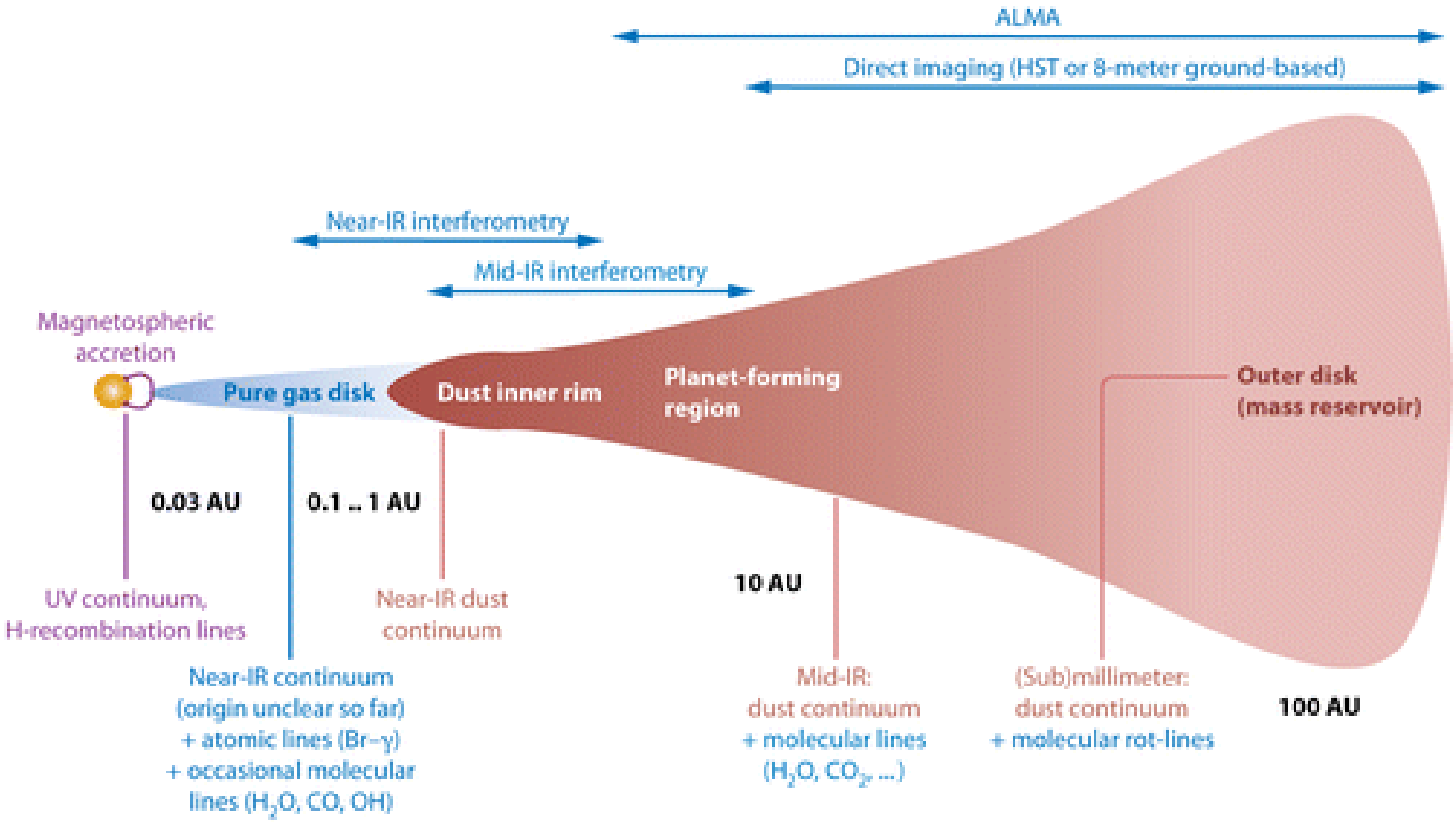
DISC EVOLUTION

<u>Life time :</u>

Most disks disappear in ~ 3 Myr .



DISC STRUCTURE SUMMARY

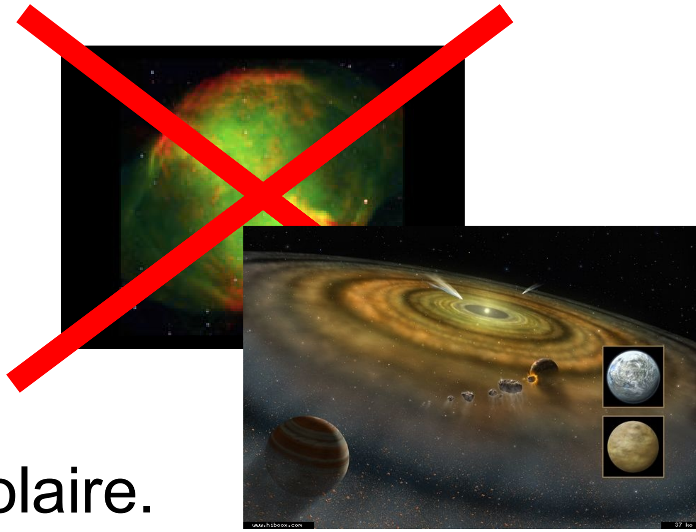


Dullemond CP, Monnier JD. 2010.
Annu. Rev. Astron. Astrophys. 48:205–39

La Nébuleuse Solaire de Masse Minimale

Minimum Mass Solar Nebula …
Nébuleuse Solaire de Masse Minimale …

Qu’est-ce que c'est que ca ?



Comme son nom l'indique (pas),
ce n'est pas une nébuleuse,
mais un disque protoplanétaire.

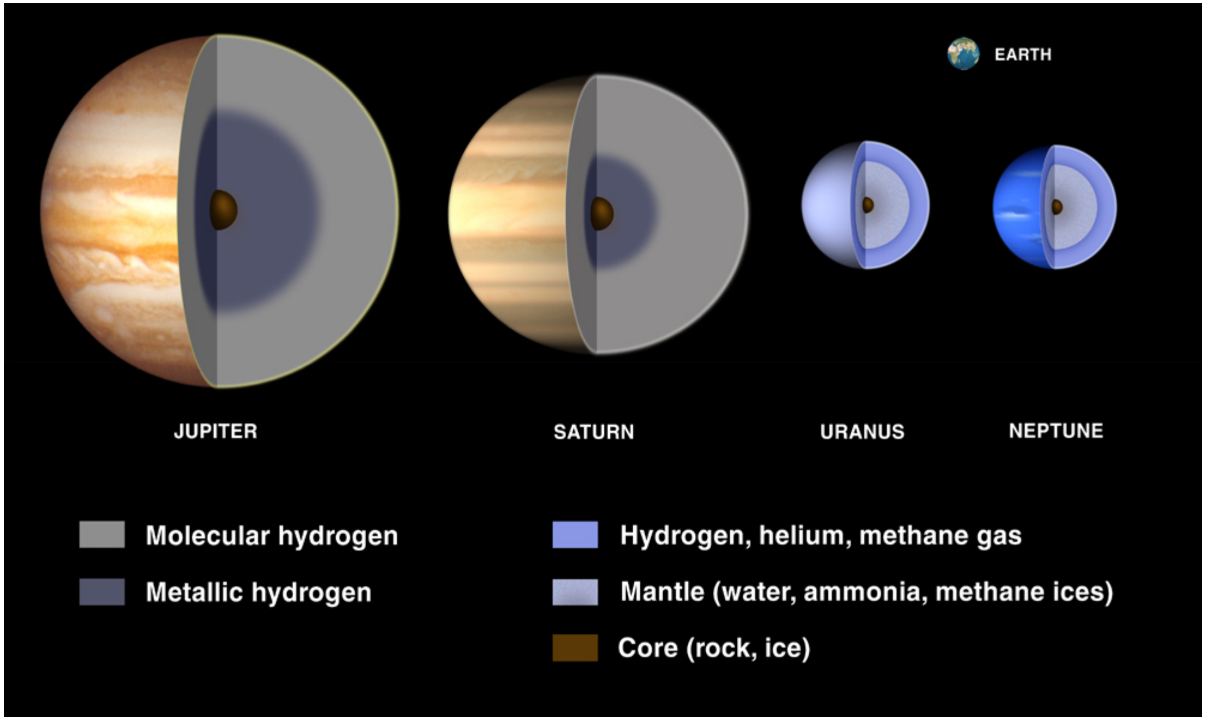
Solaire : d'où est issu de le système solaire.

De Masse Minimale : contient juste ce qu'il faut de matière pour former les huit planètes.

La Nébuleuse Solaire de Masse Minimale

Combien faut-il de solides ?

Les planètes géantes ont un coeur de ~10-15 masses terrestres.



Quels solides ?

Composition chondritique.

La Nébuleuse Solaire de Masse Minimale

Recette pour une Nébuleuse Solaire de Masse Minimale

<u>Ingredients pour 8 planètes :</u>

~ 60 masses terrestres de solides (de composition chondritique).
~ 0.01 masse solaire de la fameuse mixture H (75%), He (25%).

<u>Préparation :</u>

Répartissez les solides au fond du plat :

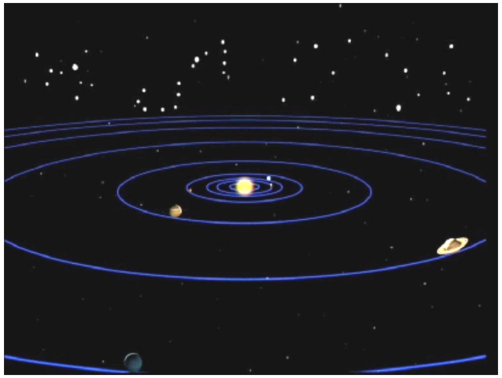
~0.05 $M_{\oplus}$ autour de 0.4 AU [0.3 : 0.5]
~0.8 $M_{\oplus}$ autour de 0.6 AU [0.5 : 0.8]
~1 $M_{\oplus}$ autour de 1 AU [0.8 : 1.3]
~0.1 $M_{\oplus}$ autour de 1.6 AU [1.3 : 2]
~15 $M_{\oplus}$ autour de 5.2 AU [2 : 7.5]
~10 $M_{\oplus}$ autour de 9,6 AU [7.5 : 15]
~15 $M_{\oplus}$ autour de 19.2 AU [15 : 25]
~15 $M_{\oplus}$ autour de 30 AU [25 : 35]

Multipliez la densité obtenue par 100 (ajout de gaz).

Couvrez le tout avec un profil en loi de puissance.

Disposez autour du Soleil pendant 10 millions d'années.

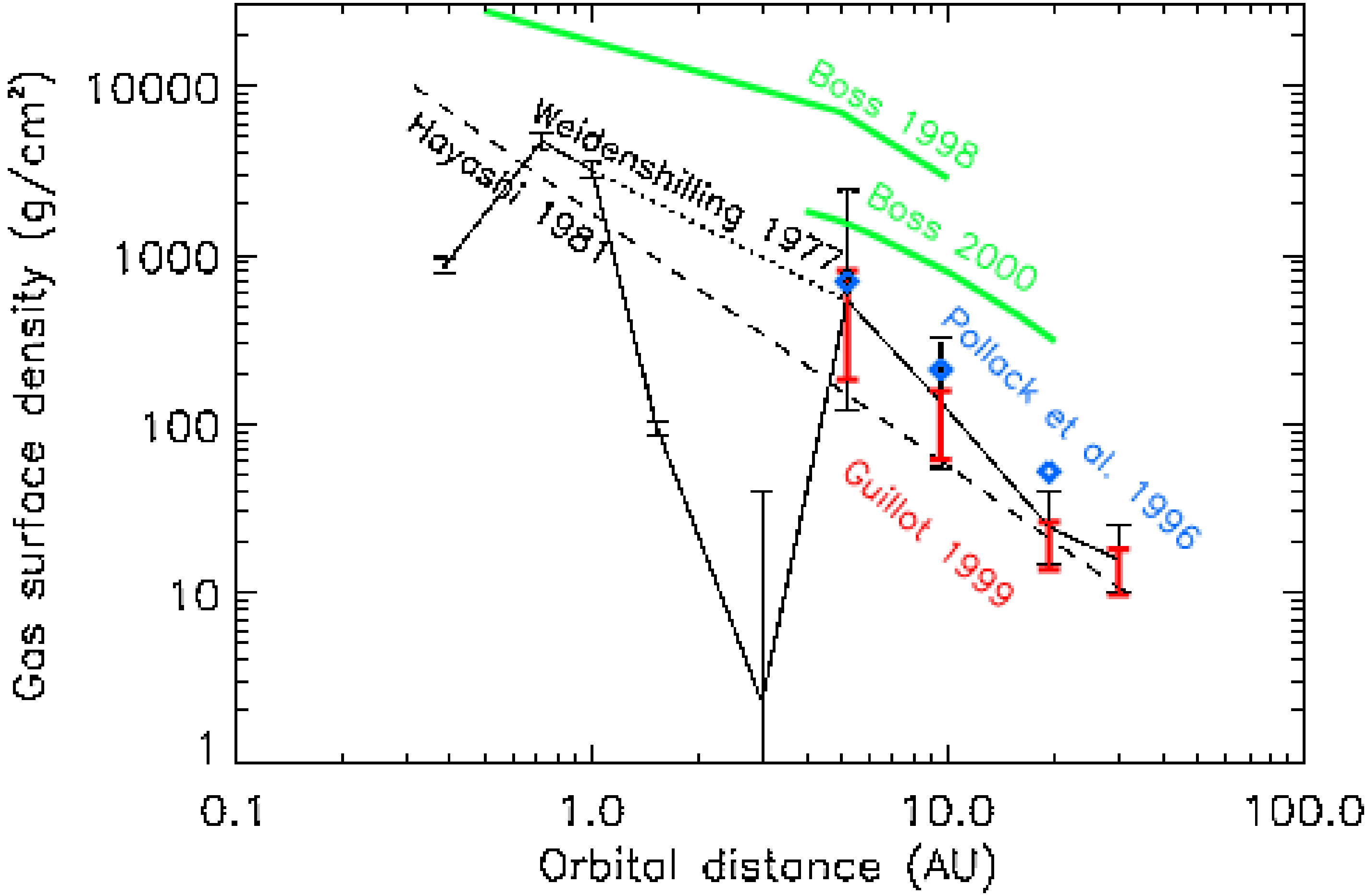
Vous obtenez le système solaire. Bon appétit !



La Nébuleuse Solaire de Masse Minimale

Hayashi (1981) : $\sigma(r) = 1700 \left(\frac{r}{1\,UA}\right)^{-1,5}$ g.cm^{-2}

et Weidenschilling (1977).



La Nébuleuse Solaire de Masse Minimale

Importance de la MMSN :

Hayashi (1981) a été cité 326 fois, plus d'une fois par mois !

Densité utilisée pour la friction dynamique sur les premiers solides, la coagulation, la migration, etc.

C'est le disque proto-planétaire étalon.

Principales hypothèses :

- Les planètes ont accrété **tous** les solides (d'où «minimale»).
- Les planètes se sont formées **localement** :
là où elles orbitent actuellement, à partir du matériel qui était là.
- La MMSN n'évolue pas : densité constante.

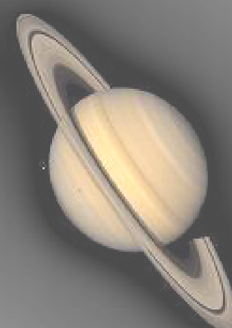
Ces hypothèses sont-elles raisonnables ?

NON !

PLANETARY FORMATION
2) DUST in DISCS

MASTER I M.A.G 2 E

Université Nice SOPHIA ANTIPOLIS

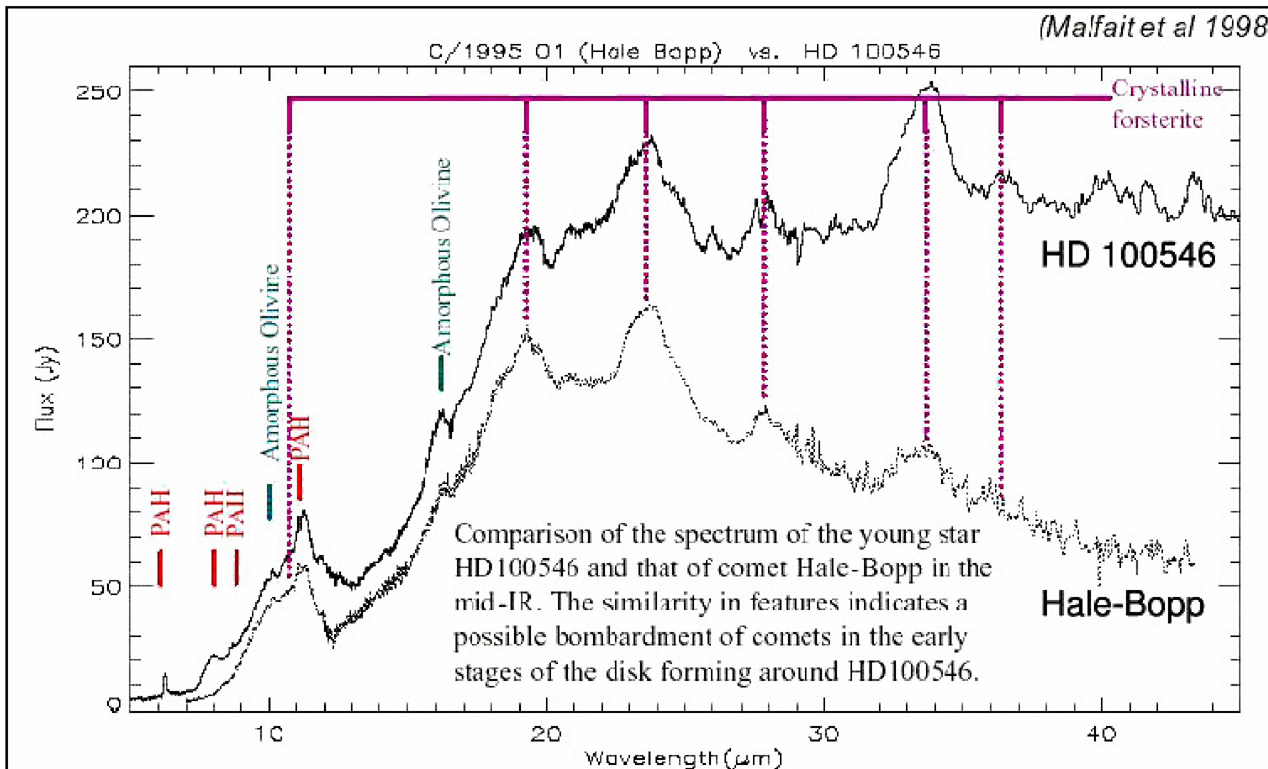


Aurélien CRIDA

DUST IN DISCS

Discs have 99 % H and He, and ~1% C, O, N, Fe, Si, Mg, S...

Dust has a similar spectrum has a comet tail :
one more indication that planets form in disks



DUST IN DISCS : gas drag

<u>Gas drag :</u> $\mathbf{a_{drag} = dv_{dust}/dt = -(v_{dust} - v_{gas}) / t_s}$

t_s = stopping time

For solid spheres of size s and density ρ_d, in a gas of density ρ, where the sound speed is c_s and the mean free path λ, t_s is :

Epstein (small grains $s < 9\lambda/4$) : $t_s = s\, \rho_d / c_s\, \rho$ (@z=0, $t_s = s\, \rho_d / \Sigma$)

Stokes (large grains, $s > 9\lambda/4$) : $t_s = (4\, s\, \rho_d / 9\, c_s\, \rho)\, (s / \lambda)$

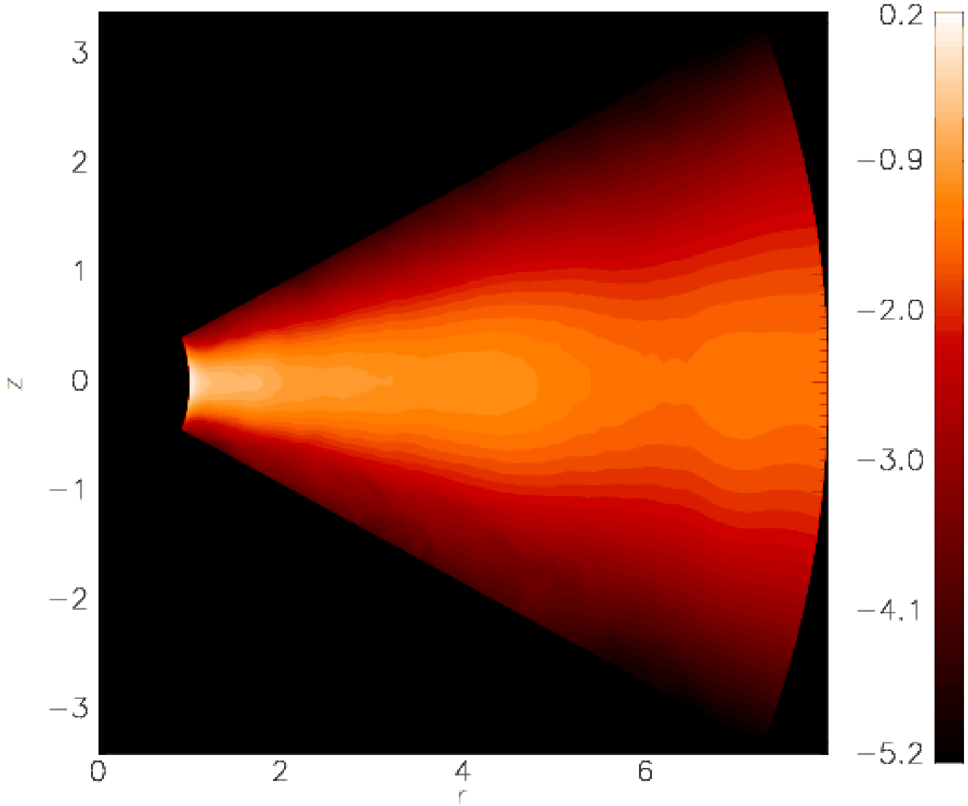
If $\Omega\, t_s \ll 1$, the stopping time is much smaller than the dynamical time ($T=2\pi/\Omega$), so the dust is coupled to the gas.

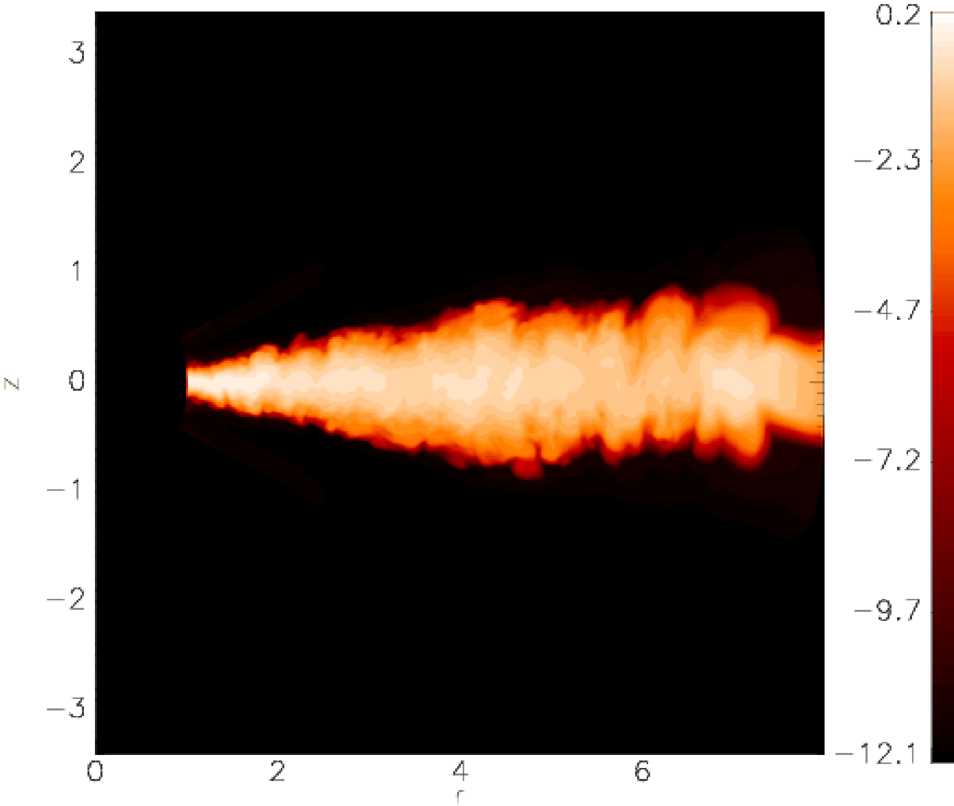
When $\Omega\, t_s \gtrsim 1$, the dust starts decoupling.

When $\Omega\, t_s \gg 1$, the dust hardly feels the gas, keplerian motion.

DUST IN DISCS : sedimentation

<u>Vertically :</u> The gas is pressure supported, but not the dust !
The grains fall onto the midplane : **sedimentation**.





Gas density

Dust density

(Fromang & Nelson 2009)

DUST IN D

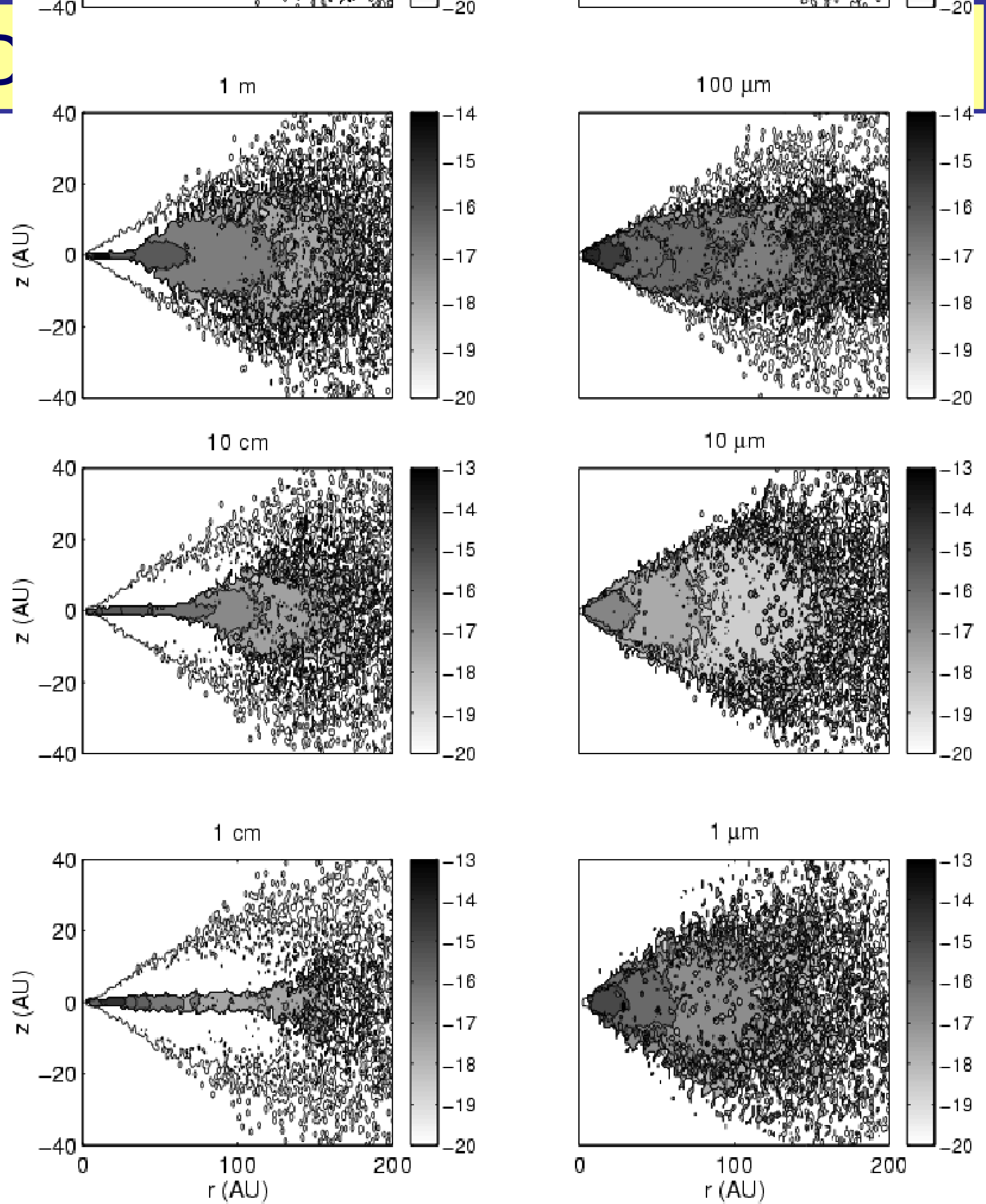
<u>Vertically :</u>

Settling depends

on the coupling, thus

on the grains size :

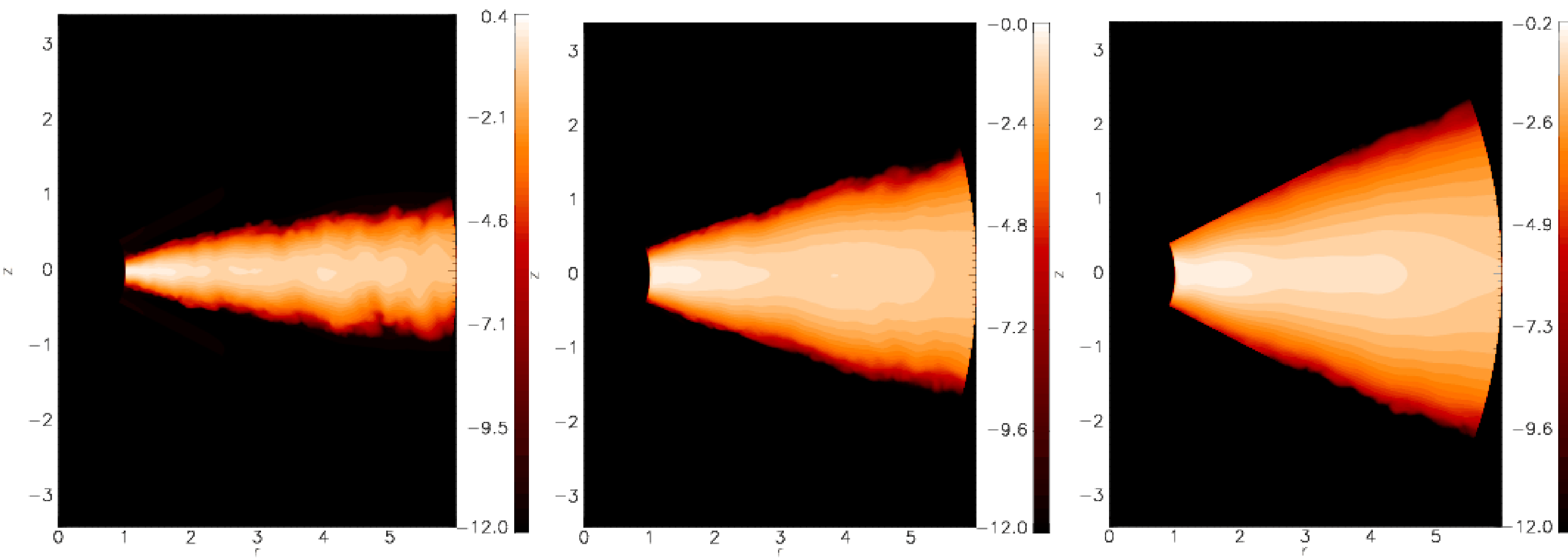
Barrière et al.



DUST IN DISCS : sedimentation

Vertically :

Turbulence => diffusion.
The equilibrium state depends on the coupling.



$\Omega\, t_s = 0.01$ $\Omega\, t_s = 0.001$ $\Omega\, t_s = 0.0001$

(Fromang & Nelson 2009)

DUST IN DISCS : radial drift

<u>Horizontally :</u>

The gas is subkeplerian → the grains feel a headwind and lose angular momentum and energy → radial drift to the central star.

<u>**Exercice :**</u> Compute the radial drift, dr/dt, assuming $v_{dust} = r\Omega_K$.

We remind that $\Omega_{gas} = \Omega_K \, [\, 1 - (1+a-2\beta)(H/r)^2 \,]^{1/2}$.

<u>**Solution :**</u>

$a_{dust} = -(r\Omega_K - r\Omega_{gas}) \,/\, t_s = -(r\Omega_K/2t_s)\,[\,(1+a-2\beta)(H/r)^2\,]$

Specific orbital energy loss rate dE/dt = power = a_{dust} x v_{dust} :

$dE/dt = -(r^2\Omega_K{}^2/2t_s)\,[\,(1+a-2\beta)(H/r)^2\,]$

But $E = -\,GM_*/2r$, so $dE/dt = (dr/dt)\,GM_*/2r^2$.

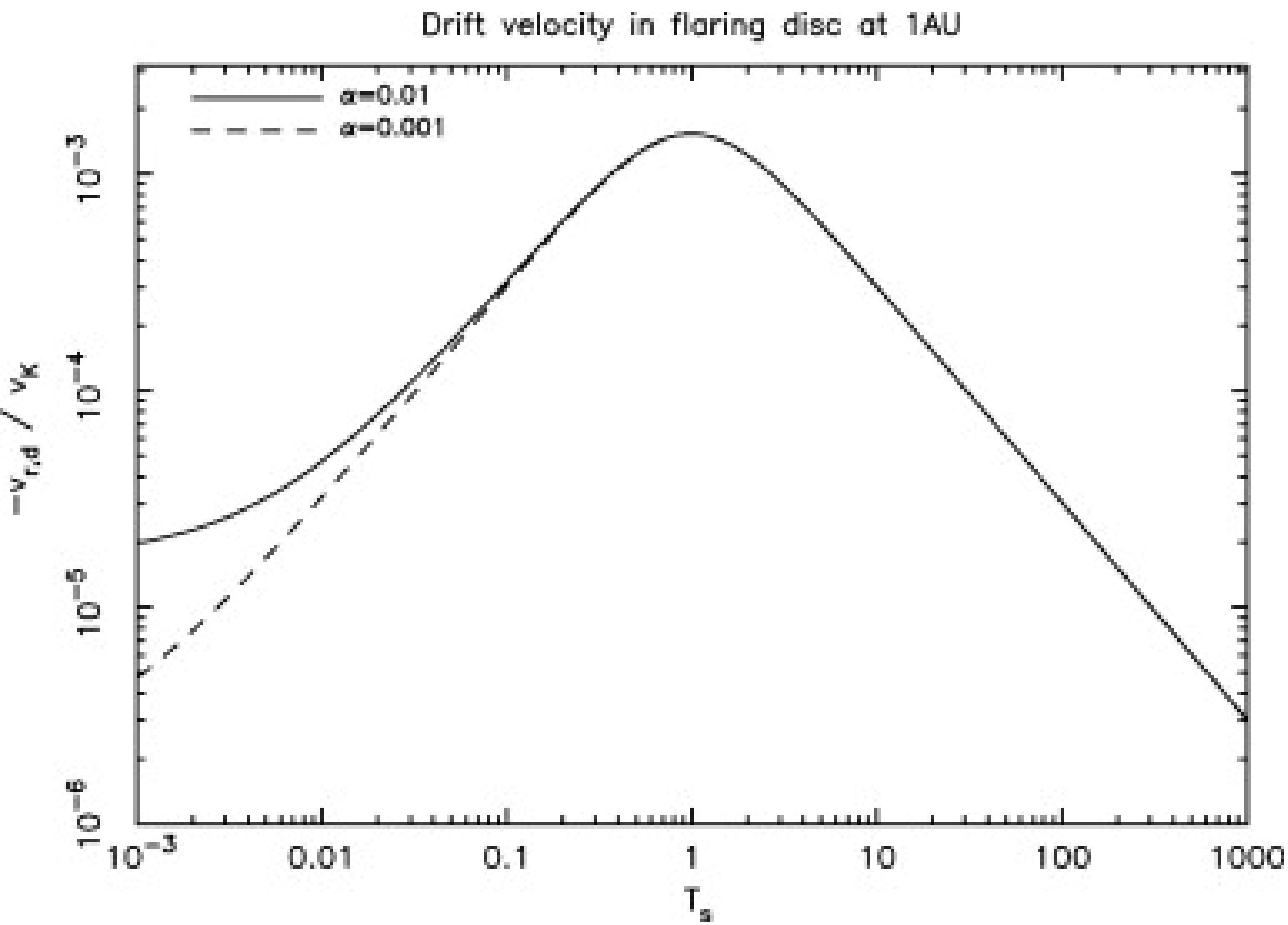
So, $\mathbf{dr/dt = (r/t_s)\,[\,(1+a-2\beta)(H/r)^2\,]}$.

Drift time : $t_{drift} = t_s \,/\, [\,(1+a-2\beta)(H/r)^2\,] \gg t_s$.

DUST IN DISCS : radial drift

Horizontally :

The gas is subkeplerian -> the grains feel a headwind and lose angular momentum and energy -> radial drift to the central star.



If $\Omega\ t_s$ = ~1 ,

fall into the star in less than 1000 orbits !

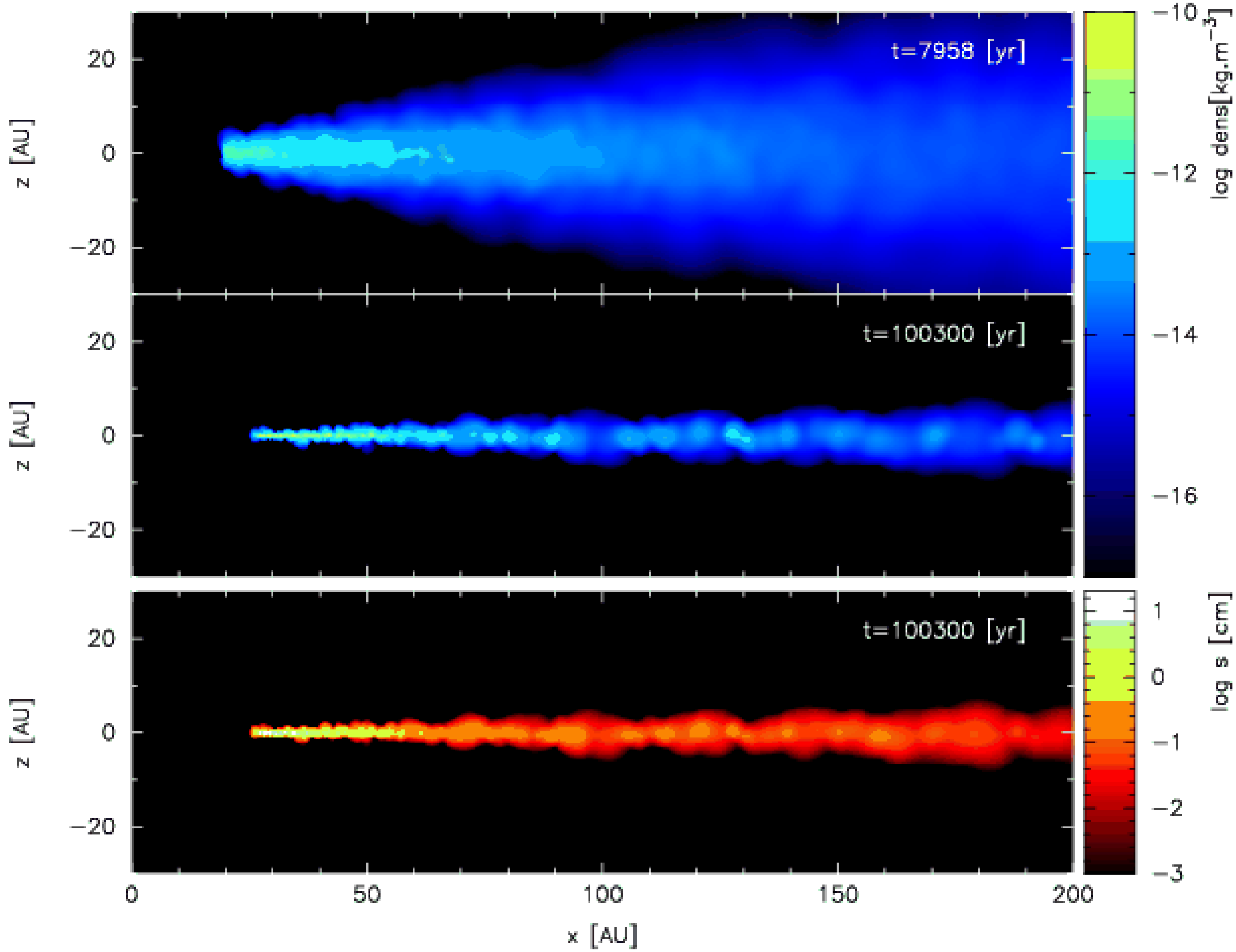
This is called :

the (centi)metre size barrier !

DUST IN DISCS : radial drift

<u>Sedimentation + radial drift :</u>

Simulations by Laibe et al (2008).



DUST IN DISCS : radial drift

<u>Horizontally :</u>

The drift is directed inwards if and only if the gas is sub-keplerian. But we have seen that

$$\Omega_{gas}^2 = \Omega_K^2 + (1/r\rho)\,(dP/dr)\ .$$

So, the dust gathers to pressure maximum.

<u>Examples :</u>

- Edge of a gap opened by a planet.

- anticyclonic vortex.

DUST IN DISCS : instabilities

In general, $\Sigma_{dust} = \sim \Sigma_{gas} / 100$.

If $H_{dust} = H_{gas}/100$, the volume densities can be equal in the midplane => a layer of gas is accelerated by the dust.

→ Possible instabilities, generating turbulence.

DUST IN DISCS : instabilities

1) <u>Kelvin-Helmotz instability :</u>
In the midplane, the gas is keplerian while it is slower in the above layers. When two fluids have different velocities, waves appear at the interface.

(movies by Anders Johansen)

Formation of a concentration of solids, gavitationnaly bound.

Formation of large asteroids from cm aggregates of dust ?

DUST IN DISCS : instabilities

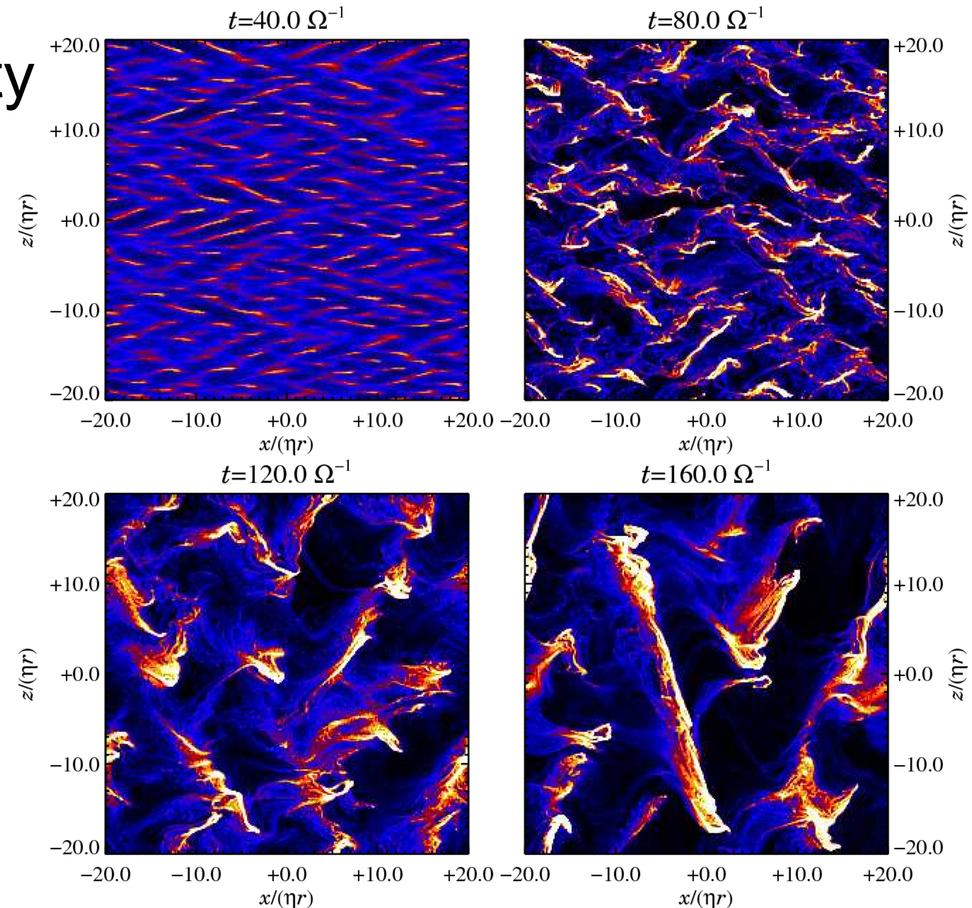
2) <u>Streaming instability :</u>

In the regions where the density of solids is larger, the gas is pulled and goes faster, so the gas drag is weaker, and the radial drift is slower.

In the regions where the density of solids is smaller, the gas is slower, and the drag is larger, so the drift is faster.

Eventually, the fast drifting dust catches up with the slow regions, enhancing the density there. The densest regions ar stable, and are even growing !

(see movie)



DUST IN DISCS : turbulence

3) External turbulence :

Even if the turbulence is not due to dust-gas interaction, it is though that proto-planetary disks are turbulent.

Then, turbulence gathers dust in anticyclonic vortices.

(see movie)

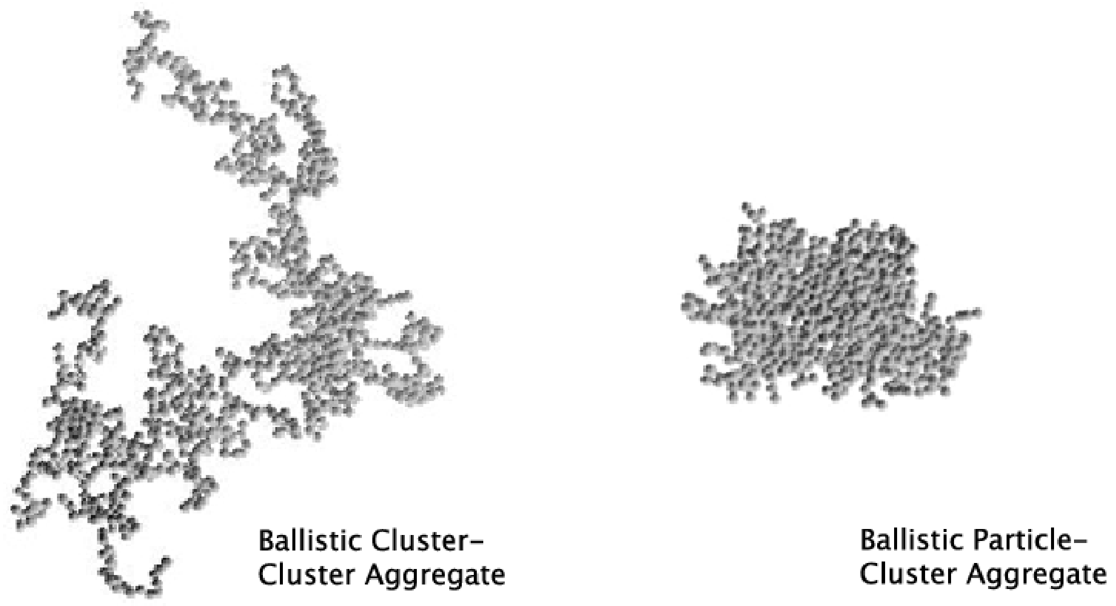
Can these local, temporary concentrations of dust really form solid bodies ?

Johansen, Youdin, and colaborators are optimistic.
No concensus so far...

DUST IN DISCS : Growth

Back to laminar flows.

Dust aggregates well from micro-meter to centi-meter (e.g. : under your bed). Formation of porous aggregates, with fractal dimension, and puffy structure.

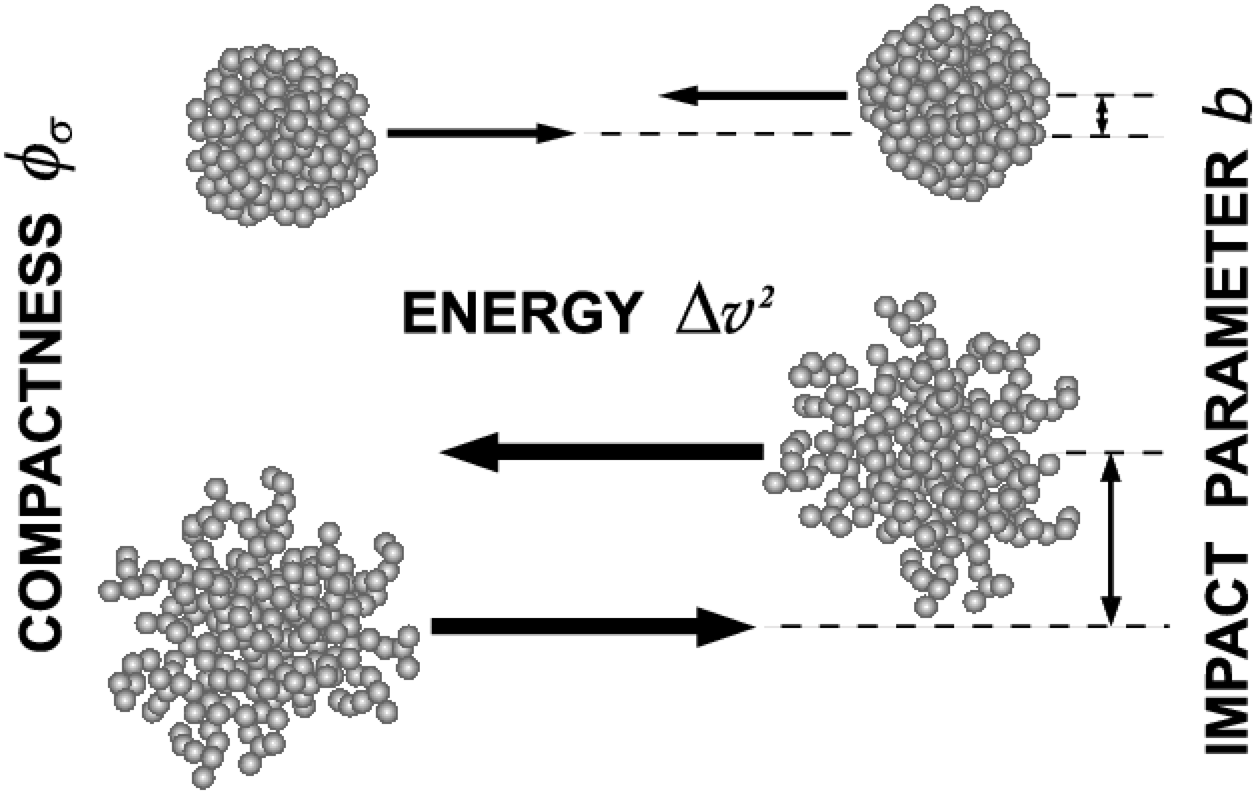


What happens when two aggregates collide ?

DUST IN DISCS : Growth

What happens when two aggregates collide ?

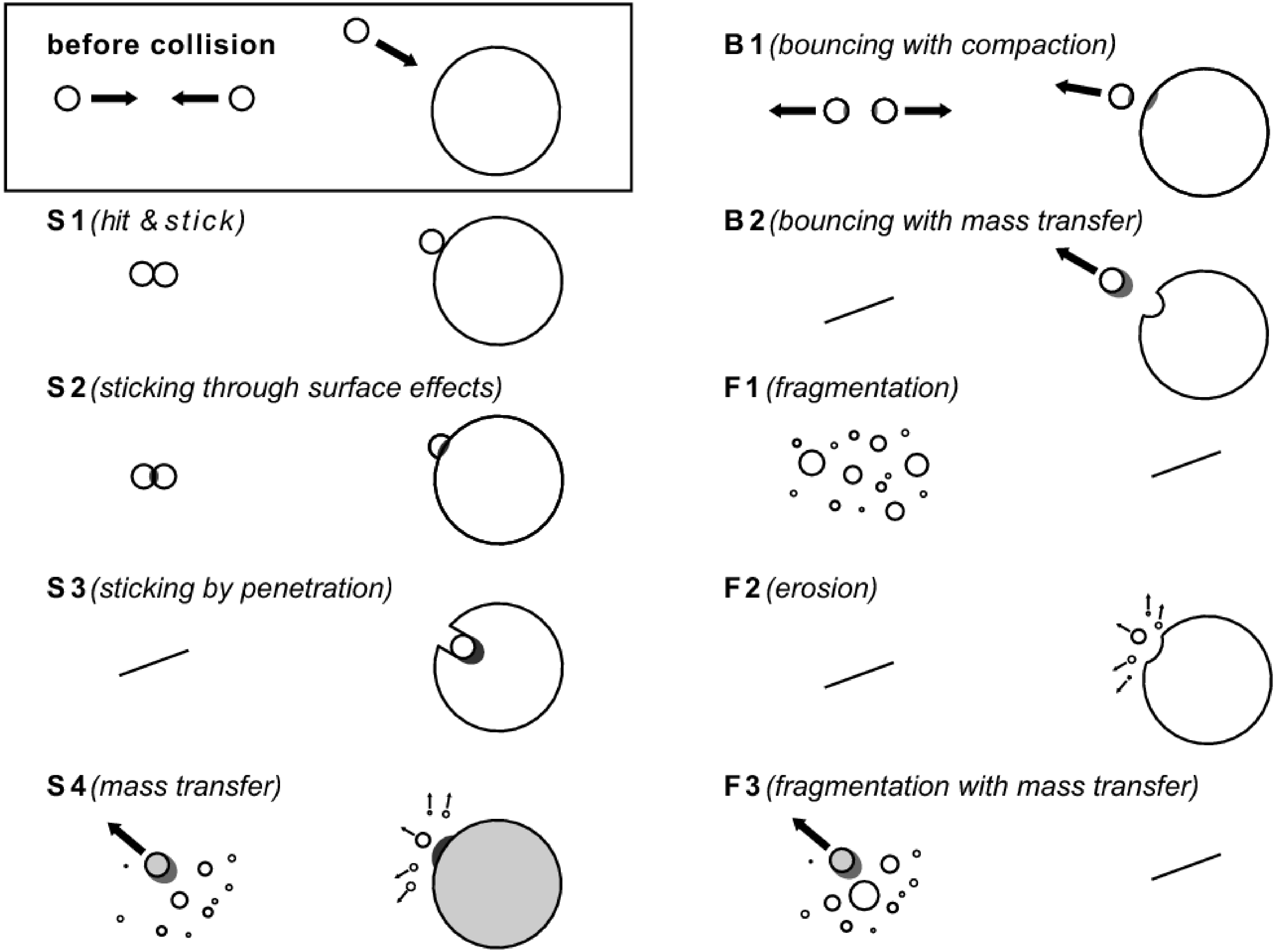
German people make experiments of dust aggregates collision in zero-g in vaccum in a drop tower.



(Ormel et al.)

DUST IN DISCS : Growth

Possible outcomes :   (Güttler et al . 2010)



DUST IN DISCS : Growth

Possible outcomes : Summary

- Sticking

  (complete merge, or mass transfer)

- Bouncing

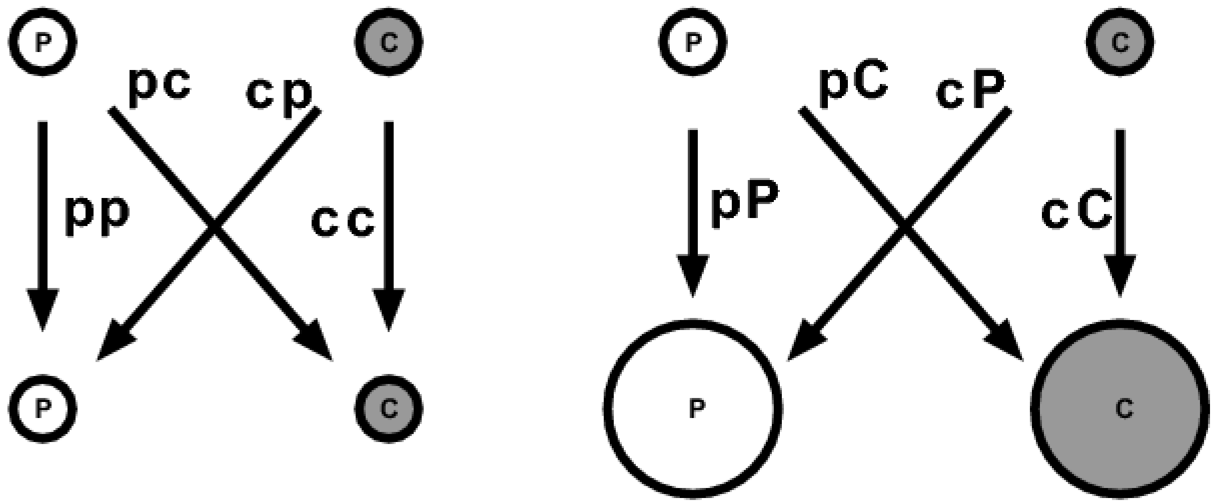
- Fragmentation

  (total destruction, or excavation of some mass)

DUST IN DISCS : Growth

<u>It depends on</u>

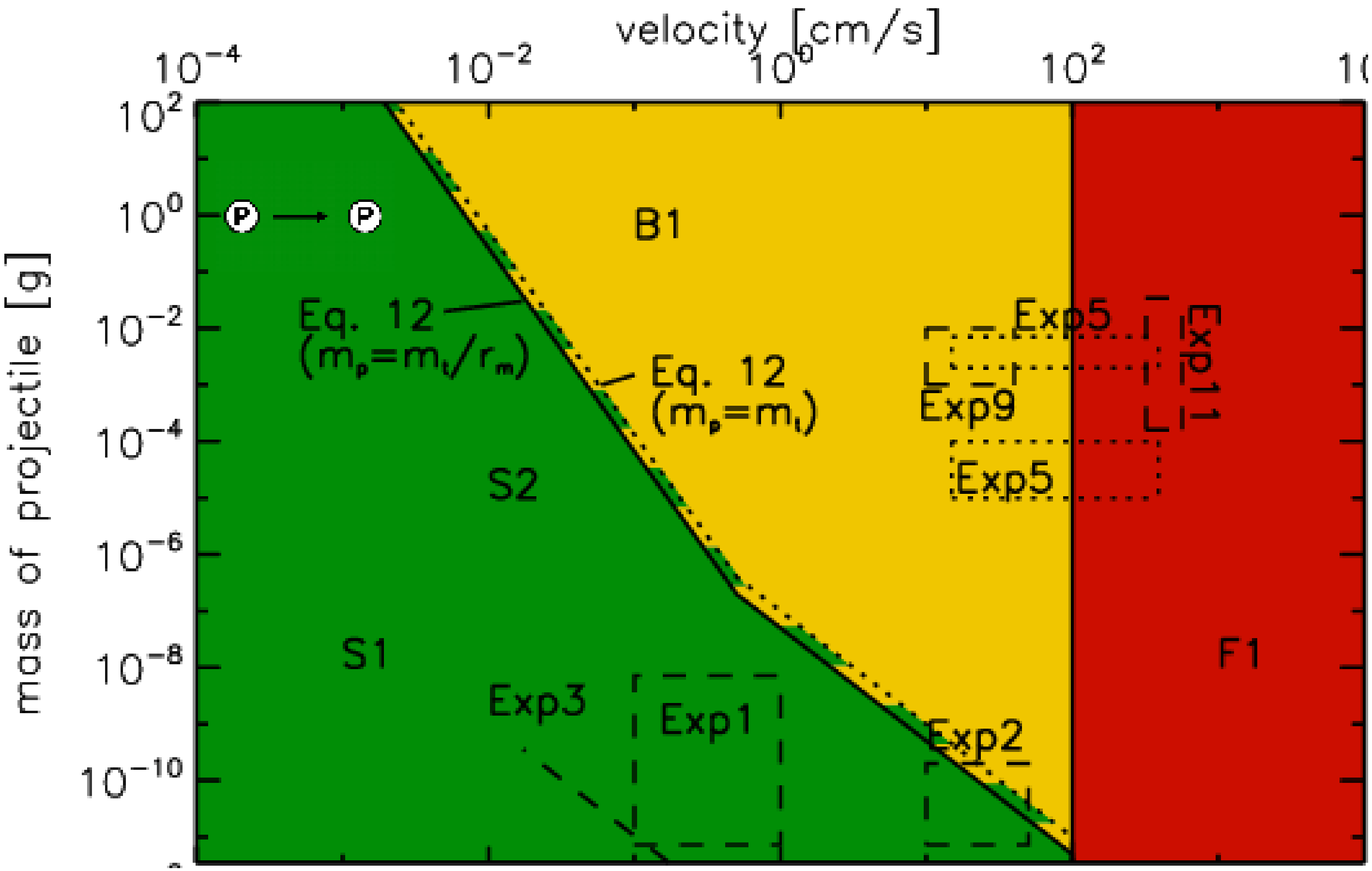
- porosity
- relative velocity
- mass ratio



(Güttler et al. 2010)

DUST IN DISCS : Growth

Result for two equal size porous aggregates :



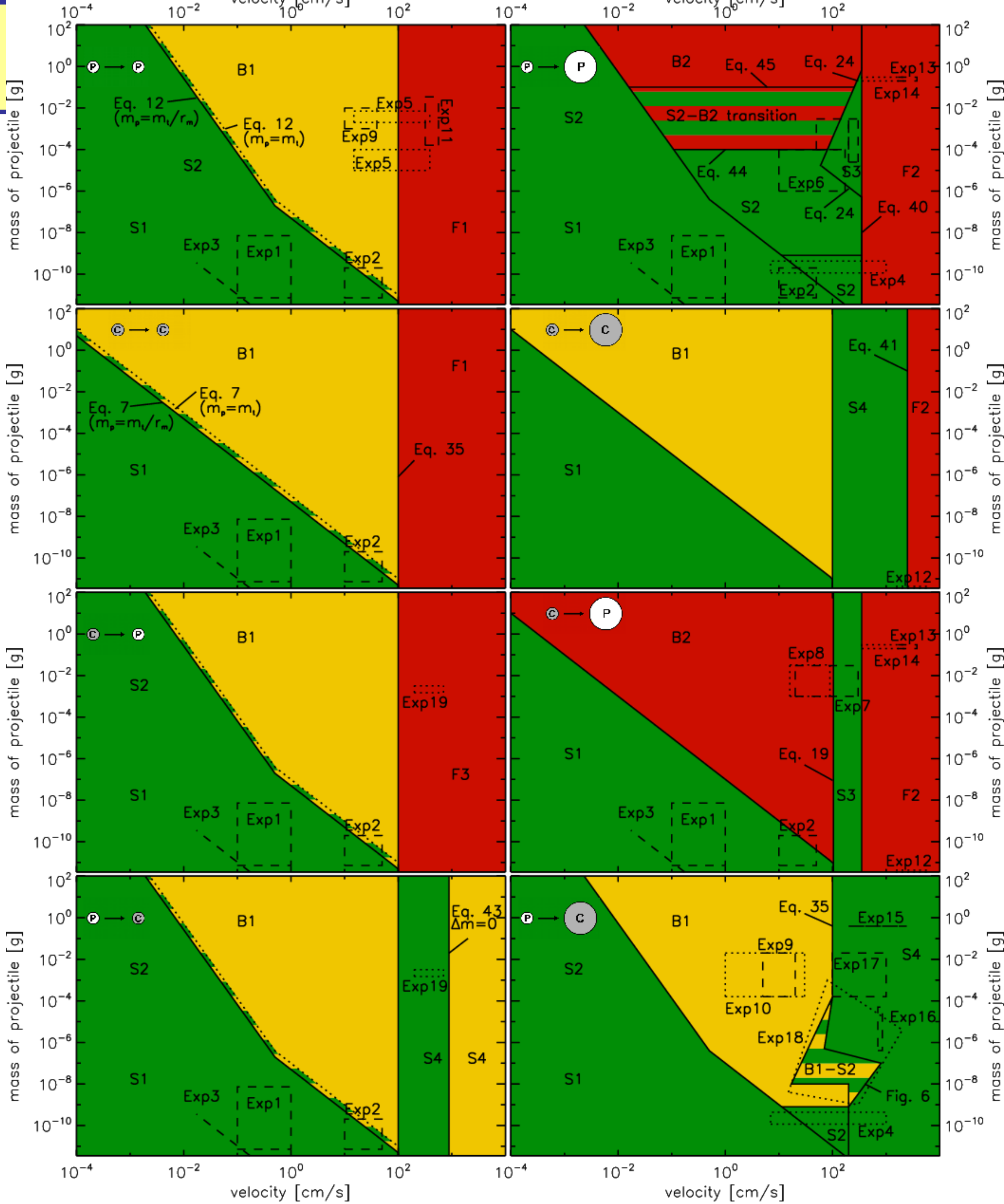
1) Too fast (v>1m/s) => Fragmentation

2) Bouncing barrier : larger bodies bounce and don't stick, unless v<1mm/s

(Güttler et al. 2010)

DUST

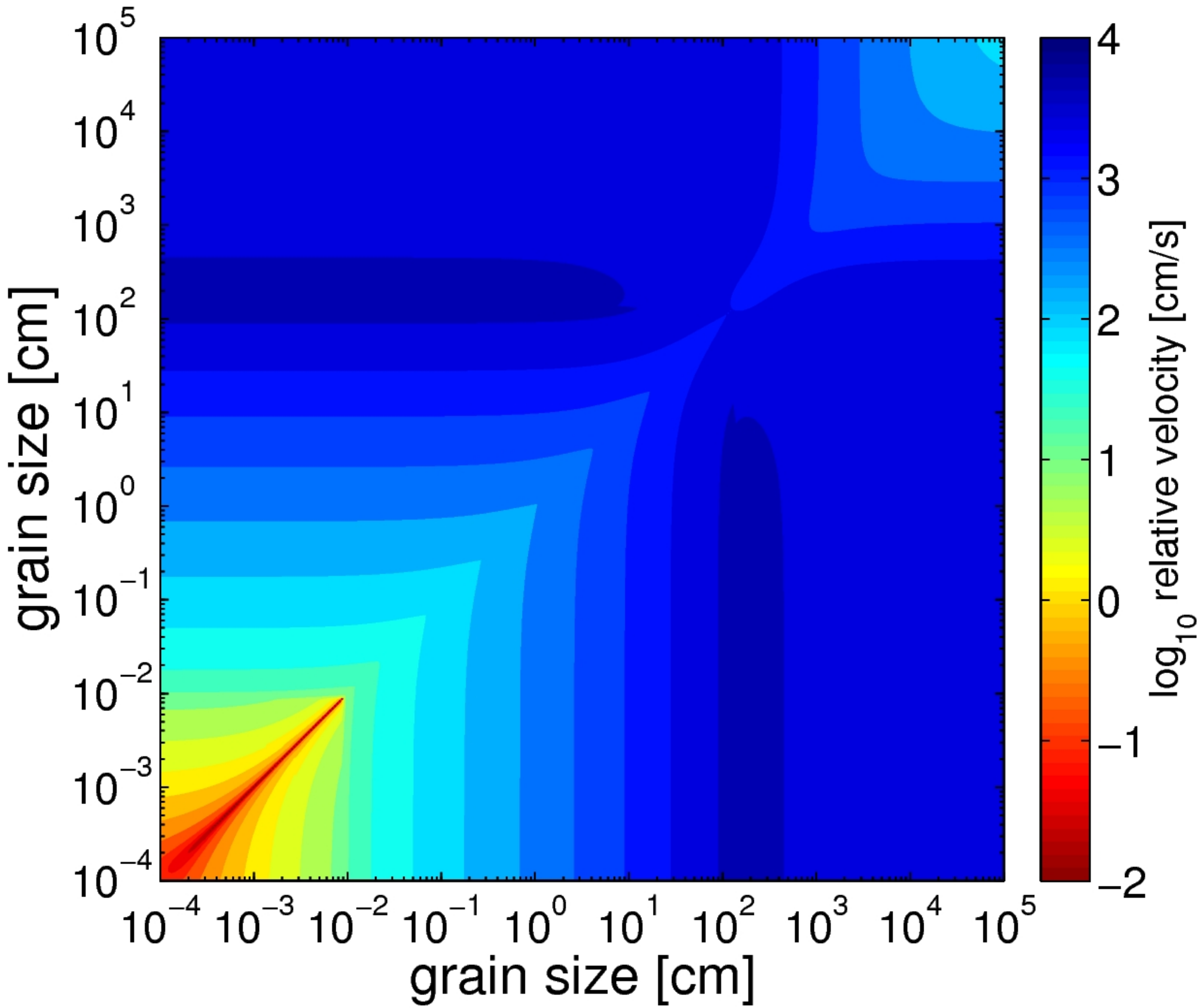
Result for all cases :



(Güttler et al. 2010)

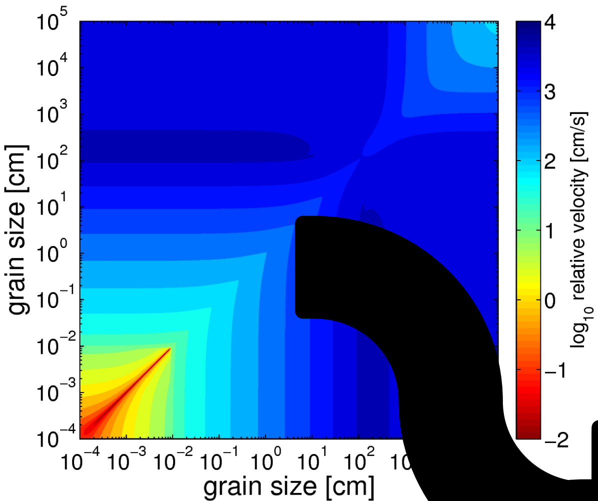
DUST IN DISCS : Growth

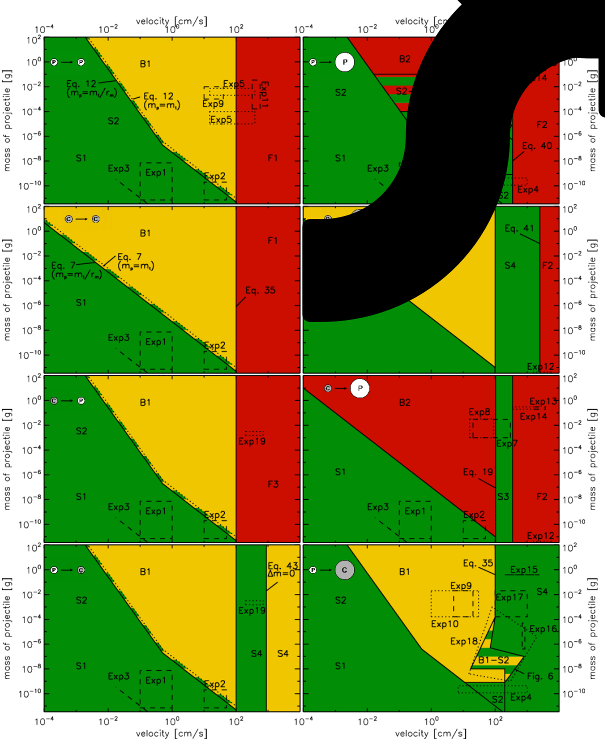
Impact velocity :

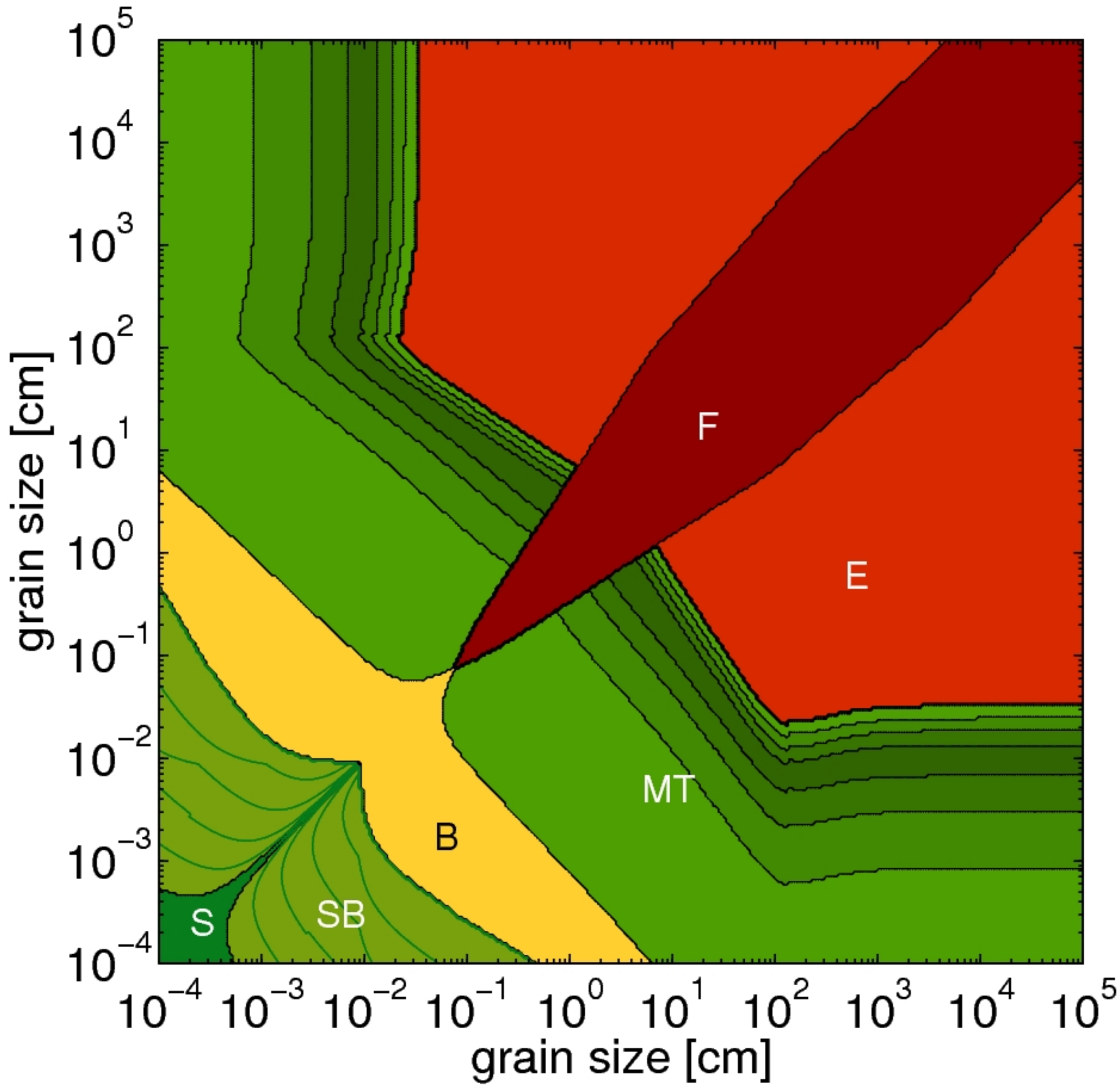


(Windmark et al. 2012)

DUST IN DISCS : Growth







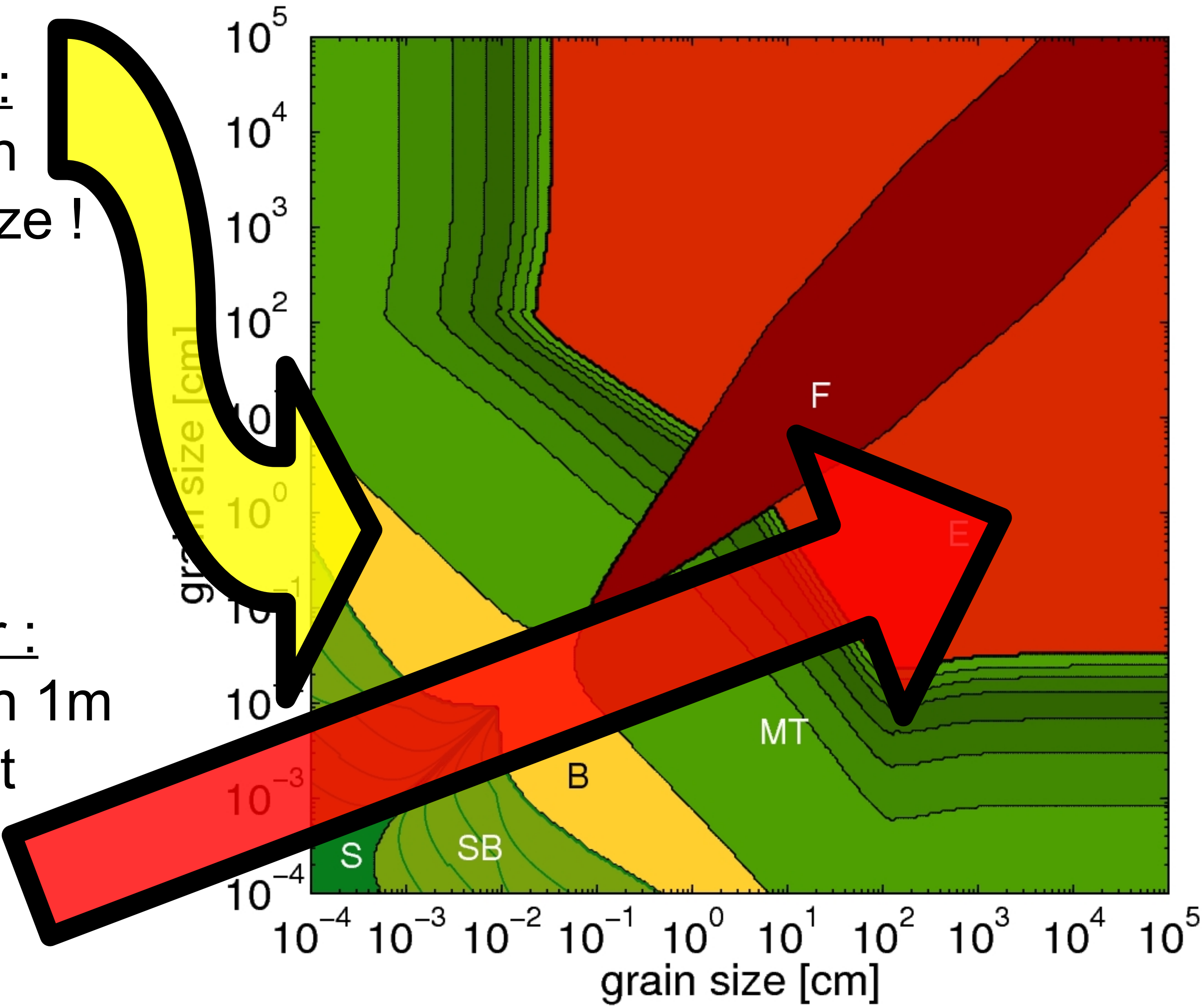
(Windmark et al. 2012)

DUST IN DISCS : Growth

<u>Bouncing barrier :</u>
impossible growth beyond the cm size !

:-(

<u>Meter size barrier :</u>
bodies larger than 1m erode or fragment each other and don't grow anymore.

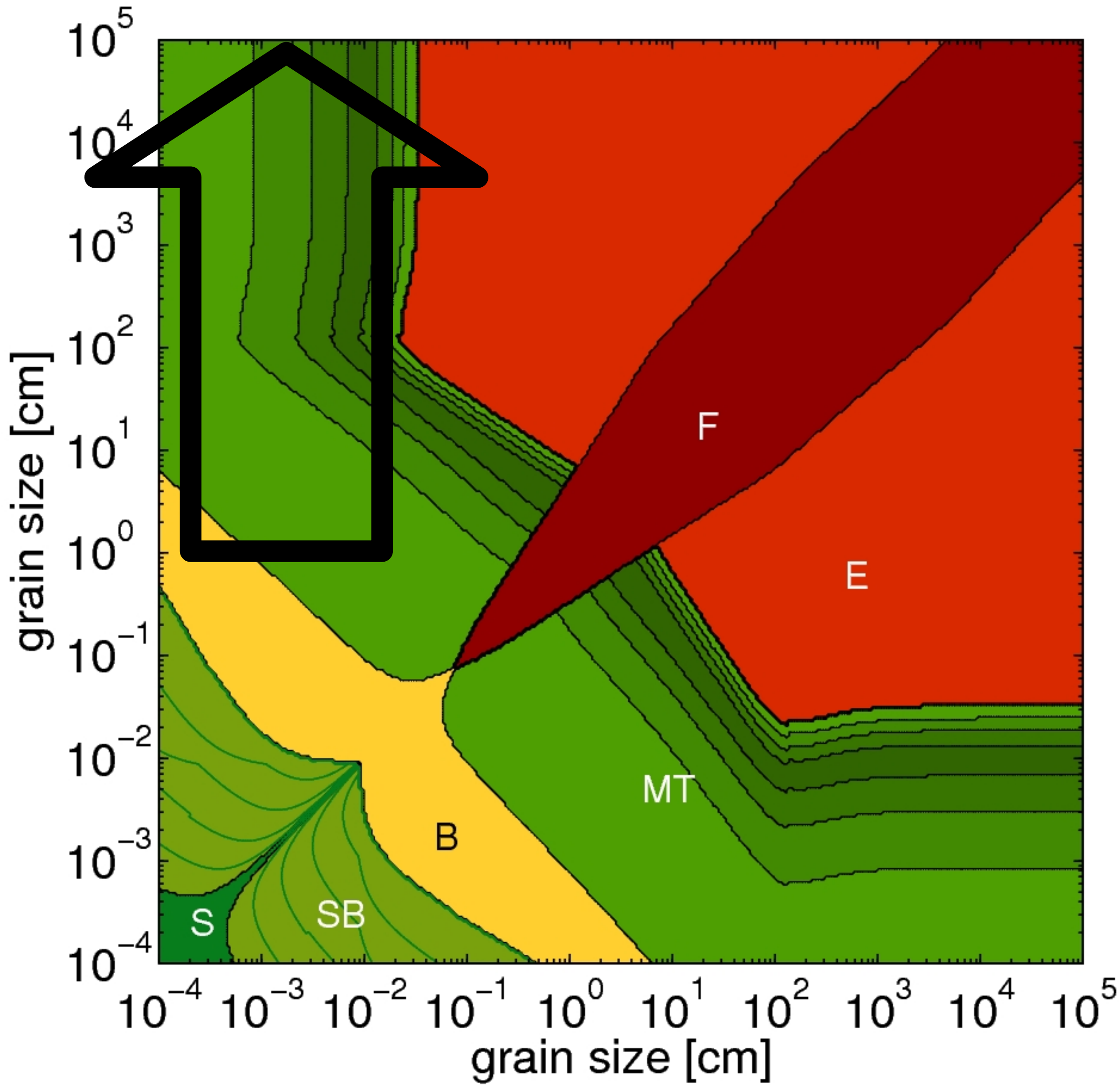


(Windmark et al. 2012)

DUST IN DISCS : Growth

Solution :

input a seed ~ 1cm into a swarm of smaller aggregates : it grows !



(Windmark et al. 2012)

DUST IN DISCS : Growth

<u>Solution :</u>

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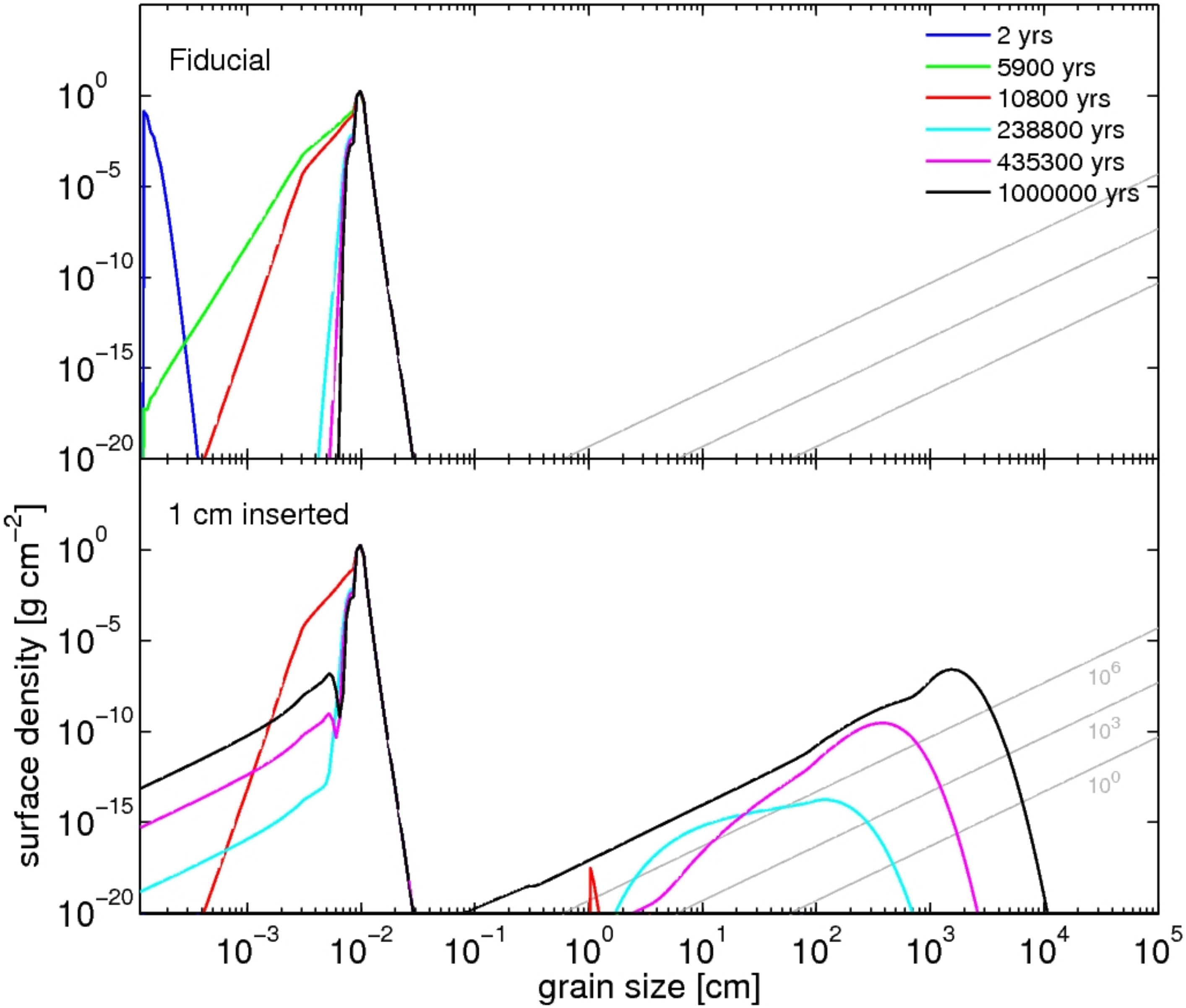


(Windmark et al. 2012)

DUST IN DISCS : Growth

Where does the seed come from ?

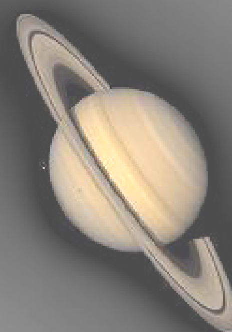
If there is a velocity distribution, it can happen that two 0.1 mm aggregates meet at vey low velocity, and stick. If you are lucky 10^6 times in a row, you get a cm size body.

It's like winning the lotery, but there are millions of players, so one of them must get the jackpot...

PLANETARY FORMATION
3) PLANETESIMALS

MASTER I.M.A.G.2.E

Université Nice SOPHIA ANTIPOLIS



Aurélien CRIDA

PLANETESIMALS

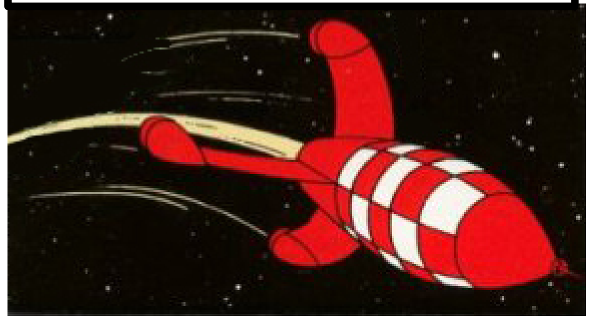
Now, we assume that we have km sized bodies, so that the gravity is the dominant force.

<u>Escape velocity :</u>

velocity that a particle should have to escape from the surface of a planet of mass M_p and radius R_p.

<u>Exercice :</u> What is v_{esc} ?

Allo, allo! Ici la Terre ! Vous venez d'atteindre la vitesse de libération, qui est de 11 km/s, vous n'êtes donc plus soumis à l'attraction terrestre.

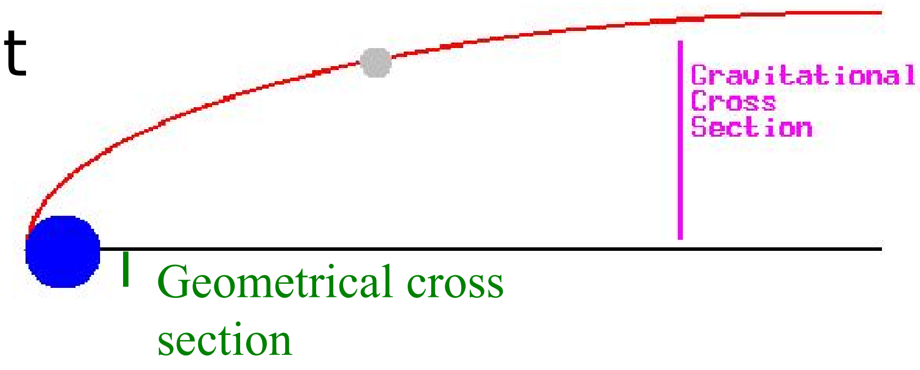


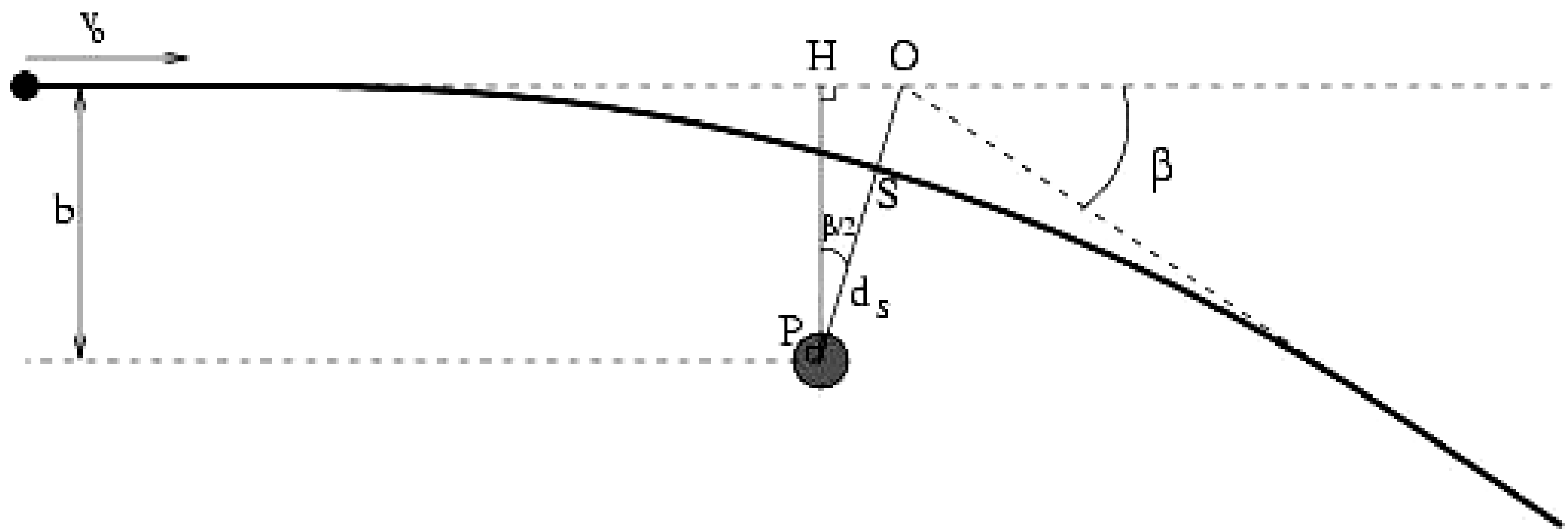
COLLISIONS

<u>Geometric cross section :</u> $\sigma_{\text{géom}} = \pi R^2$

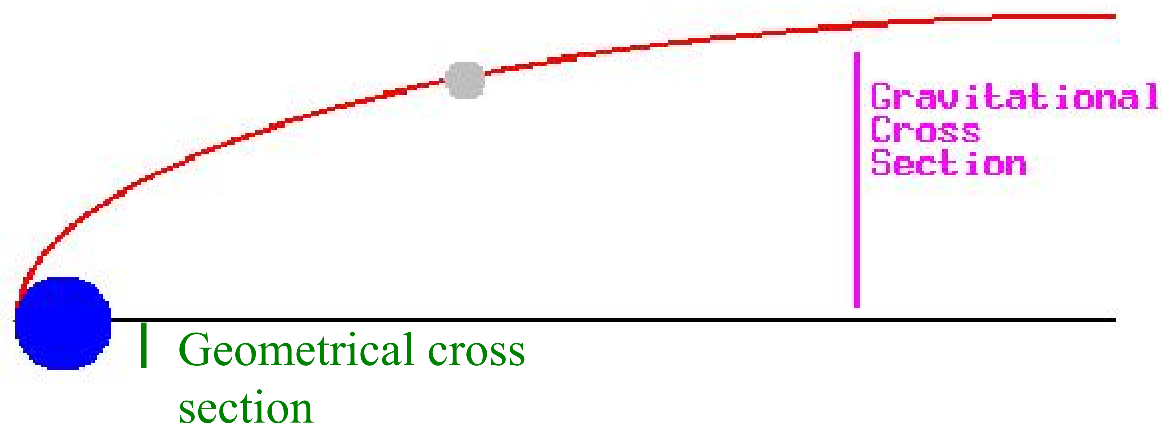
What is the gravitational cross-section ?

Which is the maximum impact parameter *b* , for a collision with the object of mass *M* and radius *R*,
if the initial velocity is v_0 ?





COLLISIONS



Gravitational Focus : $F_g = \sigma_{grav}/\sigma_{géom}$

$$= b_{max}{}^2/R^2$$

$$\mathbf{F_g = 1 + (v_{esc}/v_0)^2} \;,$$

where $v_{esc}{}^2 = 2GM/R.$

Growth : $dM/dt \sim R^2F_g \sim M^{2/3}F_g$

RUNAWAY GROWTH

1) Case $v_0 << v_{esc}$ (dynamically cold disc).

Then, $F_g \sim v_{esc}^2 / v_0^2 \sim M^{2/3} / v_0^2$.

Tale two objects of masses $M_1 > M_2$.

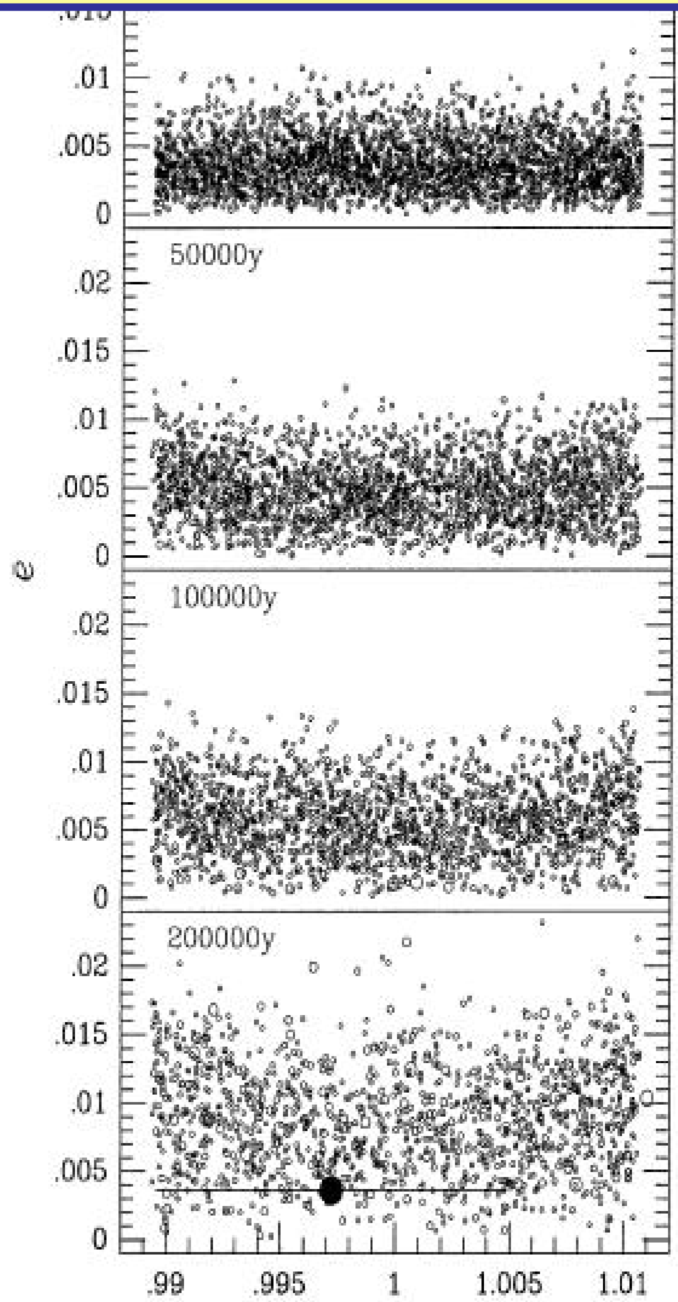
$dM_i/dt \sim M_i^{4/3} / v_0^2$;

$(1/M_i)\ (dM_i/dt) \sim M_i^{1/3} / v_0^2$.

$d(M_1/M_2)/dt = M_1/M_2\ [\ (1/M_1)(dM_1/dt) - (1/M_2)$
$(dM_2/dt)\ ] > 0.$

The mass ratios increase : the largest objects grow faster, and become even larger : Runaway growth.

RUNAWAY GROWTH

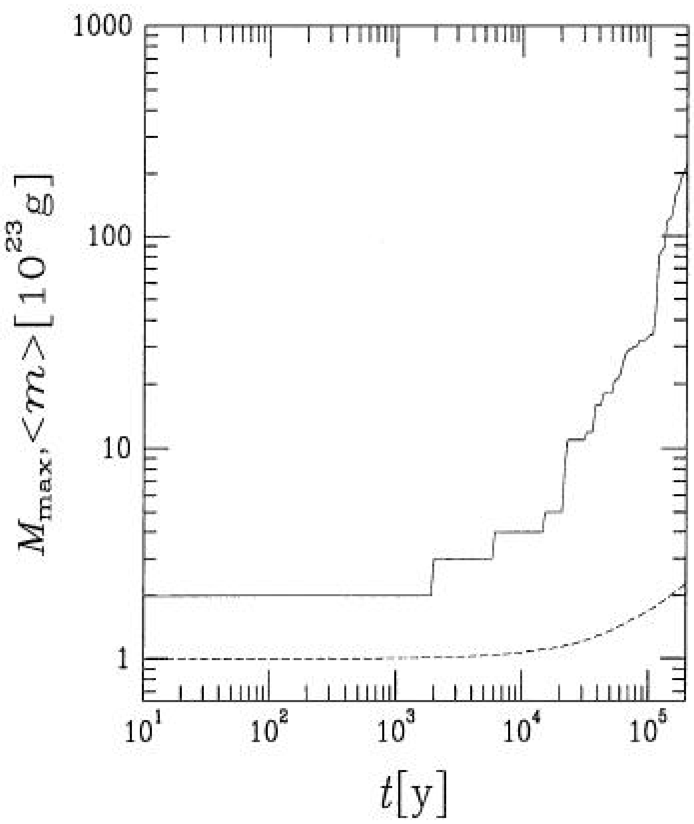


Kokubo and Ida, 2000

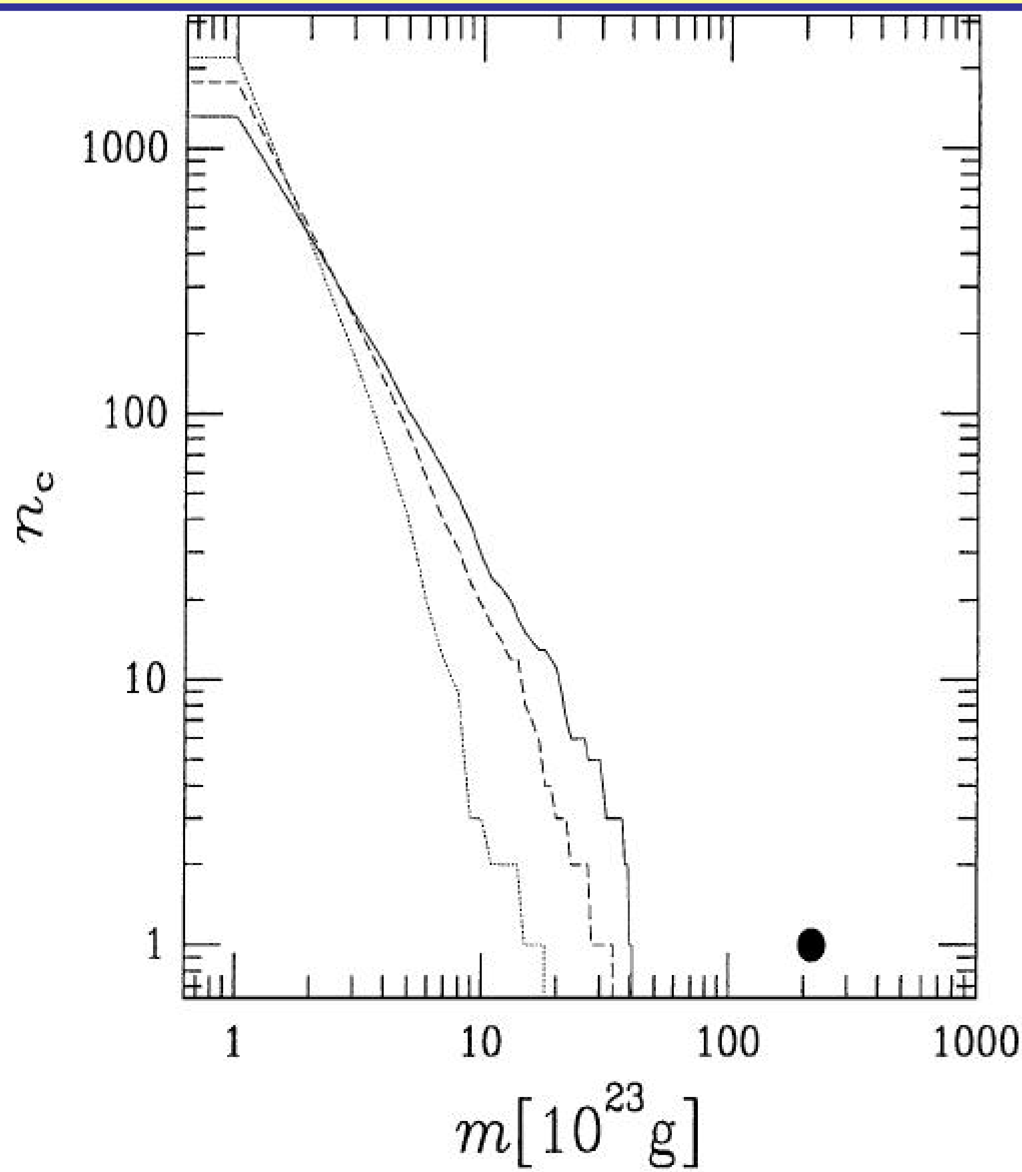
Initially 3,000 10^{23}g planetesimals

End: 1,322 planetesimals + $2\text{x}10^{25}$g `embryo'

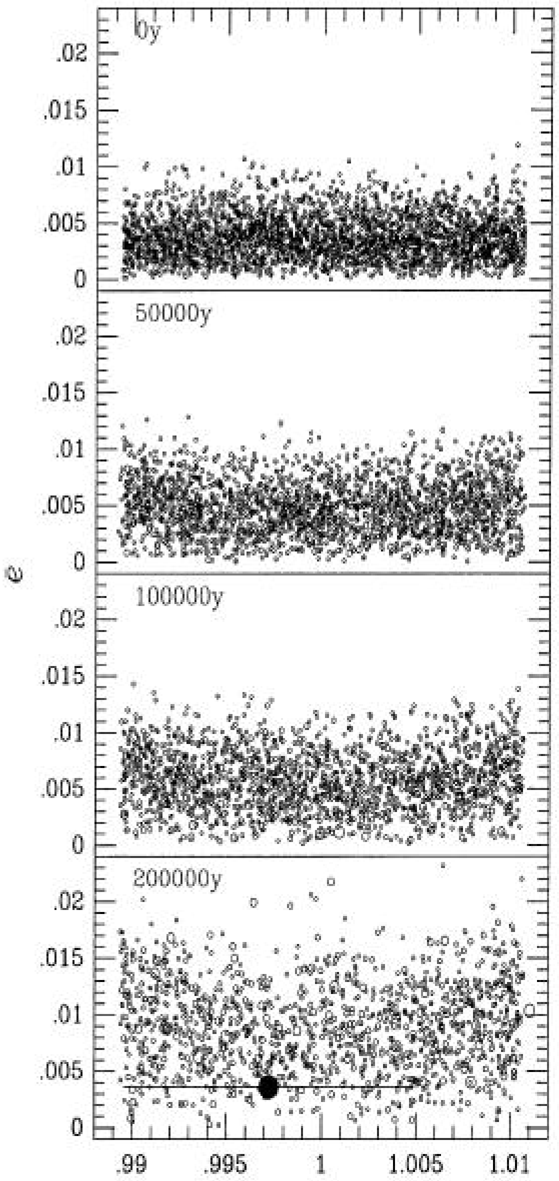
RUNAWAY GROWTH



RUNAWAY GROWTH



RUNAWAY GROWTH



Runaway growth holds as long as $v_{rel} \ll v_{esc}$

But v_{rel} grows in response to the presence of the largest bodies in the disk and it tends to become ~ v_{esc}

It takes some time to get $v_{rel} \sim v_{esc}$. Runaway growth acts only during this time. This time is short if planetesimals are big, while it can be longer if planetesimals are numerous and small (collisional damping) and if there is gas in the system (gas drag). But the runaway growth time is short or null if there is a strong turbulent stirring of v_{rel}.

OLIGARCHIC GROWTH

2) Case $v_0 \sim v_{esc}$ (dynamically hot disc).

Then, $F_g \sim 1$.

Take two objects of masses $M_1 > M_2$.

$dM_i/dt \sim M_i^{2/3} / v_0^2$;

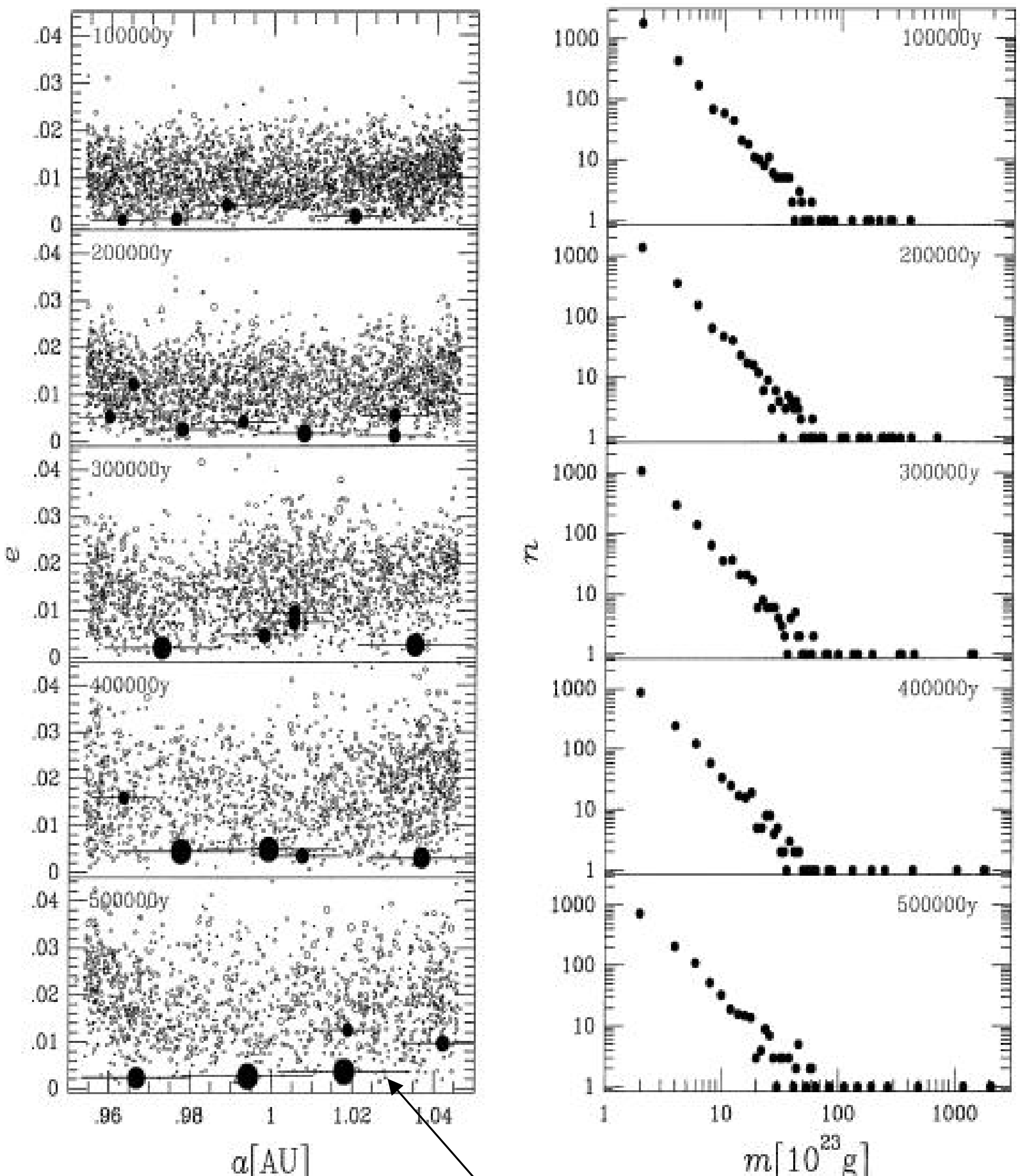
$(1/M_i)\,(dM_i/dt) \sim M_i^{-1/3} / v_0^2$.

$d(M_1/M_2)/dt = M_1/M_2\,[\,(1/M_1)(dM_1/dt) - (1/M_2)(dM_2/dt)\,] < 0.$

The mass ratios decrease.

Oligarchic growth : the largest bodies (previously formed) dominate, and grow together by accreting the small stuff, until they have absorbed all the solids in their zone of influence (feeding zone).

OLIGARCHIC GROWTH



Lines: $5r_H$

Again from Kokubo and Ida, 2000

Filled dots: mass > $2x10^{25}$g

OLIGARCHIC GROWTH

<u>Masses of the protoplanets :</u>

Zone of influence = <u>Roche lobe</u>, inside which the planet's gravity dominates over that of the star. It is close to a sphere (the Hill sphere), of radius: $R_H = a_p \, (M_p \, / \, 3M_* \,)^{1/3}$

(where the index p corresponds to the planet(esimal) and a_p is the semi-major axis, the distance to the star).

R_H : Équilibre à $a = a_p - R_H$:

$-GM^*/a^2 + GM_p/R_H^2 + a\,\Omega_p^2 = 0$ $\quad (/GM_*)$

$-a_p^{-2}(1+2R_H/a_p) + q/R_H^2 + (a_p-R_H)/a_p^3 = 0$

$- 3R_H \, / \, a_p^3 + q \, / \, R_H^2 = 0$ $\quad (q=M_p/M_*)$

Final mass : $M_p = 2\pi a_p \, 2R_H \, \Sigma_{planetesimals}$

$M_p = (\, 4\pi/(3M_*))^{1/3} \, a_p^2 \, \Sigma_{pl} \,)^{3/2}$

which is ~ 0,1 $M_\oplus$ at 1 UA, 1 $M_\oplus$ at 5 UA, in 10^4 - 10^5 years.

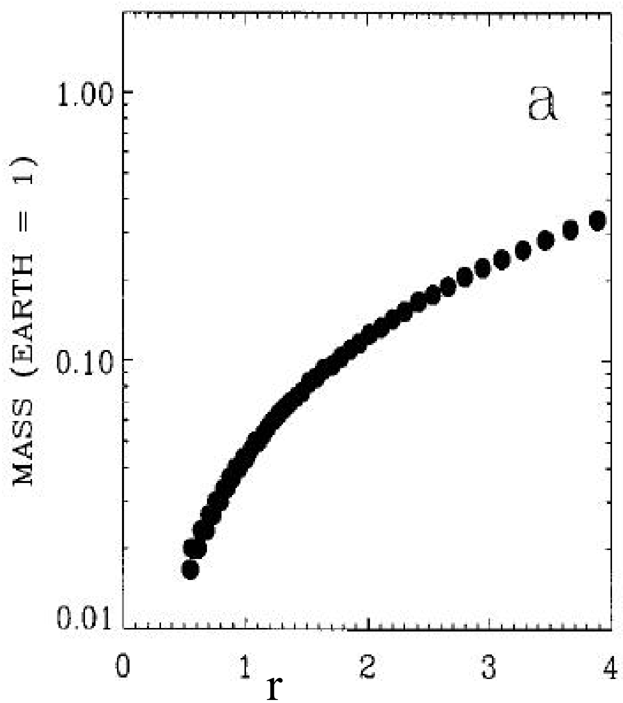
Which is good but not too good…

Every proto-planet has now absorbed everything in its feeding zone, and every feeding zone contains a proto-planet.

The system is densely packed.

OLIGARCHIC GROWTH

Masses of protoplanets: $M \sim [\, r^2\, \Sigma/(3M_{sun})^{1/3}]^{3/2}$ with $\Sigma \sim \Sigma_0/(r/r_0)^{3/2}$



Snowline: location in the disk beyond which the temperature is low enough that water is available in the form of ice. It was computed that Σ is enhanced by a factor 4-5 beyond the snowline.

This could make the oligarchs beyond the snowline as big as several Earth masses (comparable to the masses required to start to accrete gas from the disk. ???

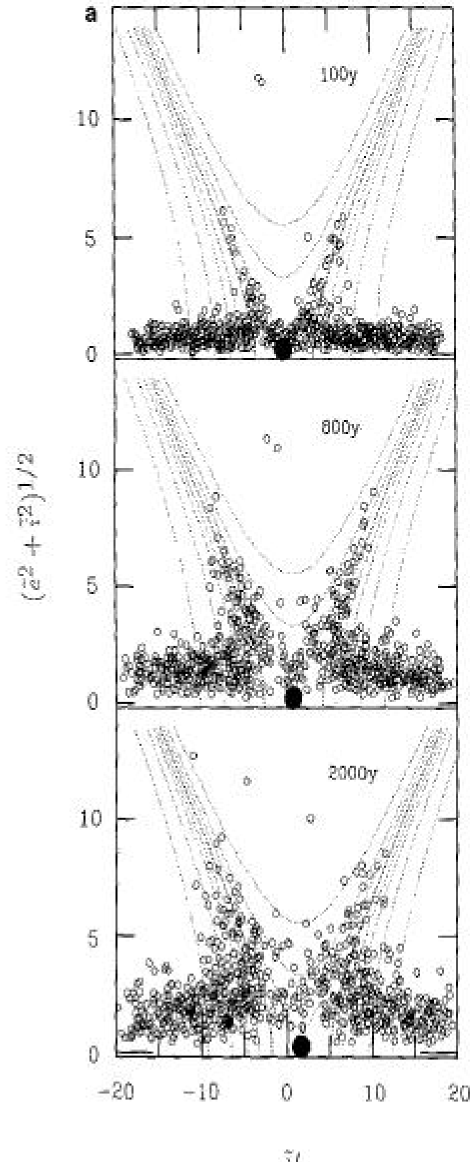
However, some recent studies suggest that Σ doesn't really increase by more than a factor 2.

In addition, accretion can't be 100% efficient.

In the end, the oligarchs are not massive enough to be terrestrial planets or cores of giant planets.

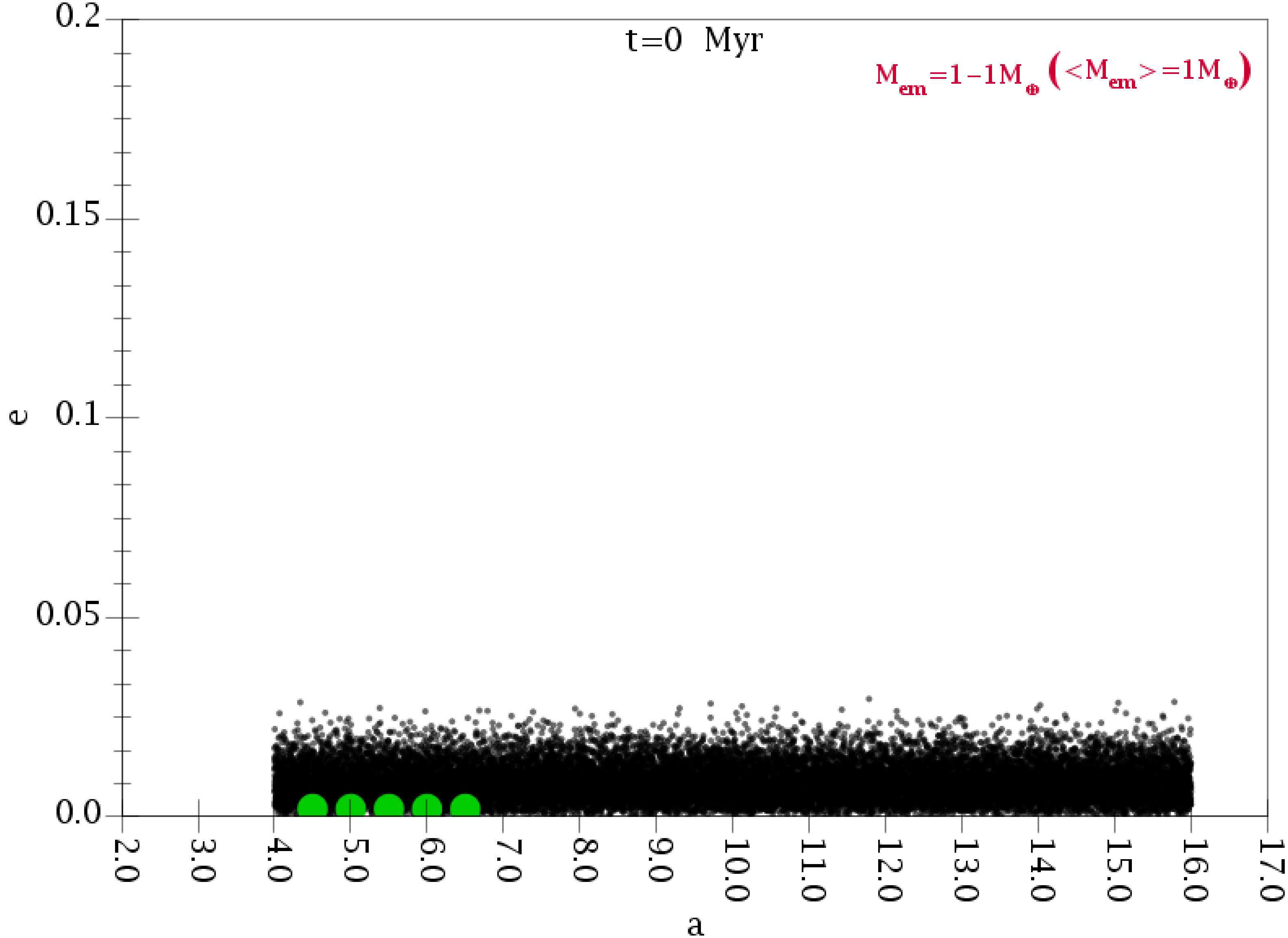
COEURS

The problem is that, even if ♦ is very large, when an oligarch becomes very massive it starts to scatter away planetesimals, rather than accreting them



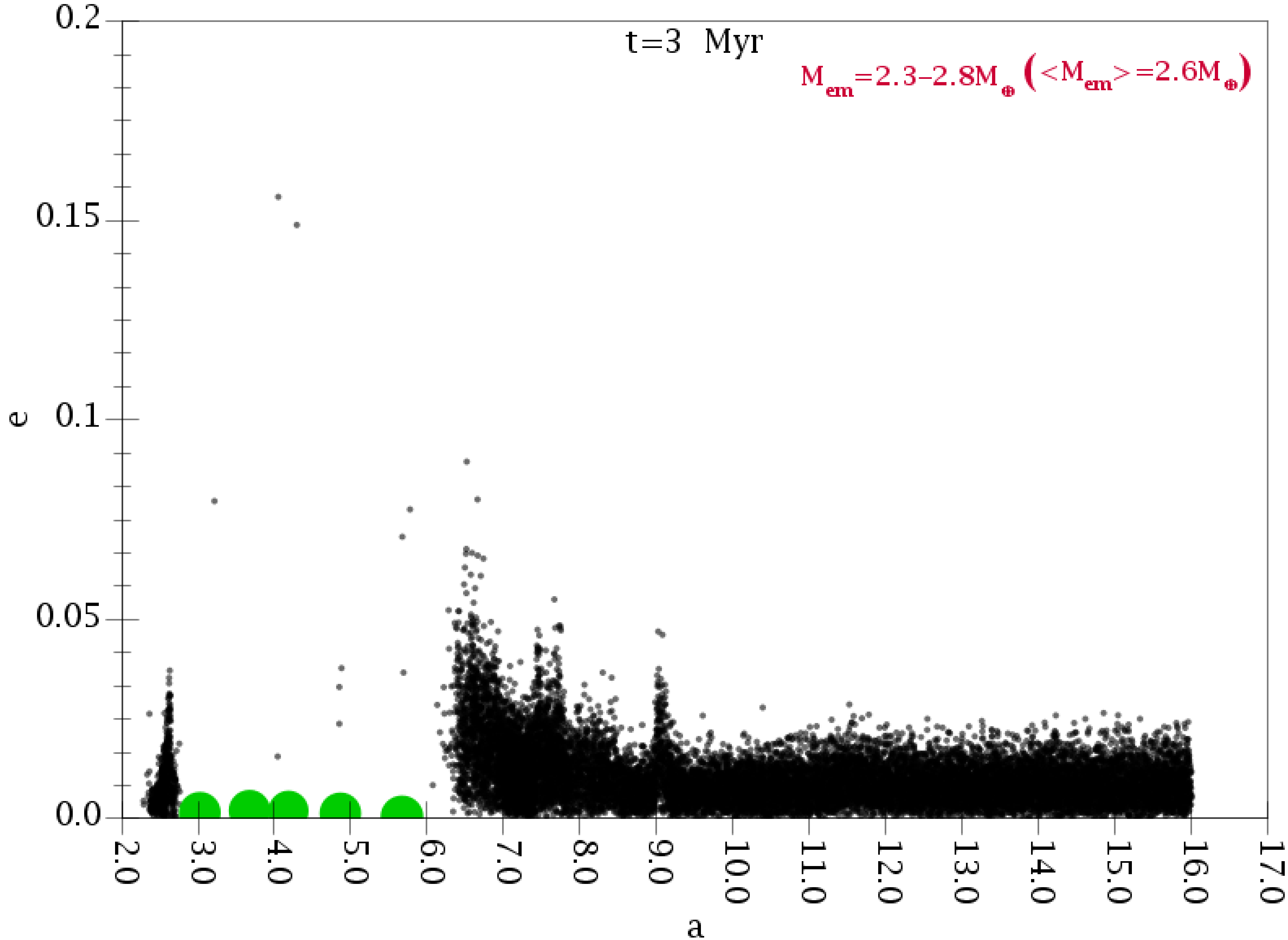
Ida and Makino, 1993

It has been proposed that damping effects on the planetesimals (due to gas drag or mutual collisions) may avoid scattering and sustain the accretion of the large body, maybe even in a runaway mode…..



COEURS

…but in this case the oligarchs open gaps and still refuse to accrete



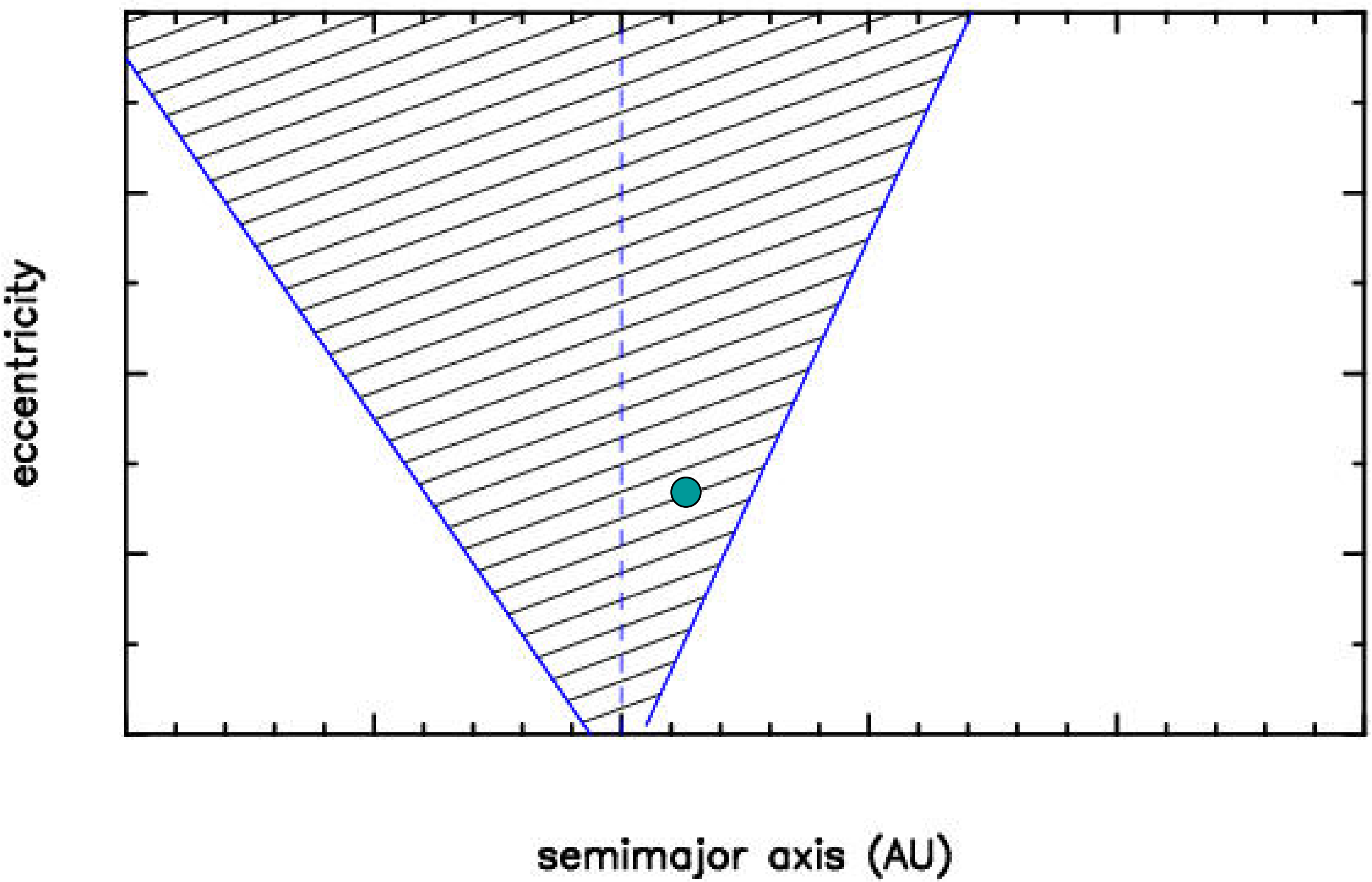
COEURS

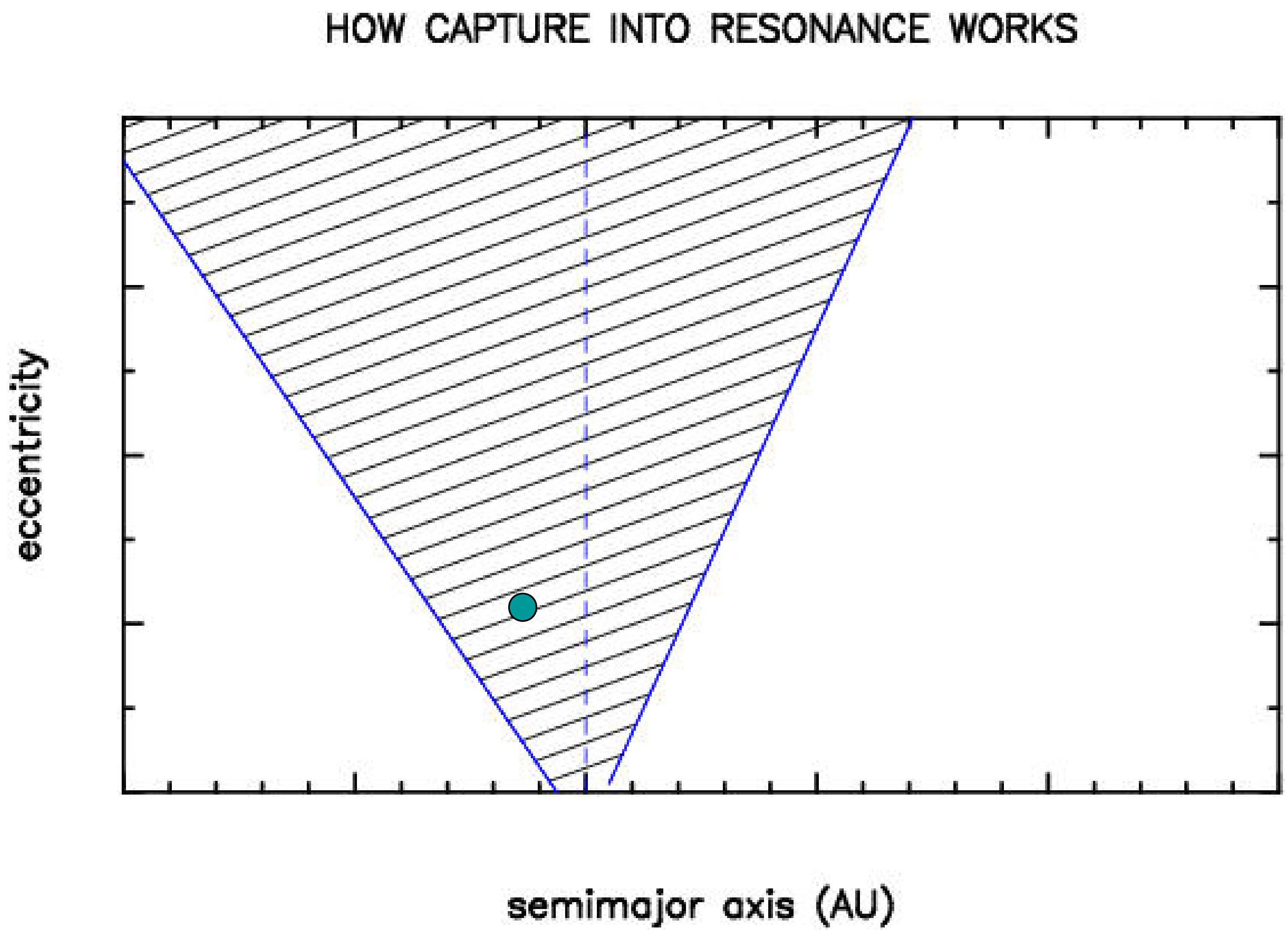
It has been proposed that migration (either of the planetesimals due to gas drag or by the oligarchs themselves (see later)) helps to avoid the gap, but this does not work either because of shepherding :

during convergent migration, capture in mean motion resonance is likely.

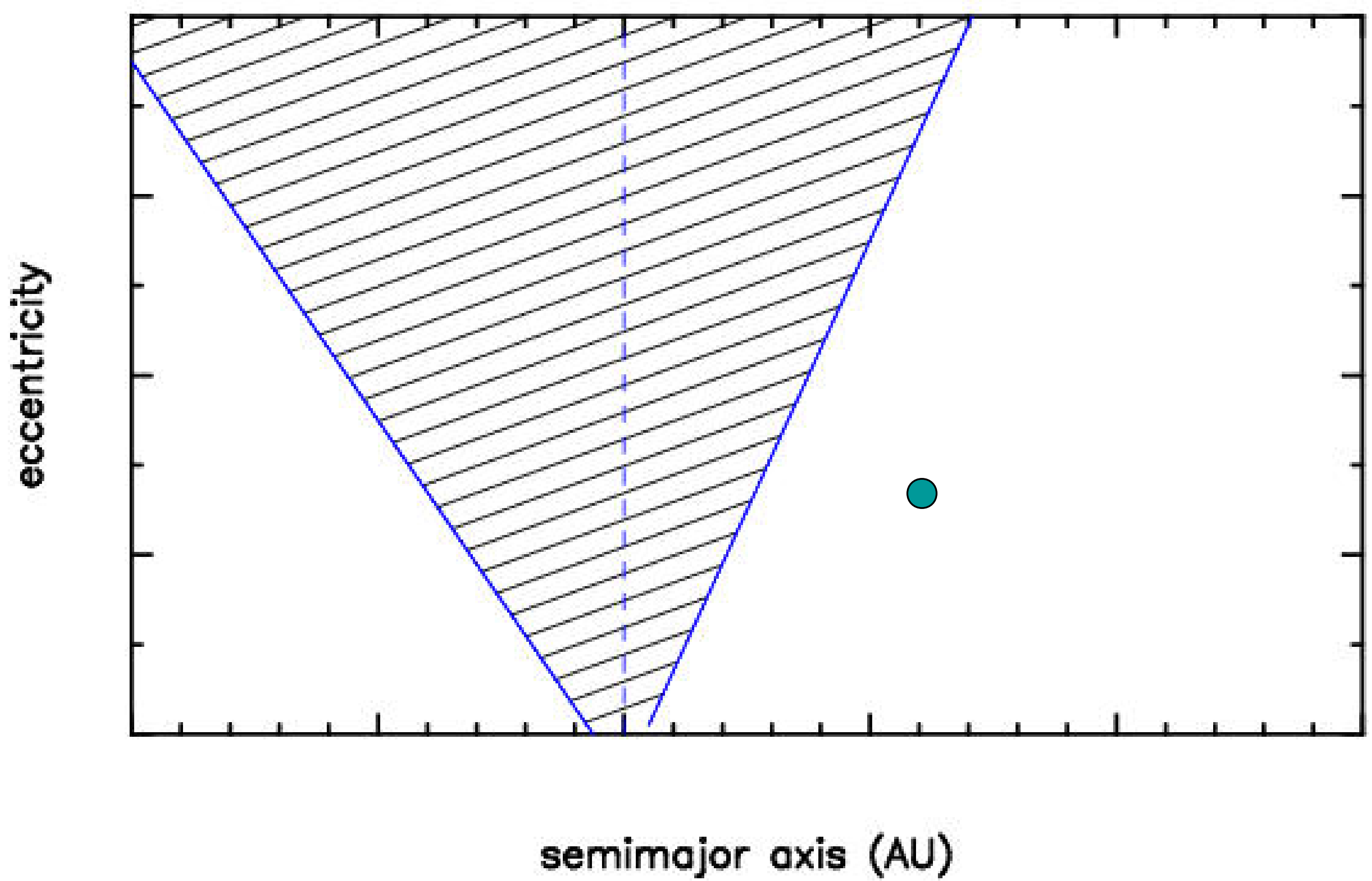
Dynamics of a particle moving relative to a mean motion resonance

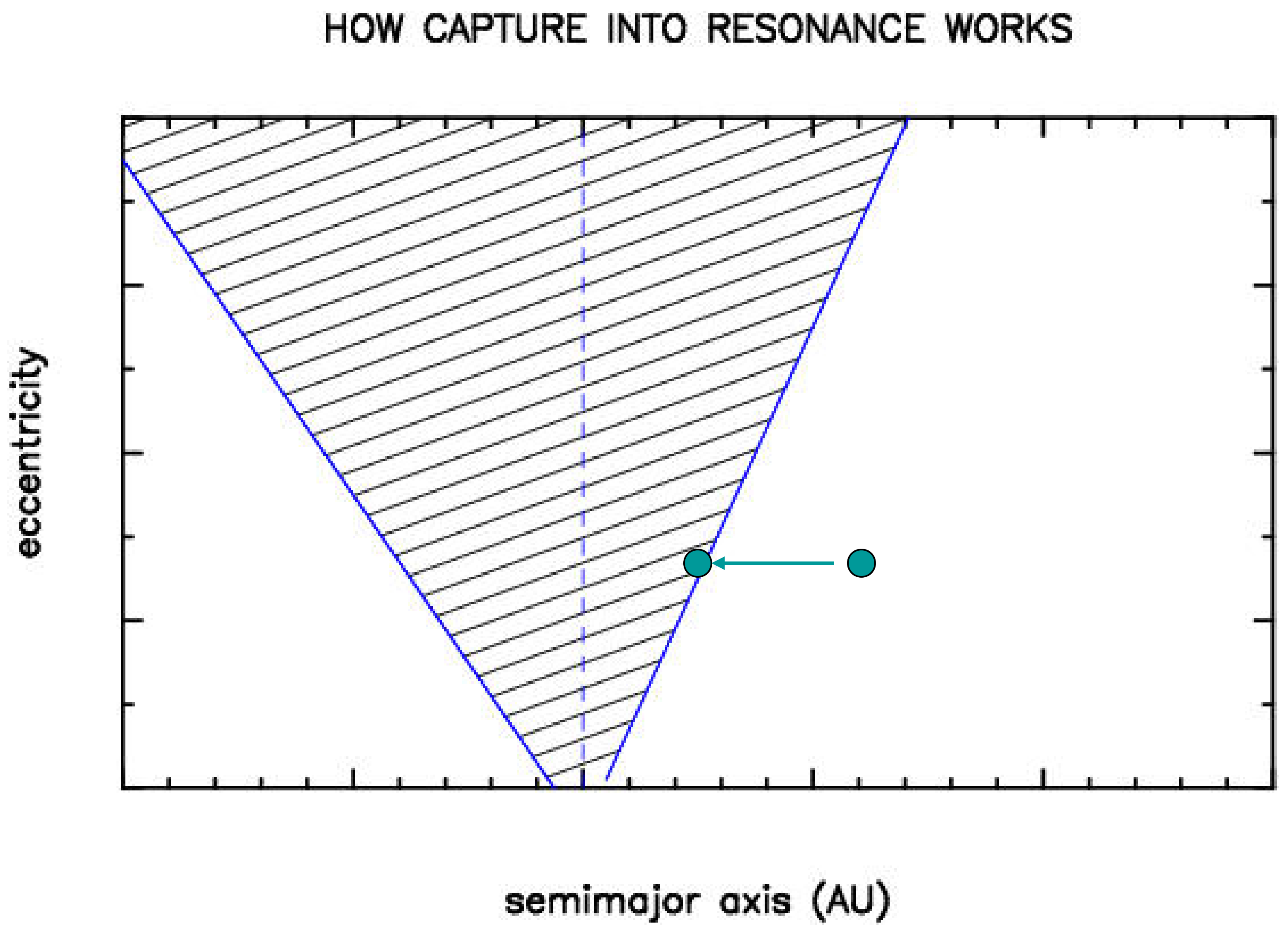
HOW CAPTURE INTO RESONANCE WORKS




HOW CAPTURE INTO RESONANCE WORKS
eccentricity
semimajor axis (AU)

HOW CAPTURE INTO RESONANCE WORKS

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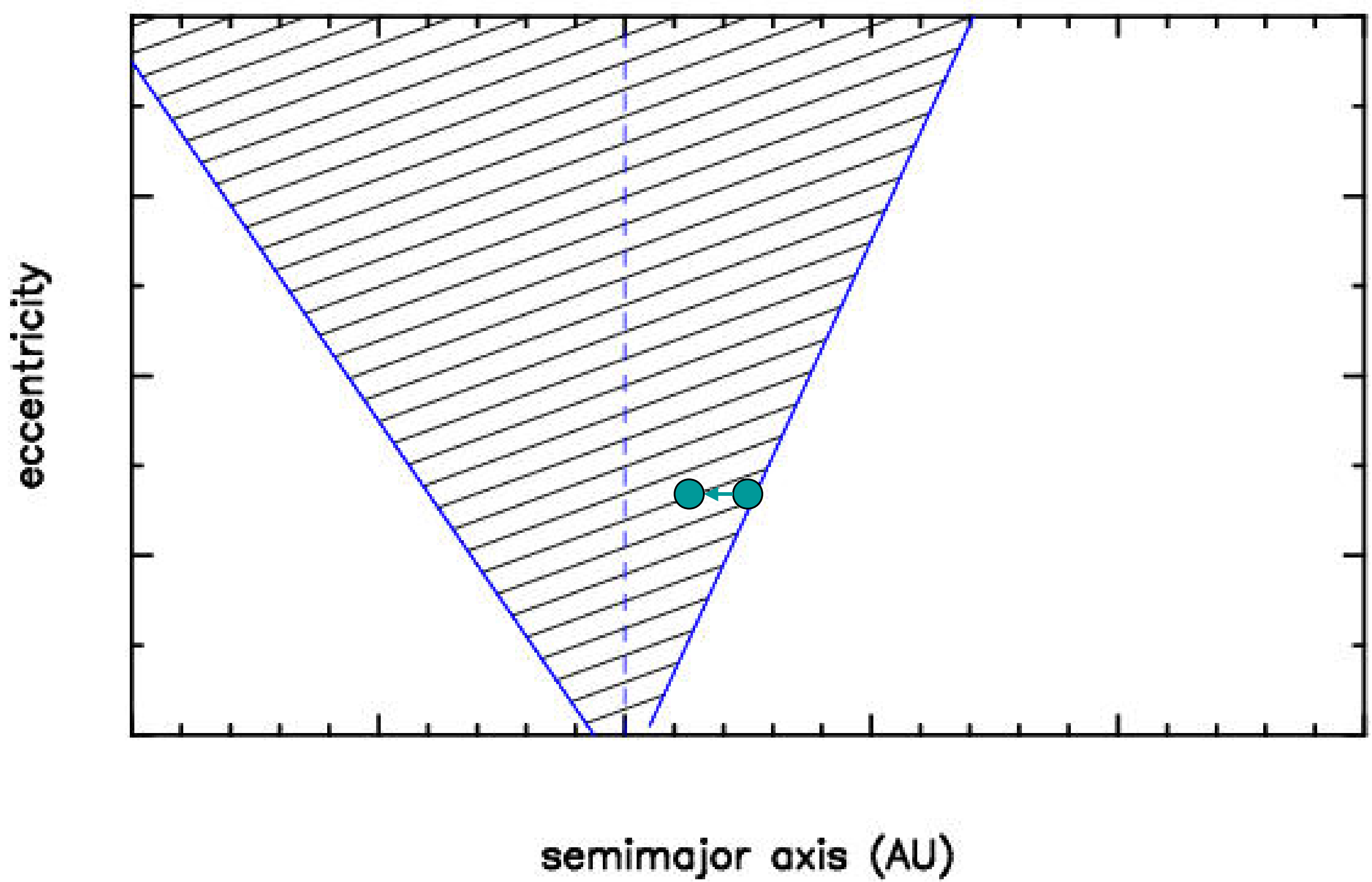

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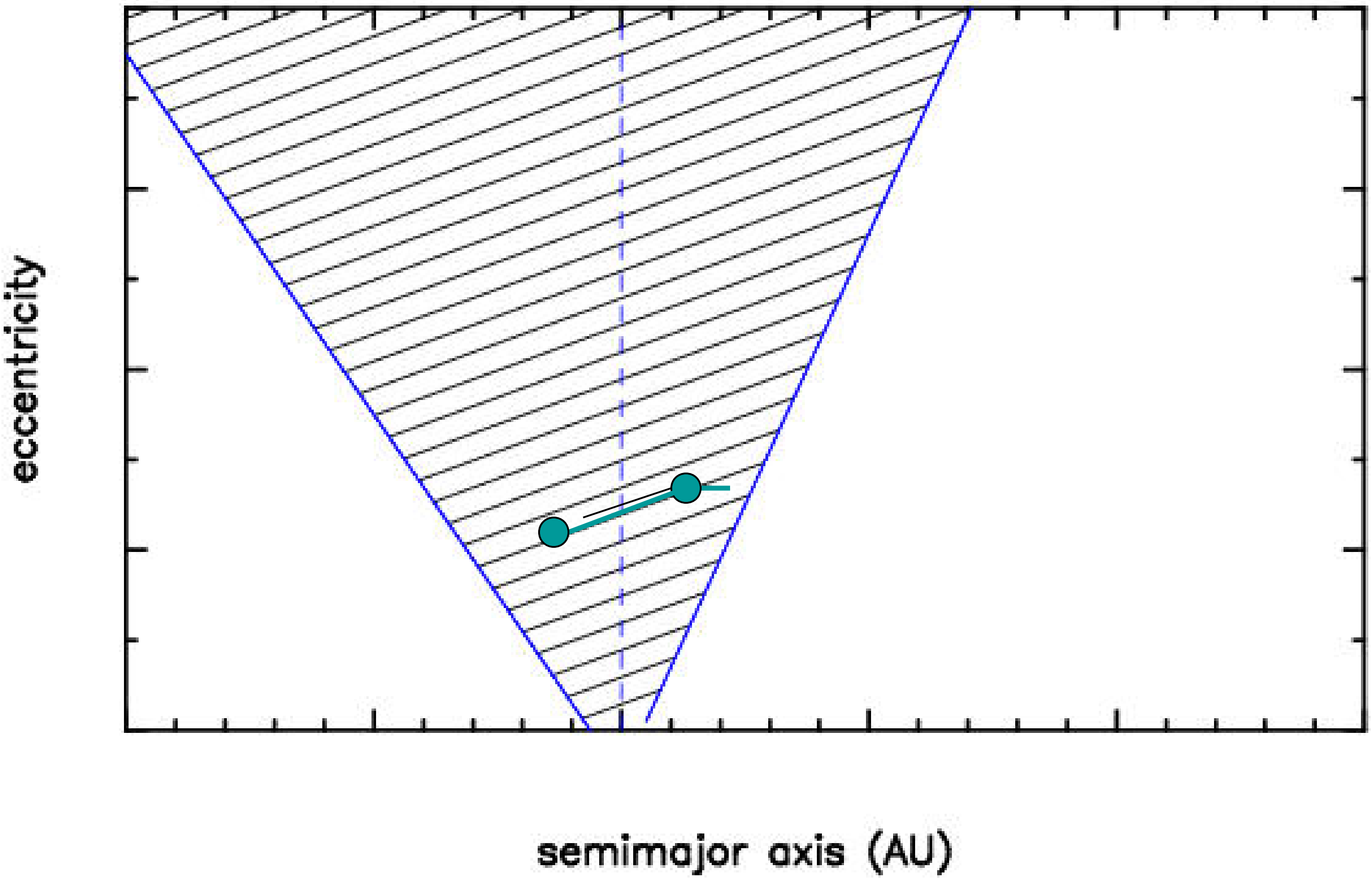

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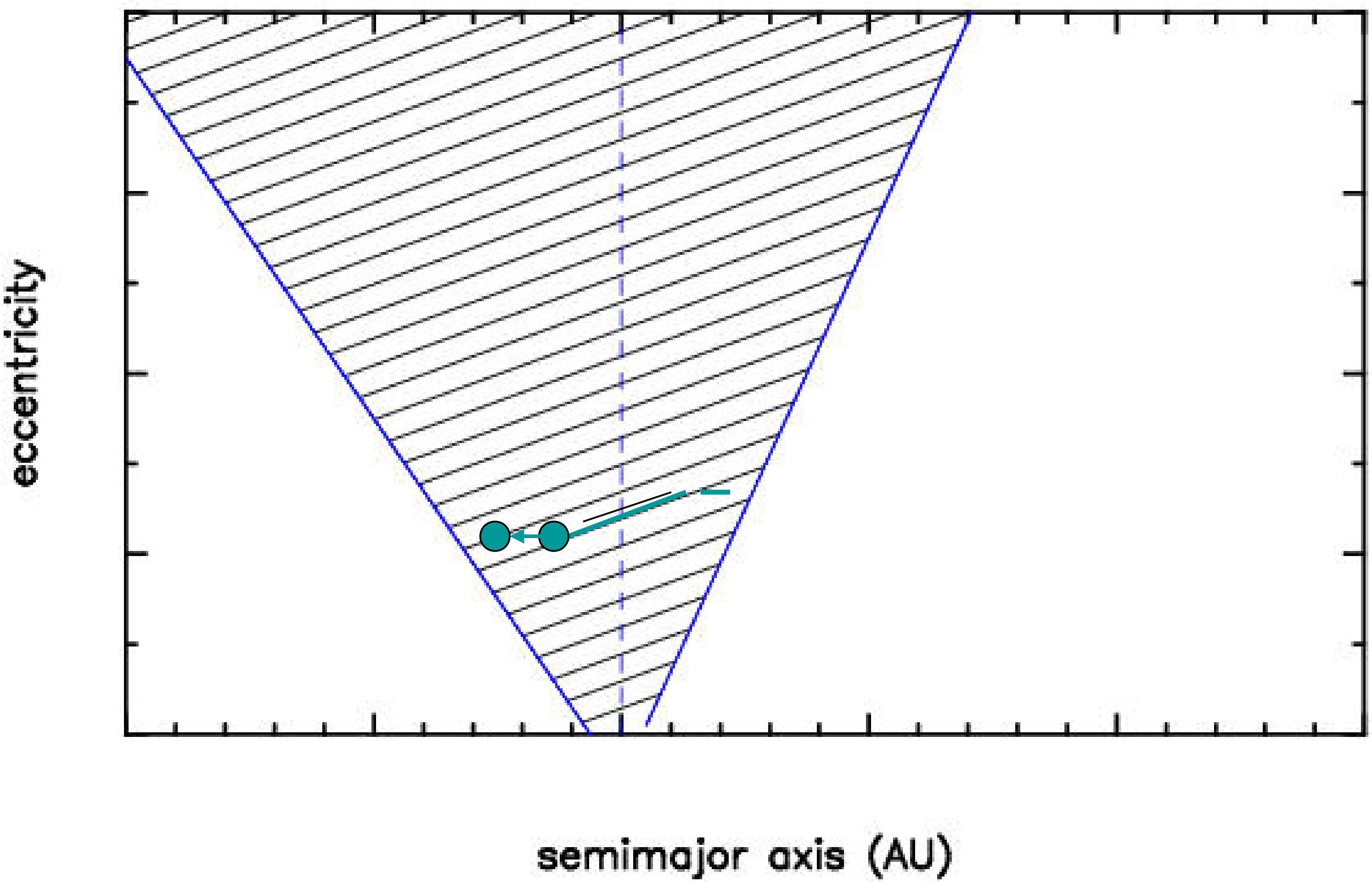

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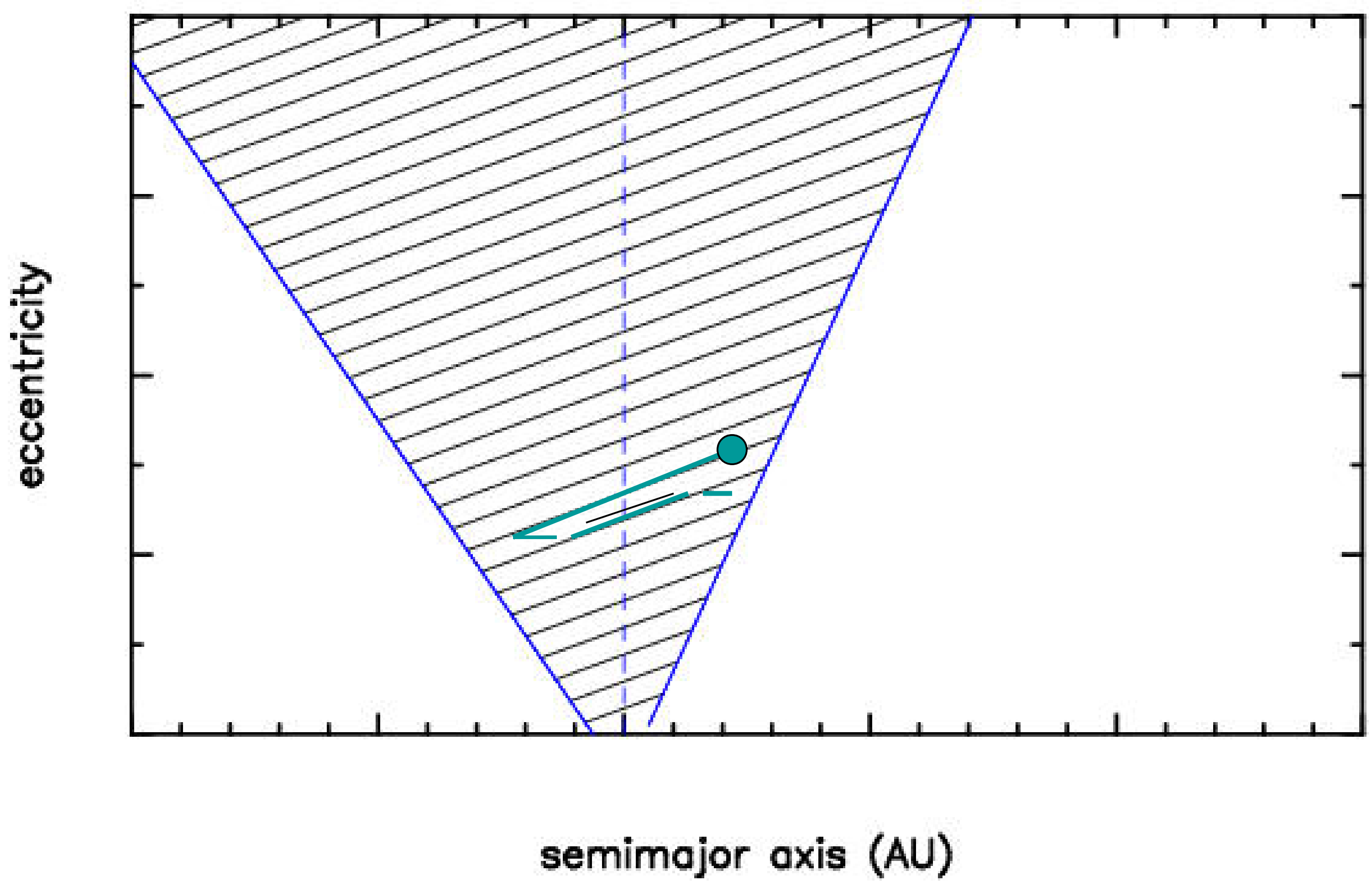

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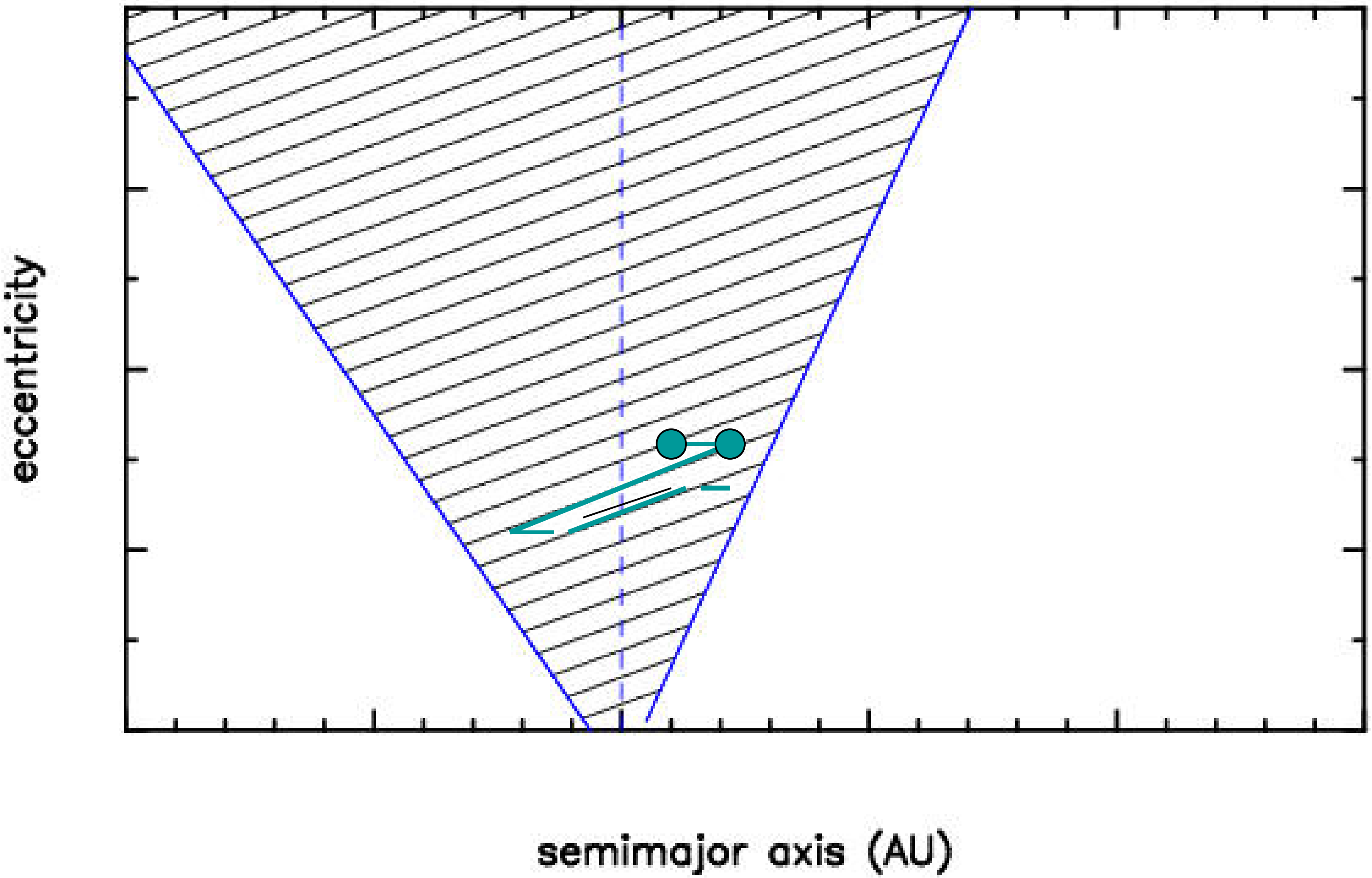
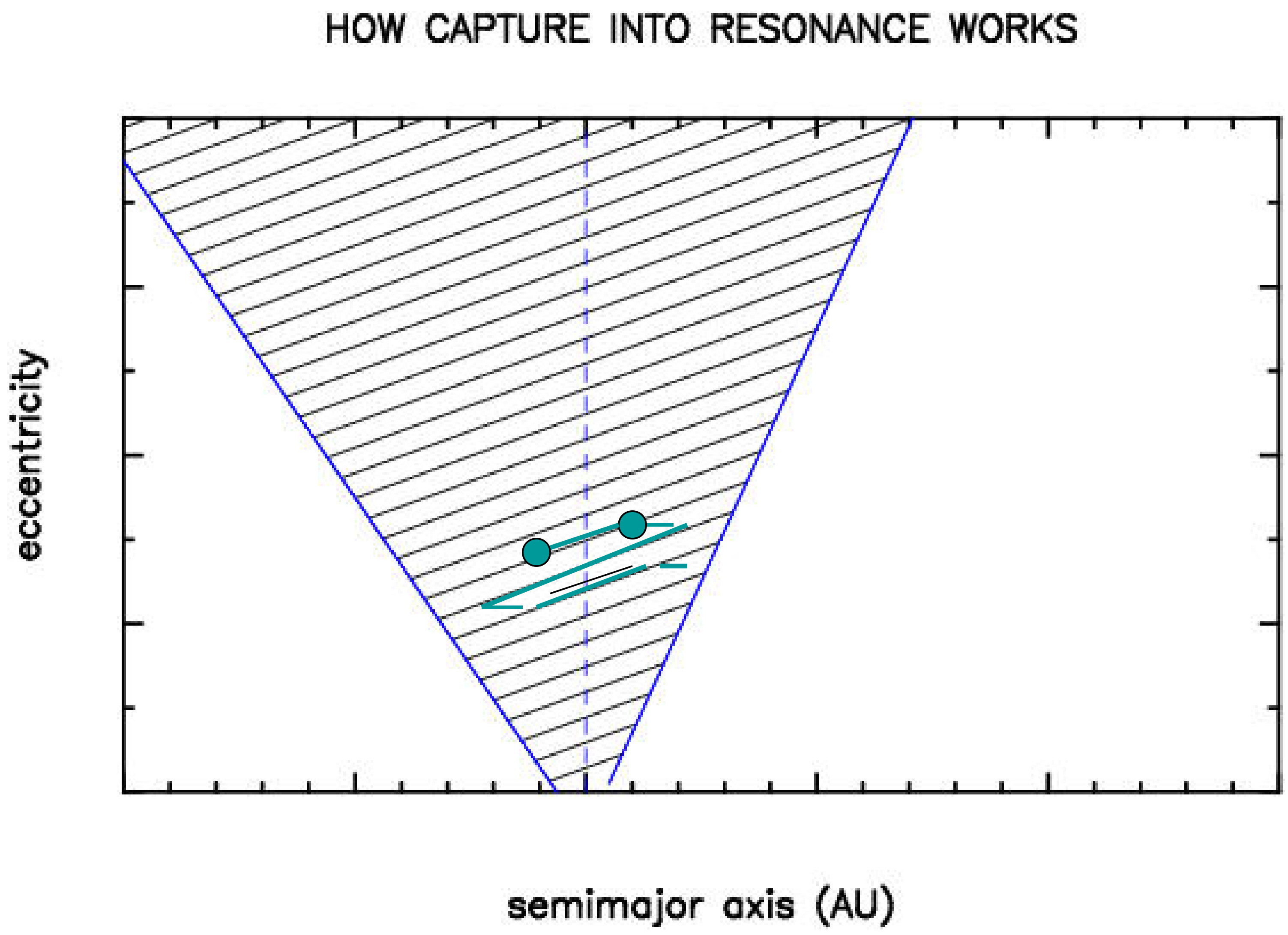
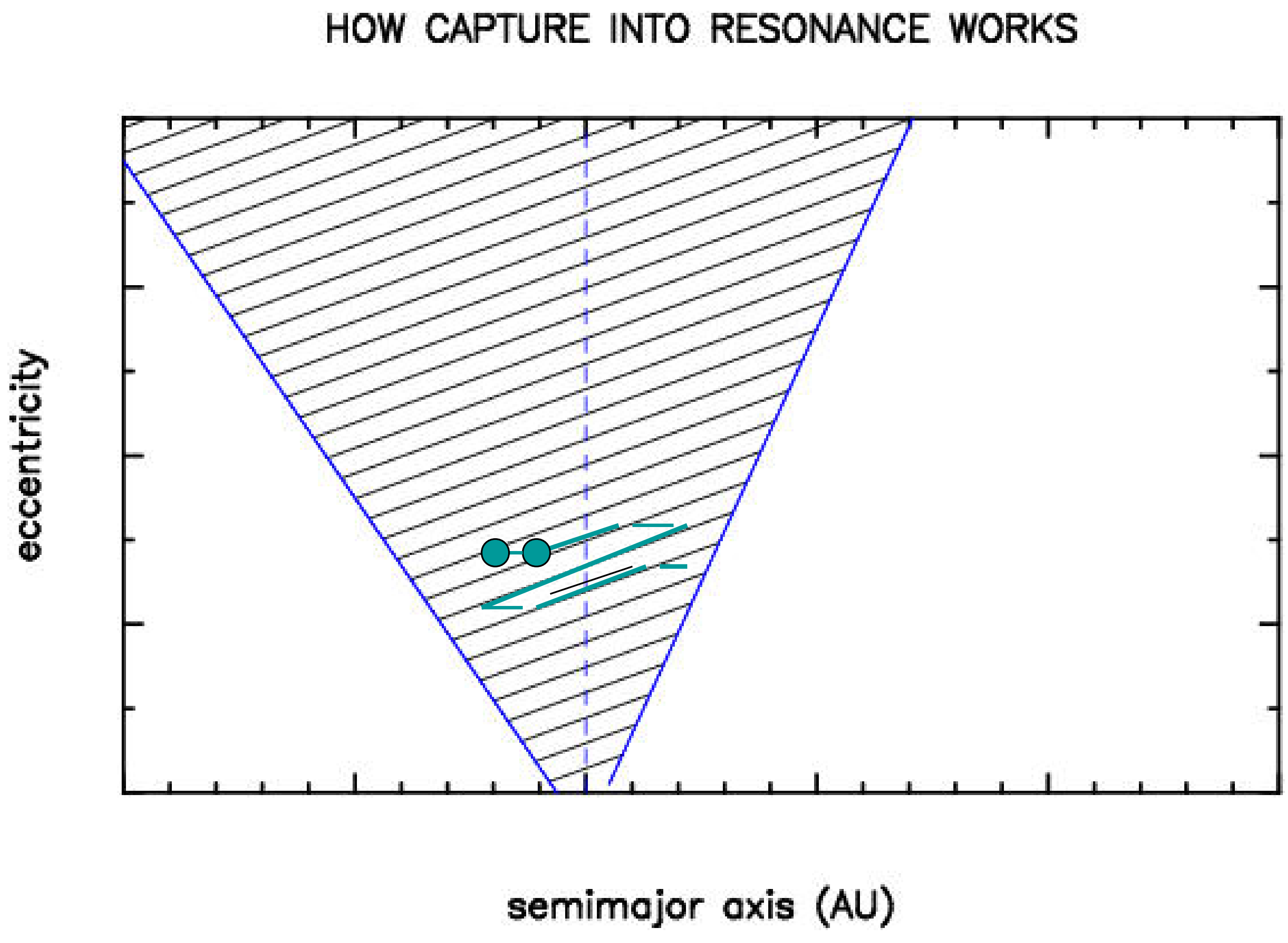



HOW CAPTURE INTO RESONANCE WORKS
eccentricity
semimajor axis (AU)


HOW CAPTURE INTO RESONANCE WORKS
eccentricity
semimajor axis (AU)

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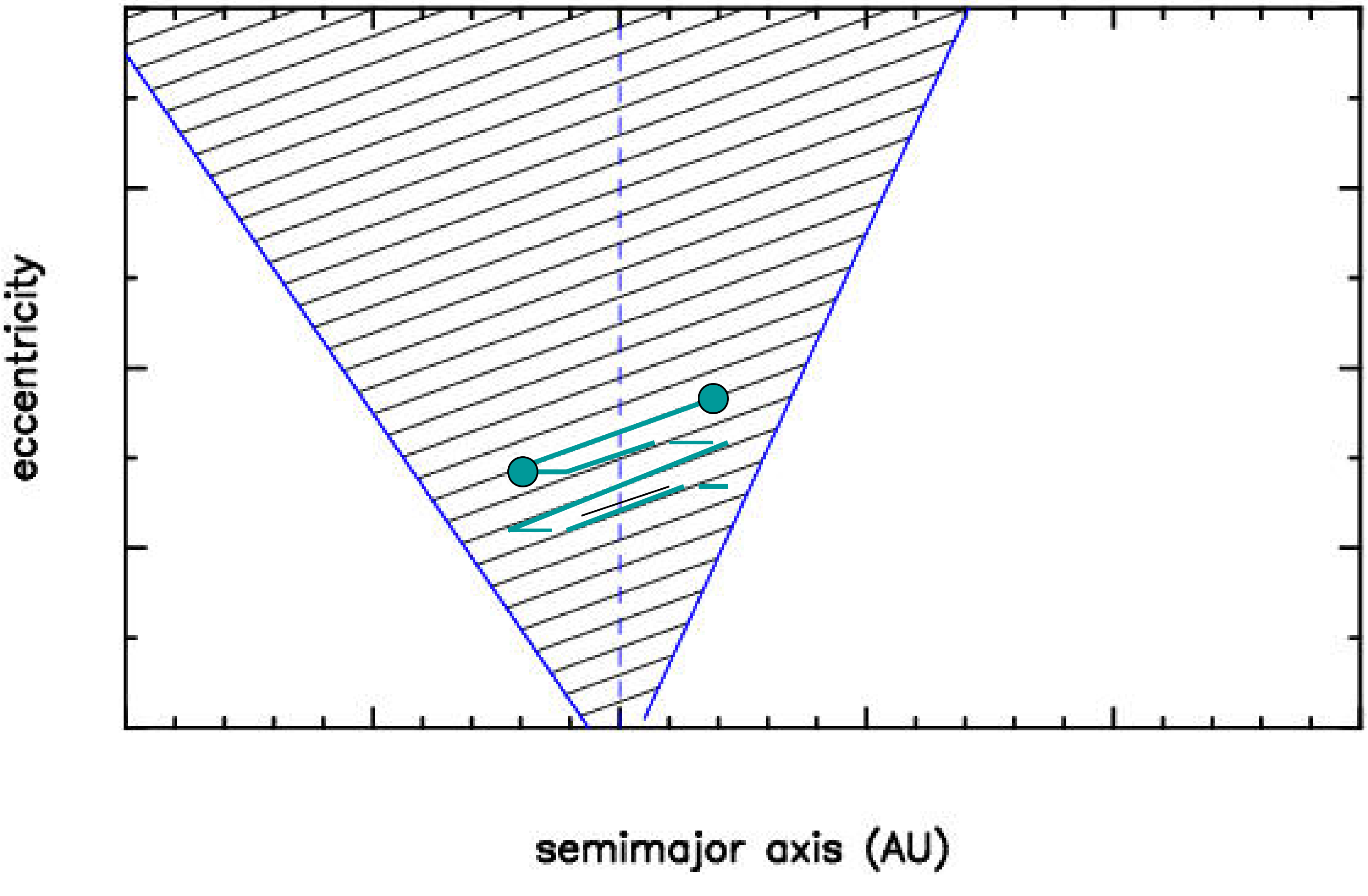

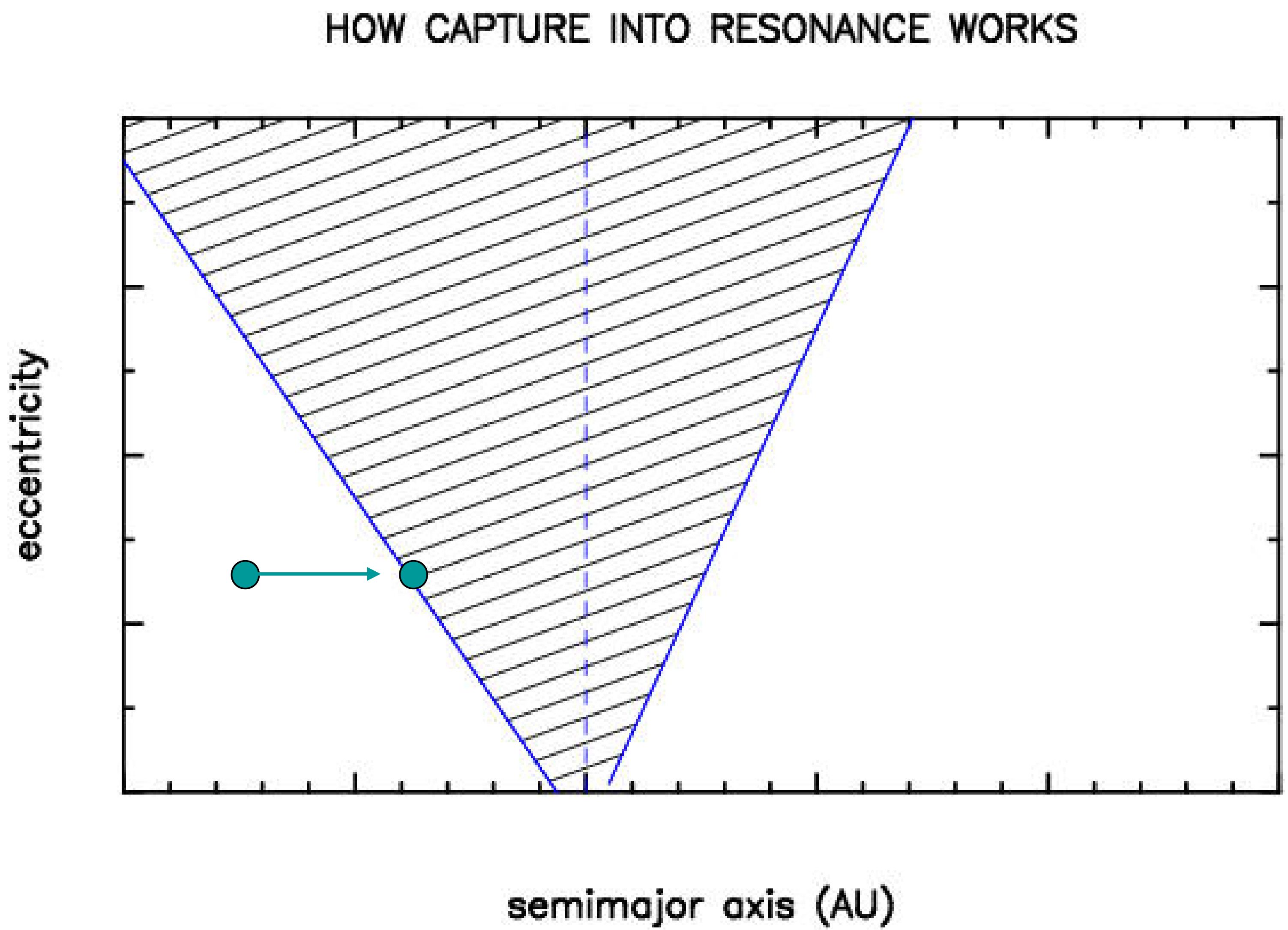
HOW CAPTURE INTO RESONANCE WORKS
eccentricity
semimajor axis (AU)

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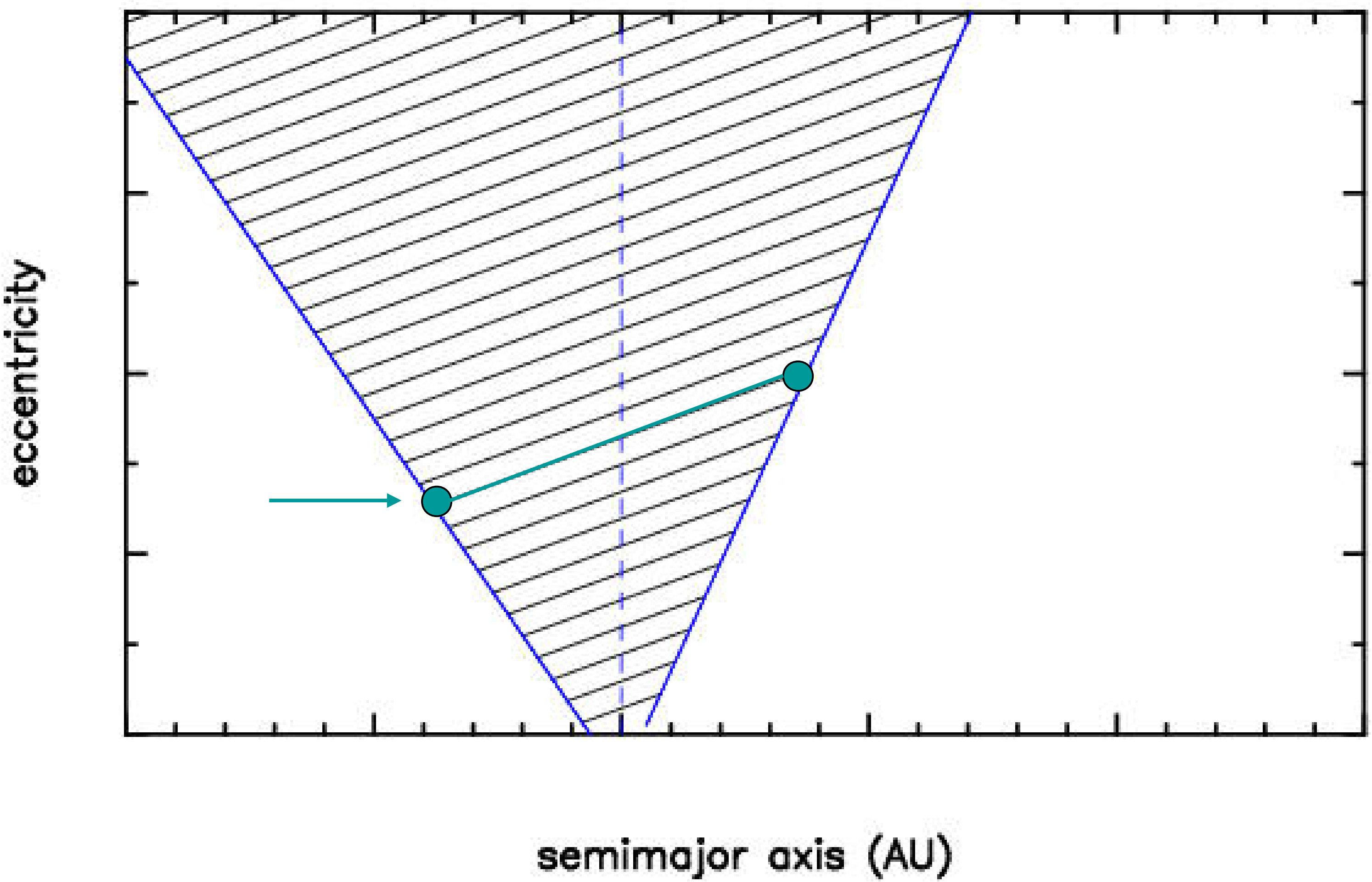
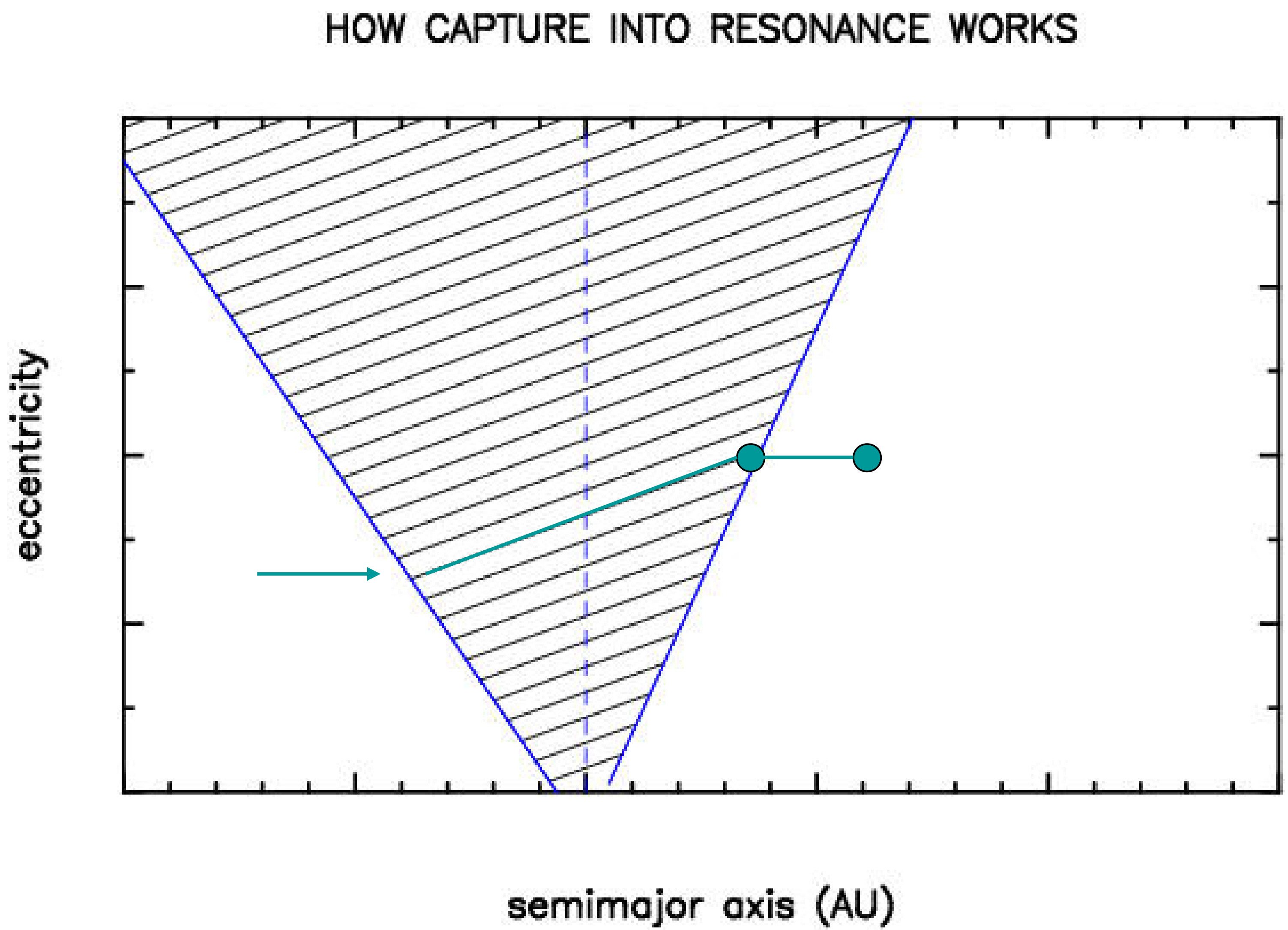
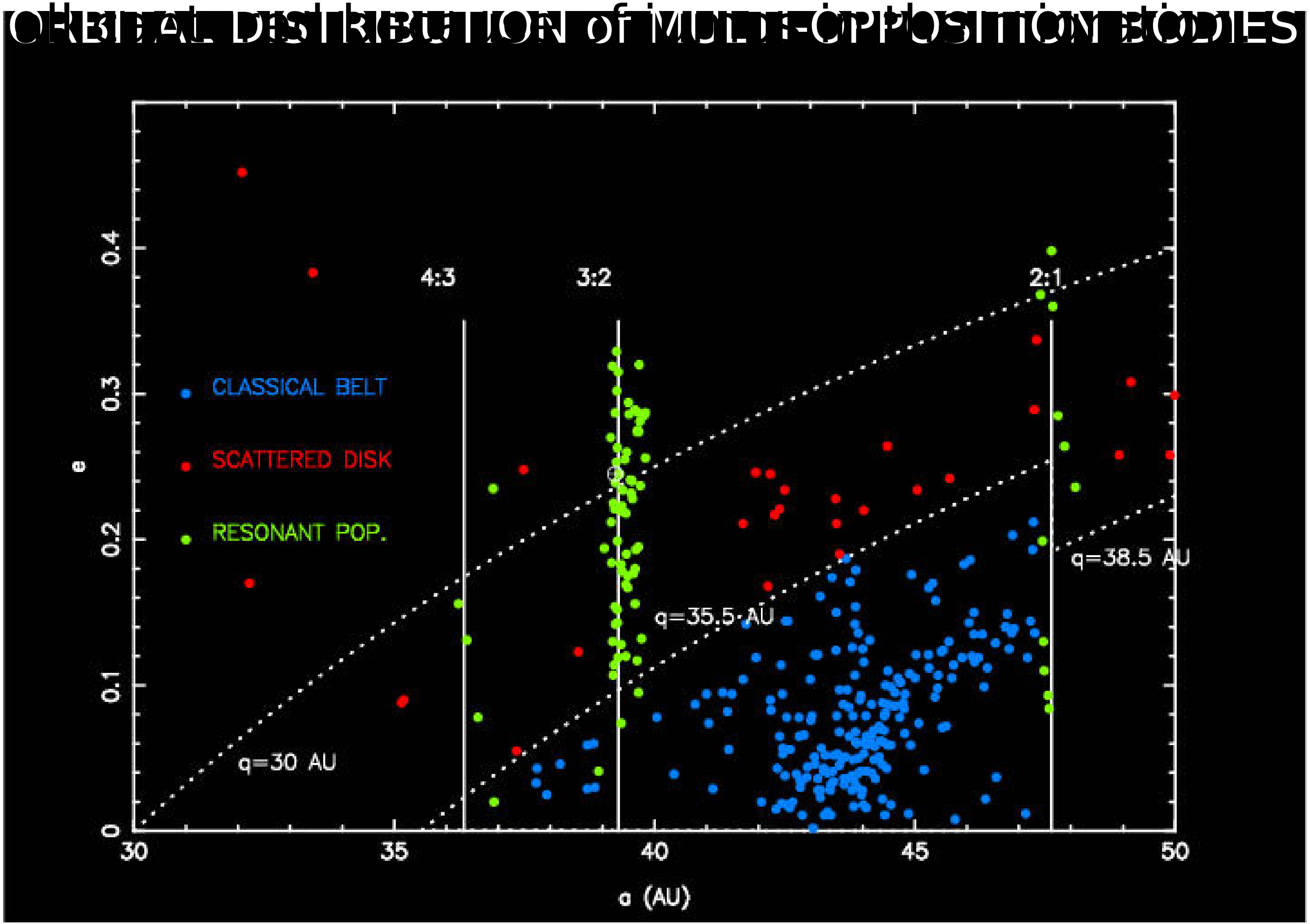



HOW CAPTURE INTO RESONANCE WORKS
eccentricity
semimajor axis (AU)

Ex: The Kuiper belt.

Resonance capture indicates an outwards migration of Neptune.

Not [illegible]



FORMATION of EMBRYOS

Conclusion :

Embryos (or cores) form in two phases from planétésimals.

Runaway growth : the most massive grow the fastest.

Oligarchic growth : the mass differences between oligarchs get smaller.

Final mass = isolation mass = $\sim 15\pi r^2\Sigma \sim r^{1/2}$.

If Σ increases suddenly (snowline), easier formation of massive cores, but probably not massive enough.

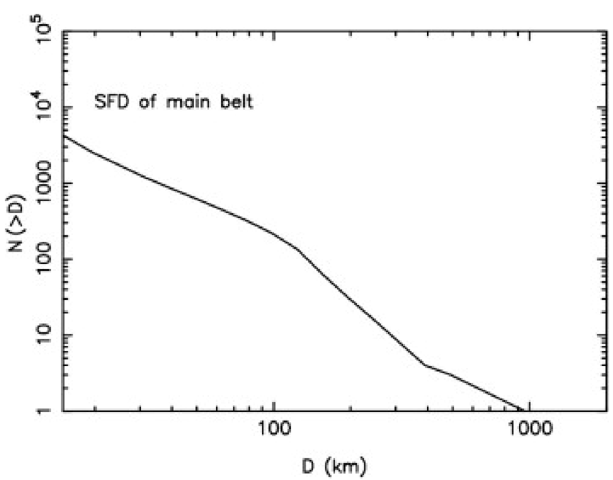
FORMATION of EMBRYOS

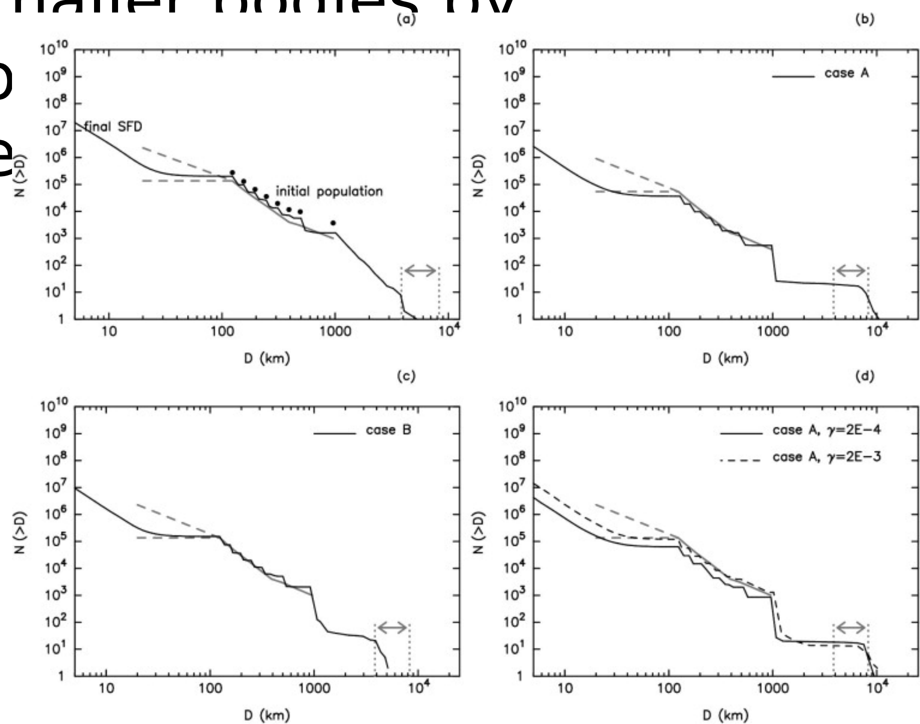
<u>Alternative solution :</u>

Direct formation through vortices, in a turbulent disk.

Real ?

Some people suggest that asteroids form big, and then are grained in smaller bodies by collisional evolution (Morb
But this is not yet accepte





SUMMARY

5 STEPS :

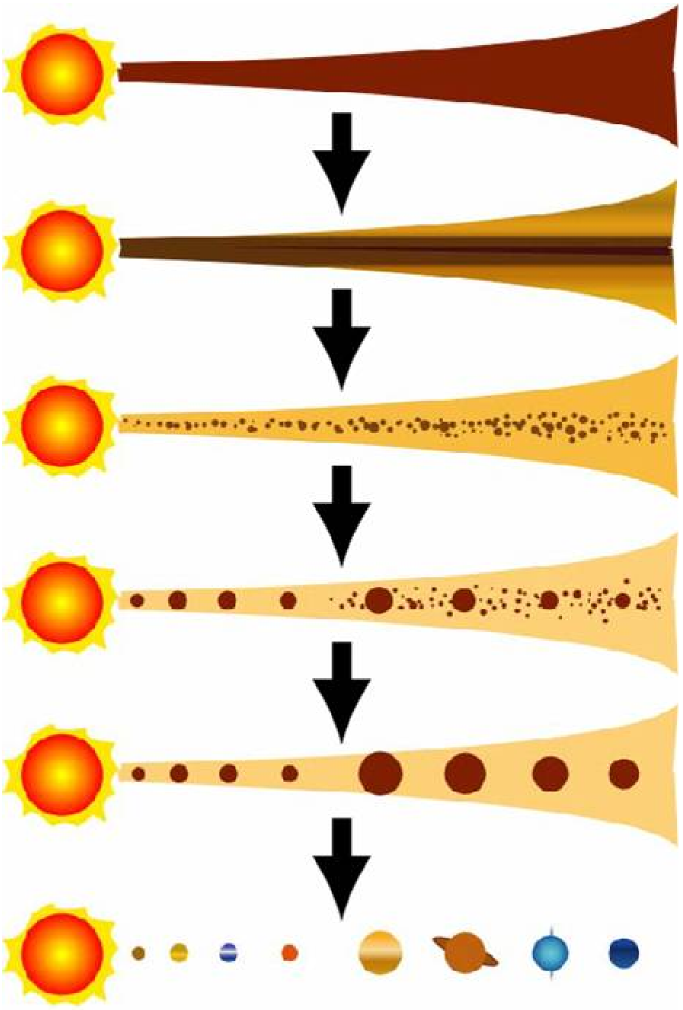
a/ Condensation.

b/ Sedimentation.

c/ Formation of planetesimals
(~1km, 10^{-9} $M_{\oplus}$).

d/ Formation of embryos
runaway growth, oligarchic growth
$M \sim 10^{-1} - 10^{-2}$ $M_{\oplus}$, $M \sim \Sigma^{3/2} r^3$.

e/ Formation of terrestrial planets or cores



PLANETARY FORMATION
4) PLANETS

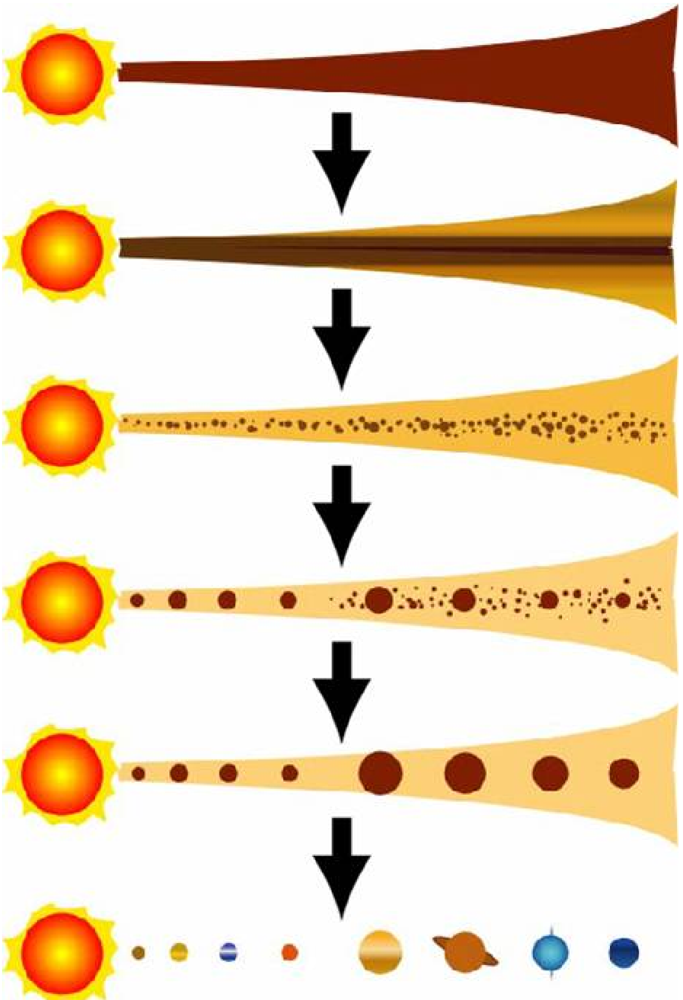
MASTER I.M.A.G 2.E
Départements Physique & Terre Environnement Espace • Université Nice Sophia-Antipolis •

Université Nice SOPHIA ANTIPOLIS

Aurélien CRIDA

SUMMARY

We have seen that, in a Proto-Planetary Disk ,
- dust settles,
- aggregates form, upto ~cm-size,
- then larger for a few lucky guys,
- then planetesimals grow,
- in runaway then oligarchic mode.



We now have a lot of oligarchs,
of masses 0.01 – 0.1 Earth mass,
densely packed.

This is unstable. N-body simulations show that chaos, orbit crossing, and collisions take place, until only a few bodies are left.

It takes about 10^8 years at 1AU.

STABILITY CRITERION

A system of many planets around a star (or satellites around a planet) is stable if and only if

the distance between the orbits is larger than 5 mutual Hill radii,

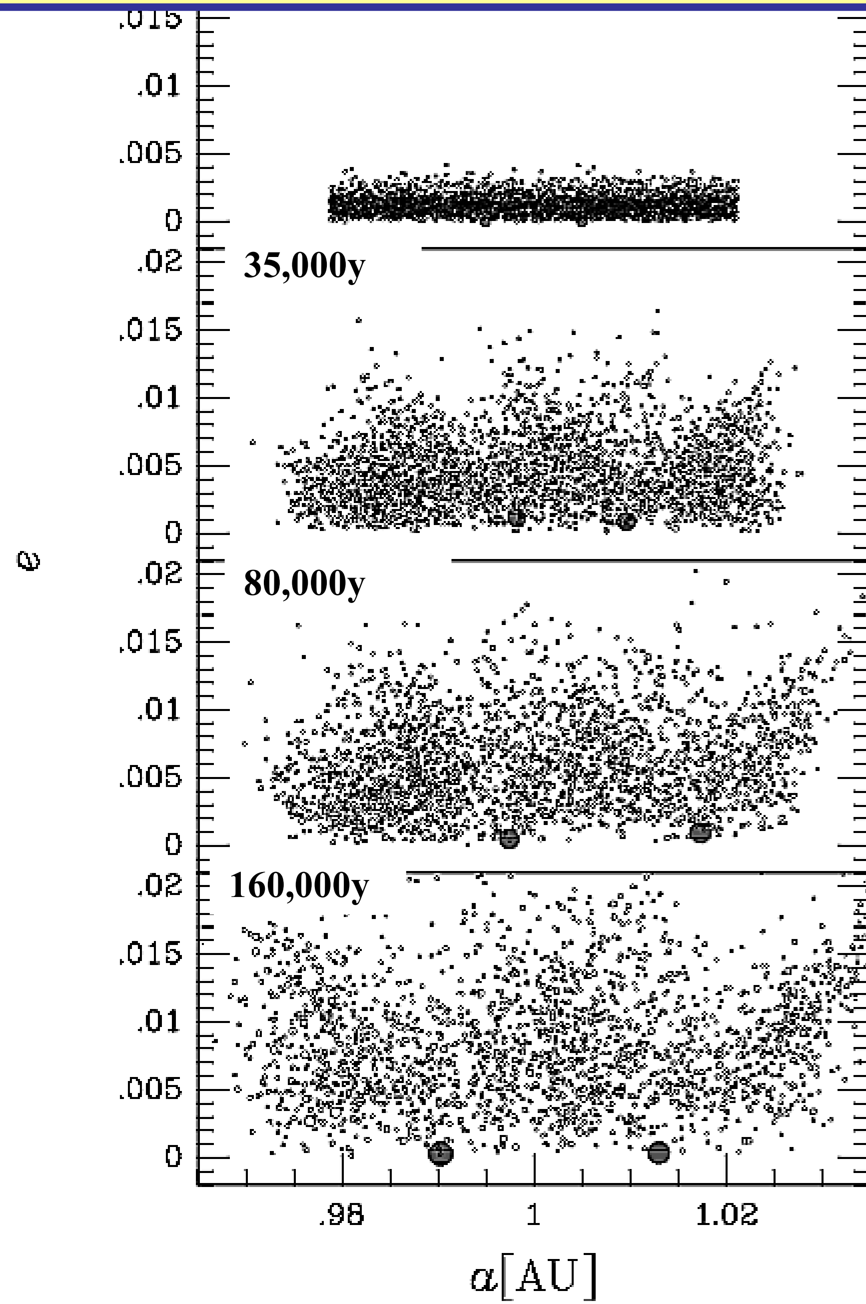
where $r_{H\,i,j} = [(M_i+M_j)/(3M_*)]^{1/3}\ (a_i+a_j)\ /\ 2$

Application: Is the Solar System stable now ?

GIANT IMPACTS

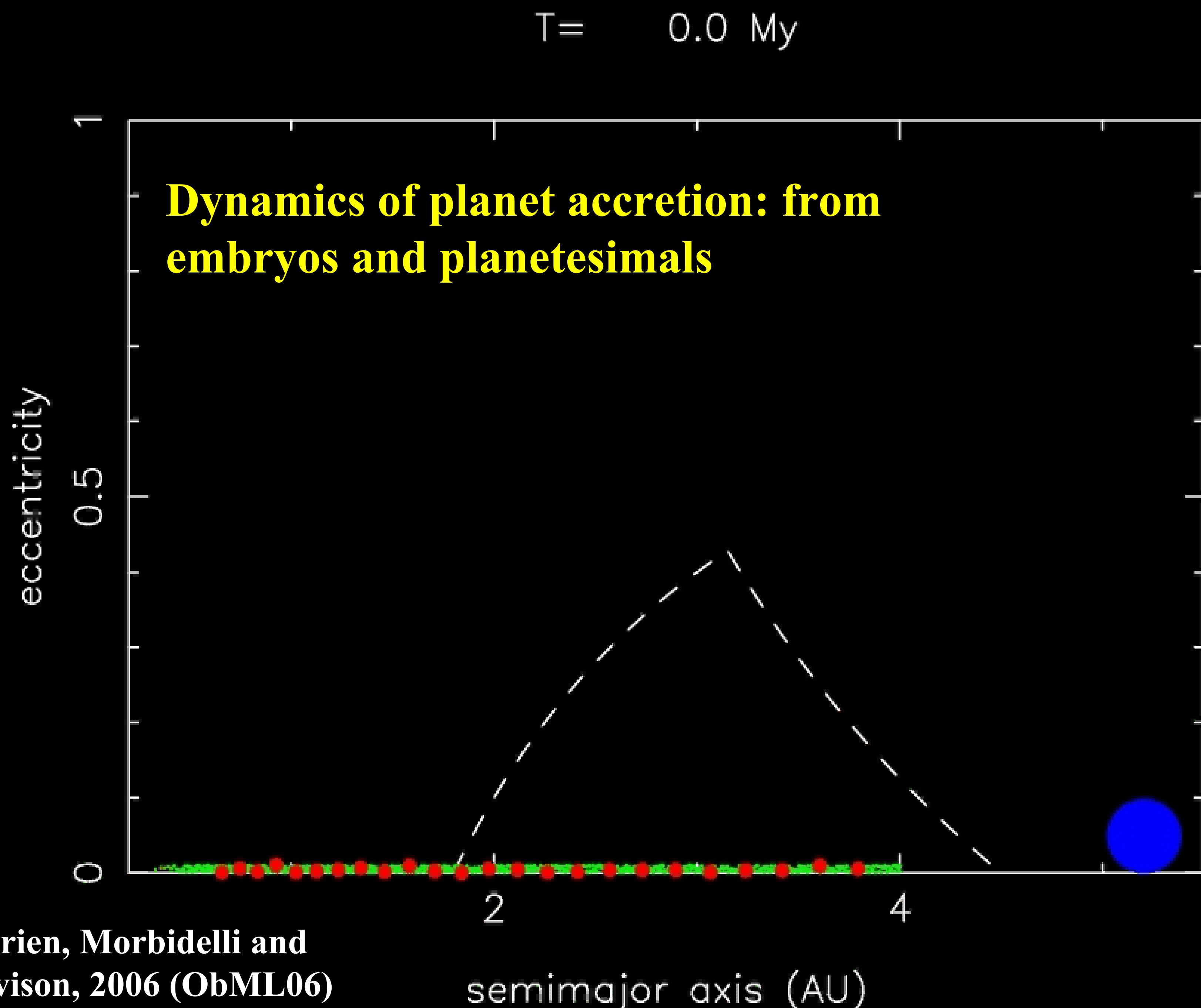
In 10^5-10^6 years embryos are formed, separated by only a few mutual Hill radii, surrounded by planetesimals.

These embryos are NOT planets. Their size is typically that of the Moon.



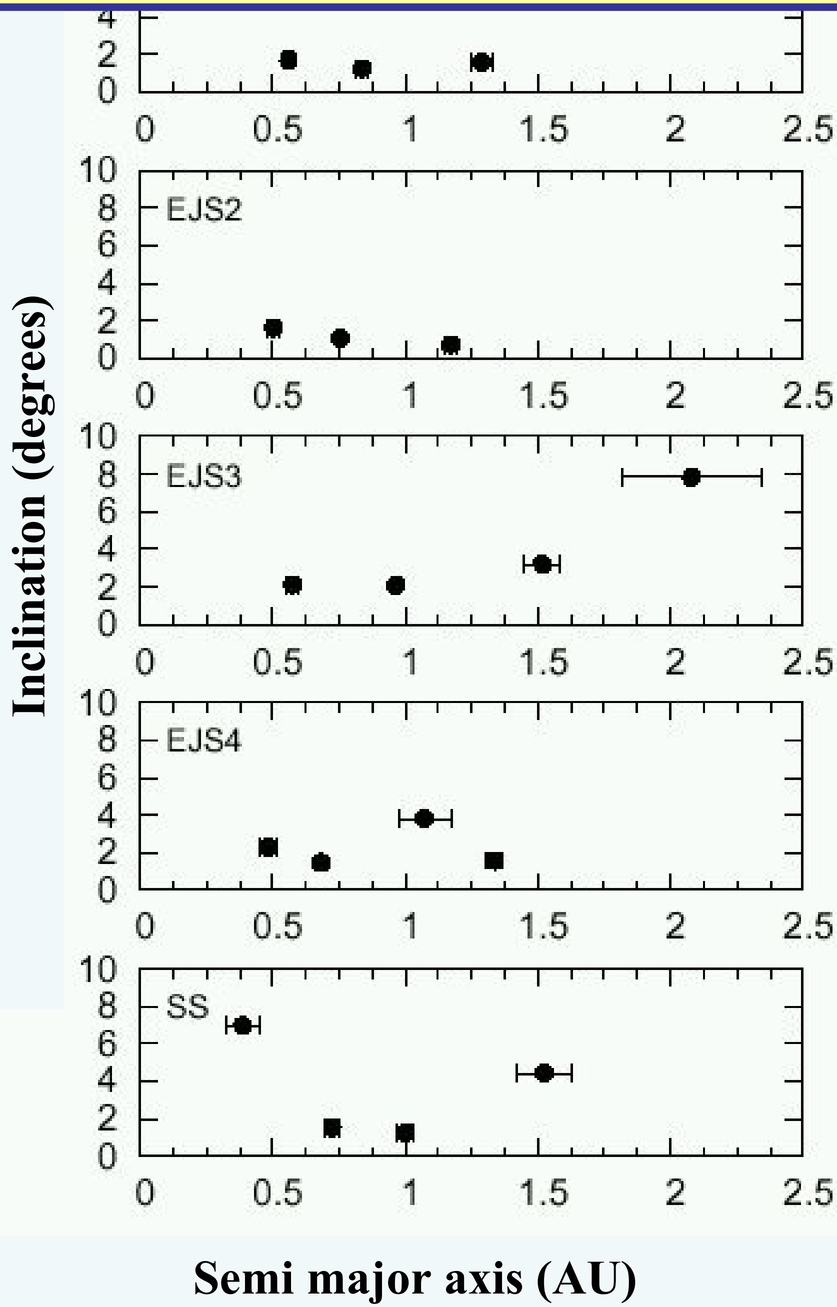
Ida and Makino (1993), Kokubo and Ida (1995, 1996, 1998), Thommes et al. (2003), Chambers (2006) .....

Dynamics of planet accretion: from embryos and planetesimals



Obrien, Morbidelli and Levison, 2006 (ObML06)

FINAL ORBITS



Angular Momentum Deficit (AMD)

$$= \frac{\sum_j m_j \sqrt{a_j(1-e_j^2)}\cos i_j - \sum_j m_j \sqrt{a_j}}{\sum_j m_j \sqrt{a_j}}$$

$\mathbf{AMD_{ss} = -0.0018}$

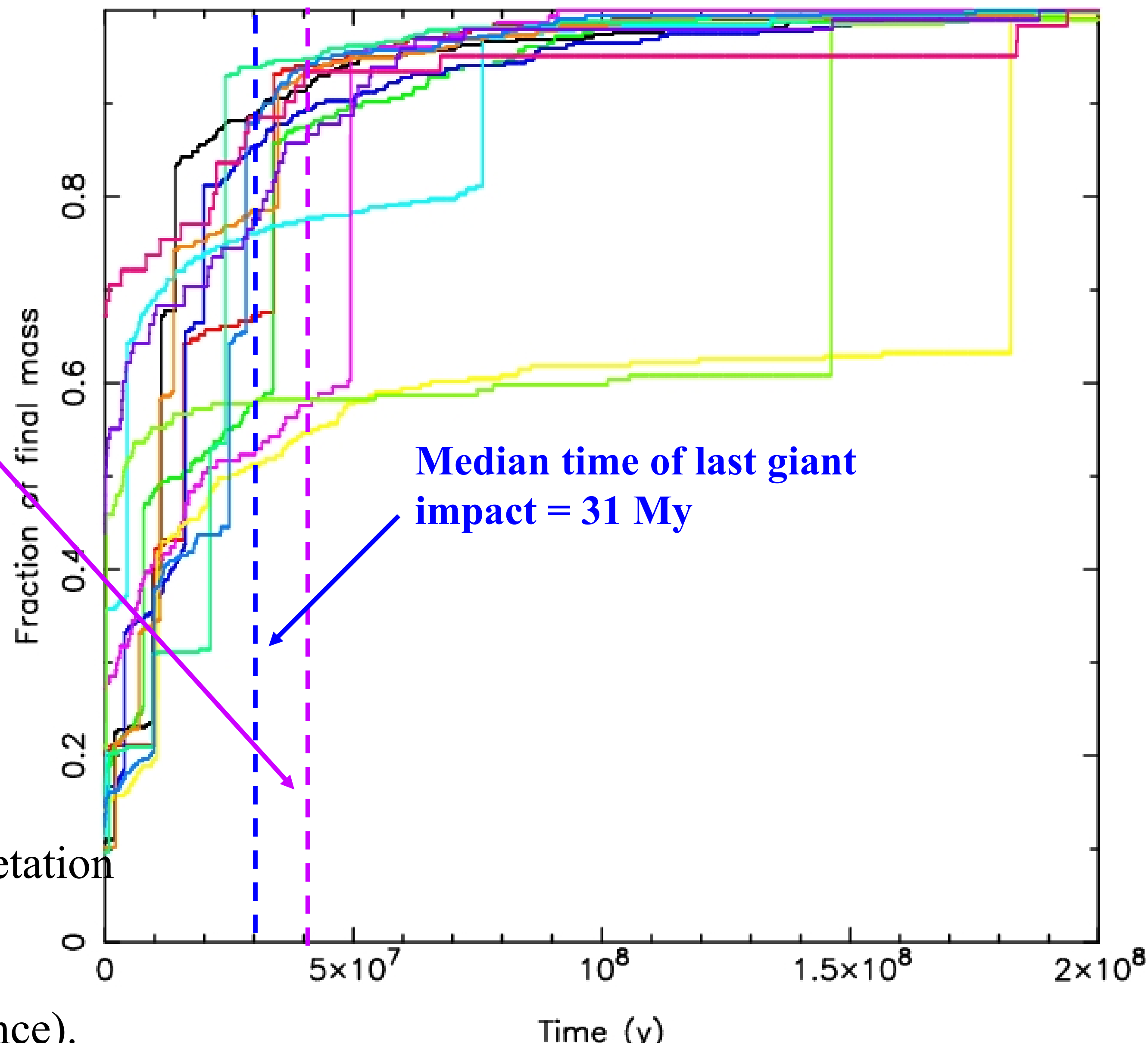
$\mathbf{AMD_{simul} = -0.0010}$

$\mathbf{AMD_{chambers} = -0.0070}$

Difference with previous work: the consideration of a large number of individually small planetesimals (*Dynamical Friction !*)

FORMATION TIME-SCALE

Median time for acquiring 90% of final mass = 40My



Median time of last giant impact = 31 My

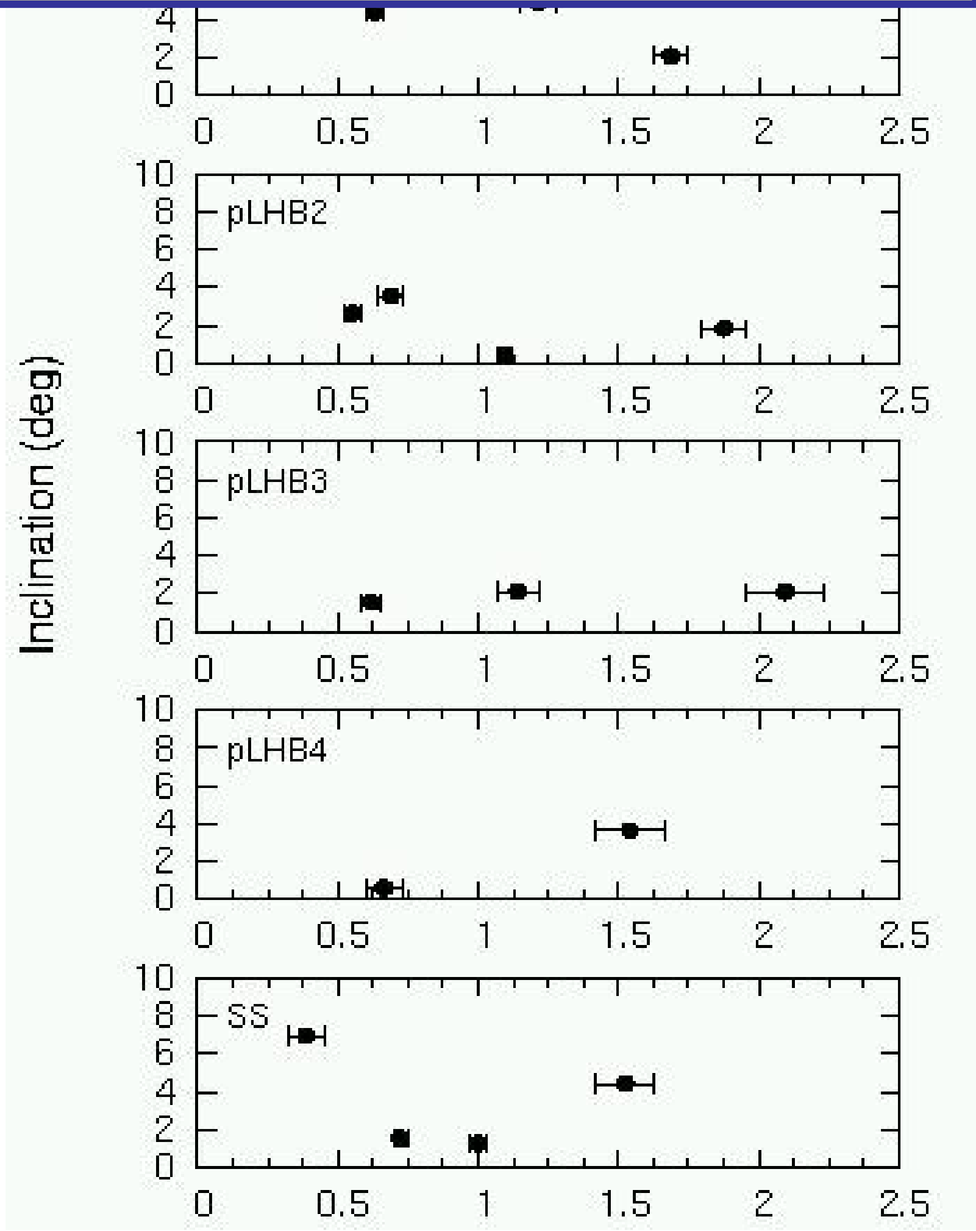
Very good agreement with the pre-2007 interpetation of $^{182}Hf/^{182}W$ chronology (Kleine et al. 2005, Science).

FINAL ORBITS

The previous results have been obtained assuming that Jupiter and Saturn had initially their current (eccentric) orbits.

If we assume that the orbits of Jupiter and Saturn were quasi-circular, as expected from models of their formation/evolution, the results of terrestrial planets formation simulations are less good in terms of AMD :

AMDss = - 0.0018
AMDsimul = - 0.0030

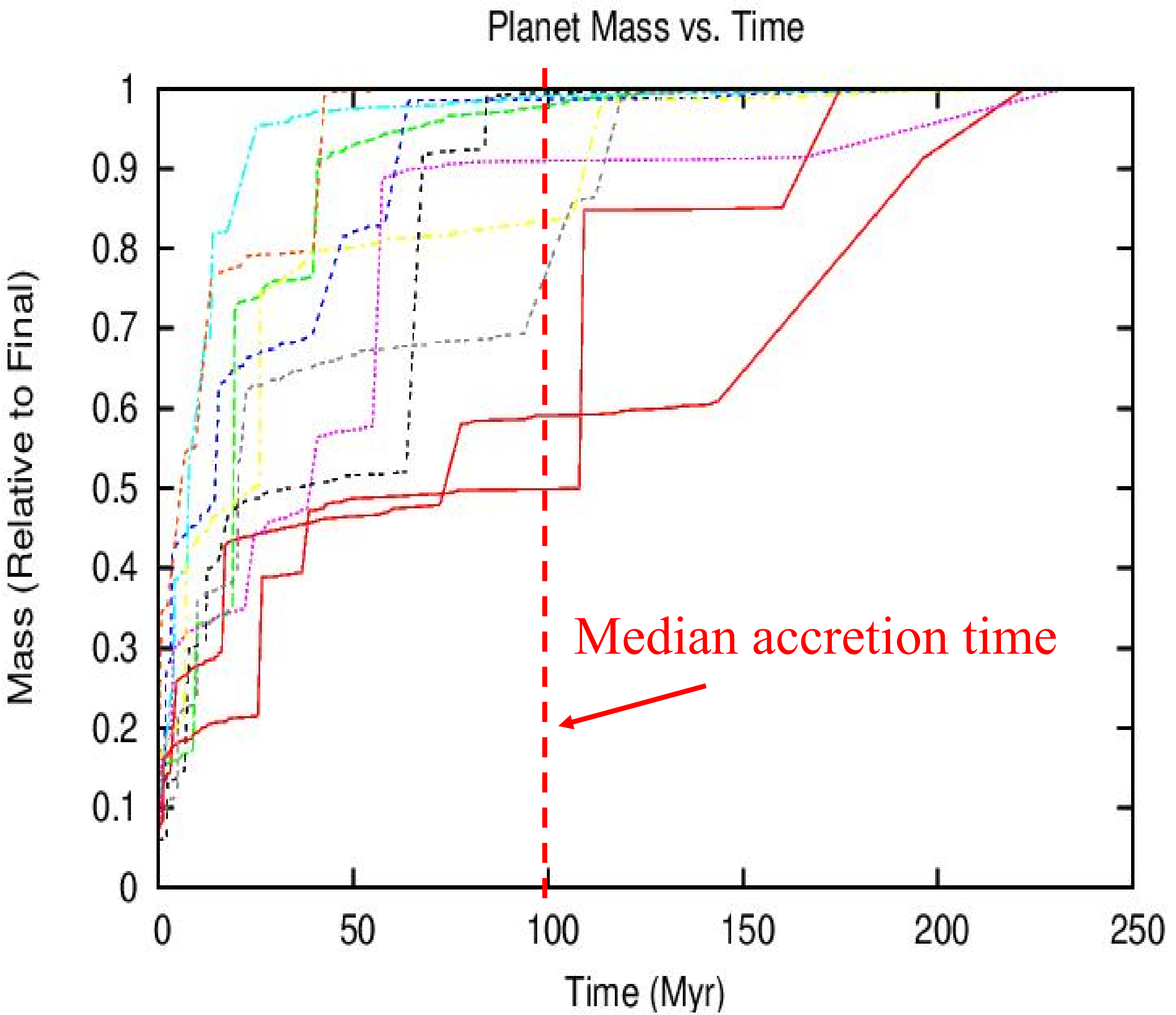


Semi-major axis (AU)

FORMATION TIME-SCALE

But the terrestrial planets accretion timescale also becomes longer (~100 My),

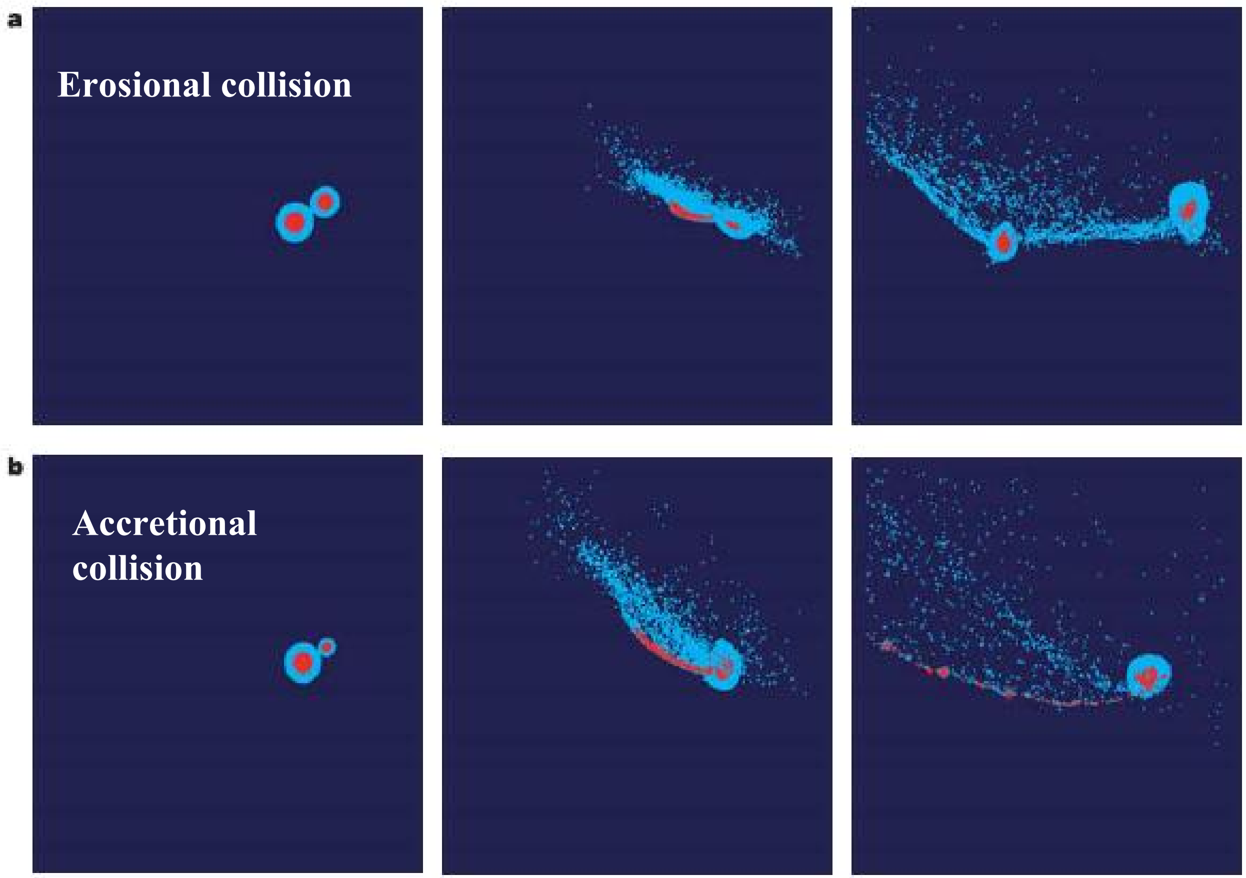
in agreement with the post-2007 datation of the Moon (Touboul et al. 2007, Nature)



ANGULAR MOMENTUM DEFICIT PROBLEM

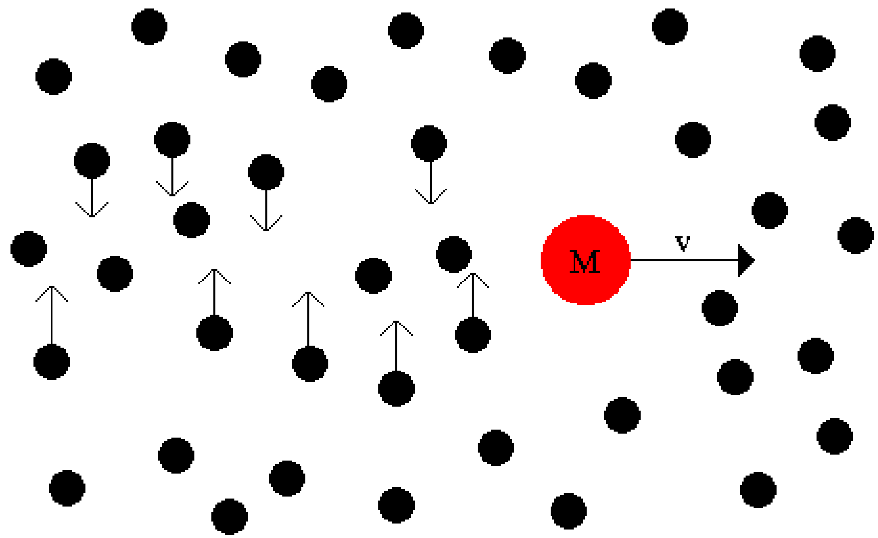
Partial regeneration of the planetesimal population when embryos collide with each other should allow dynamical friction to remain effective as long as giant collisions occur (to be tested with simulations)

Asphaug et al., 2006

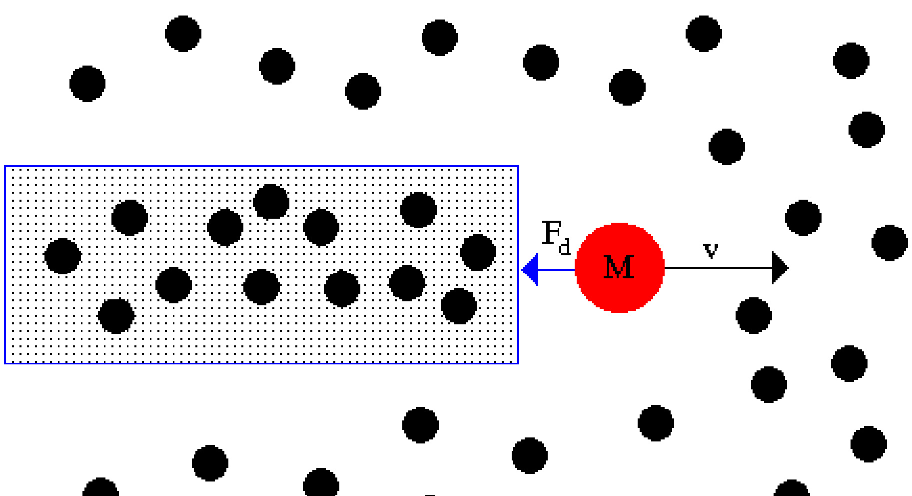


DYNAMICAL FRICTION

consider a mass, M, moving through a uniform sea of stars. Stars in the wake are displaced inward.



this results in an enhanced region of density behind the mass, with a drag force, F_d known as dynamical friction



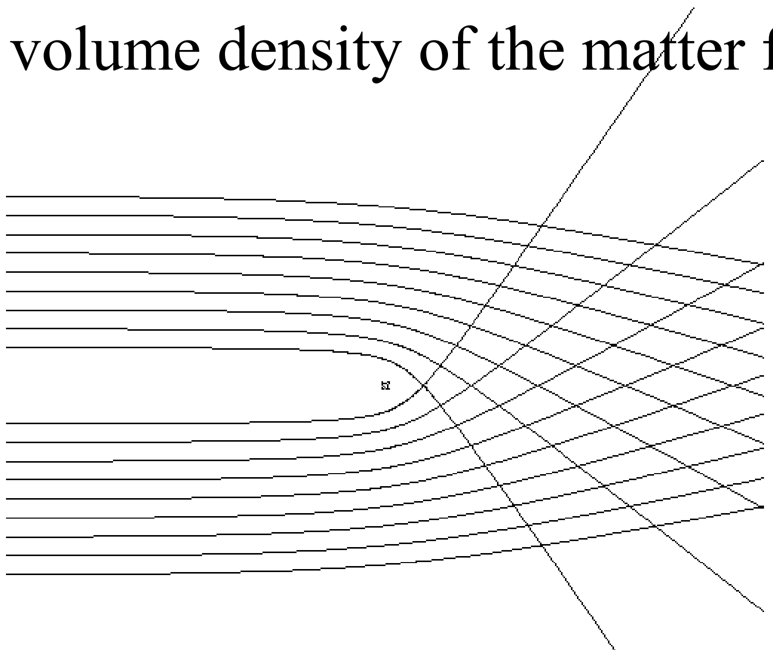
DYNAMICAL FRICTION

When an object of mass M goes through a “gas” of many objects of smaller masses with velocity v, it feels a drag force, called dynamical friction, given by :

$$f_{dyn} = C\, G^2 M^2 \rho / v^2 \quad ,$$

where C is a dimensionless constant, dependent on how v compares to the dispersion velocity of the surrounding matter,

and ρ is the average volume density of the matter field.



SUMMARY

KEY PARAMETERS for the final configuration :

- orbits of Jupiter (and Saturn) : circular or eccentric ?

- amount of dynamical friction (hard to guess!)

Remark:
Circular Jupiter favors correct AMD + late last giant impact...

Jacobson & Morbidelli (in preparation):

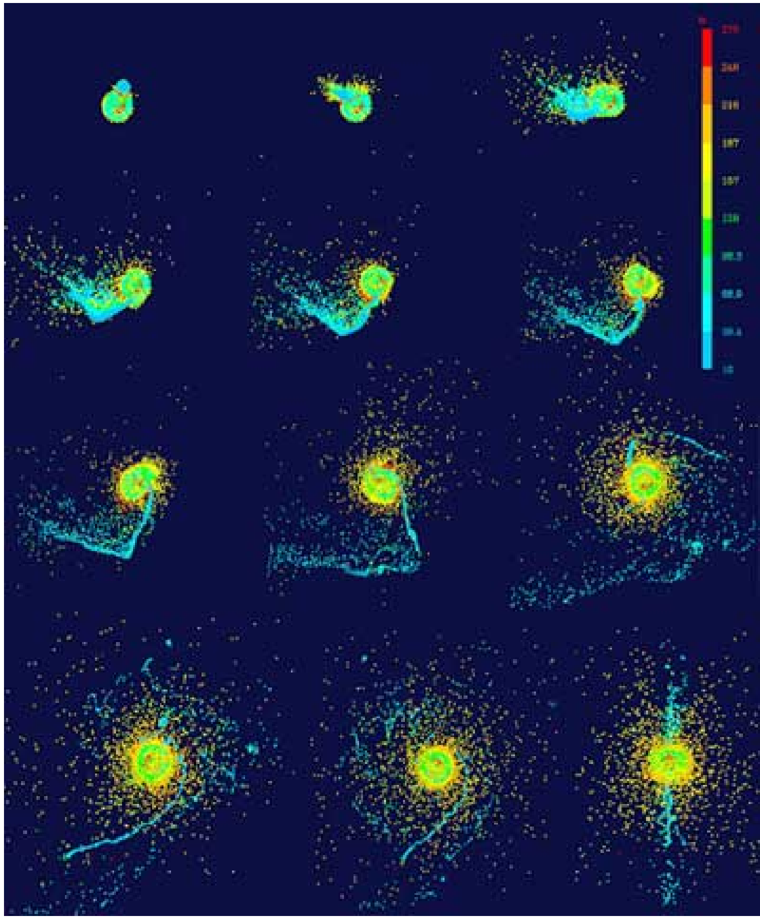
More dynamical friction => later last impact + less late veneer.

EARTH'S MOON FORMATION

MOON FORMATION

Giant impacts between embryos are their principal way of growing.

It is considered that one of these impacts on the proto-Earth led to the ejection of a lot of mass around the earth, which then gave birth to the Moon.



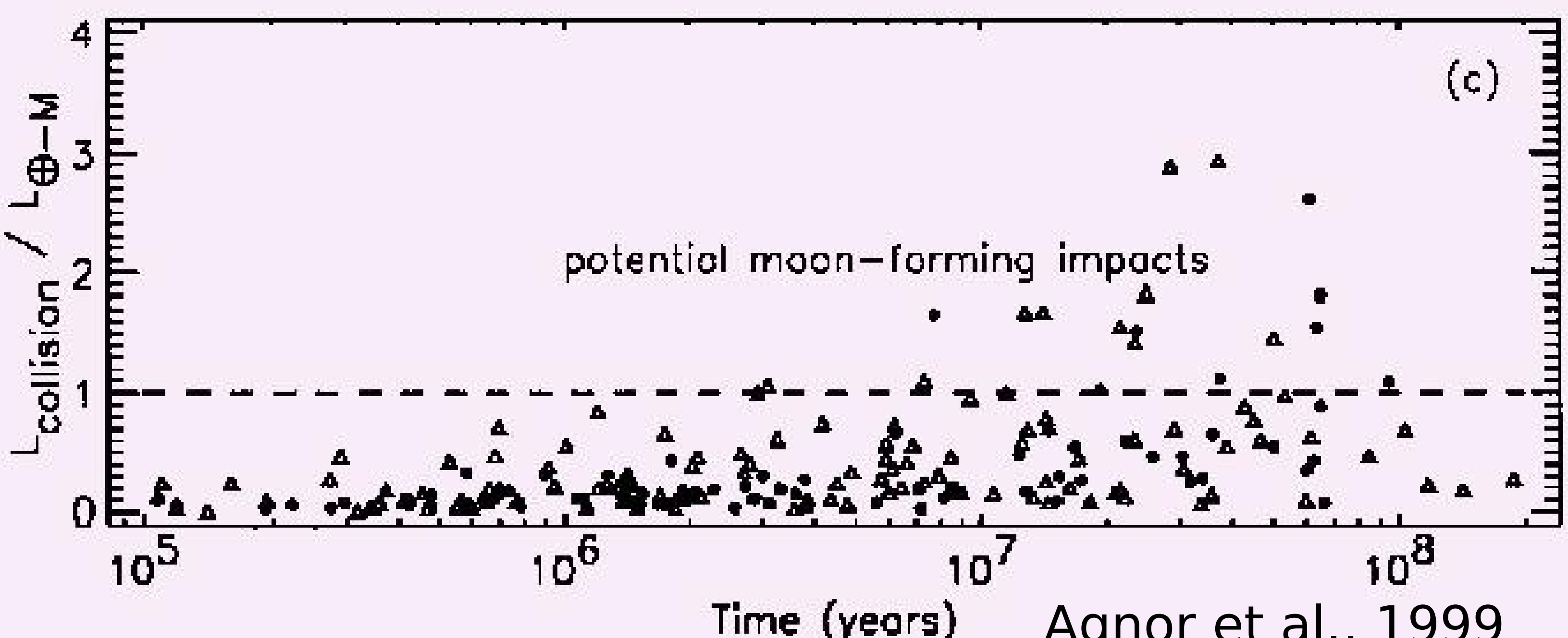
The “moon-forming impact” was NOT an exceptional chance, but a likely event.
(Agnor et al., 1999)

Moon Forming Impact

Potential Moon-forming impacts are those that carry an angular momentum larger than that of the Earth-Moon system

They are not rare: They occur for 25% of the planets that have a final mass > 0.5 M_{earth}

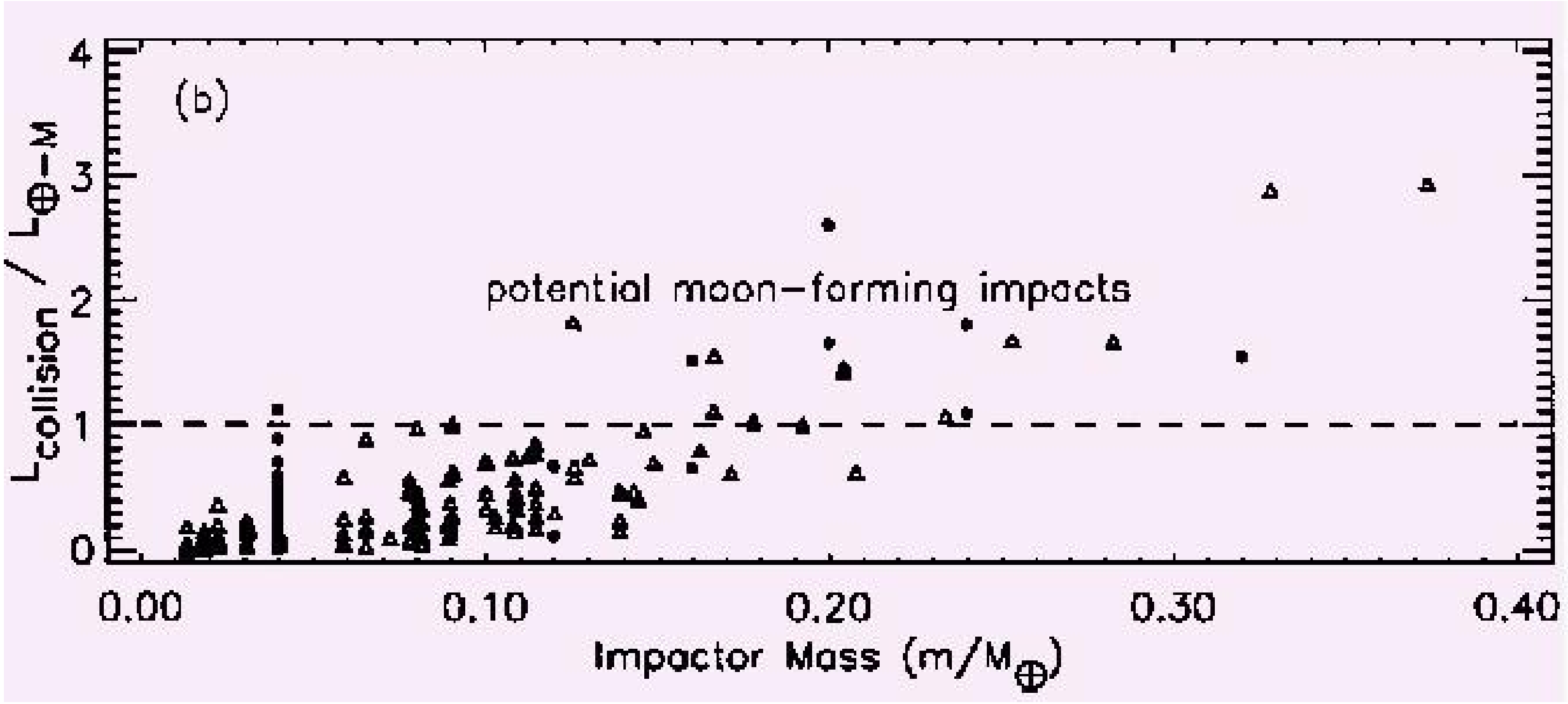
They tend to occur towards the end of the accretion.



Agnor et al., 1999

Moon Forming Impact

They involve impactors which are at least as massive as Mars.



Agnor et al., 1999

Moon Forming Impact

Successive simulations are those that:

Carry an angular momentum larger (but not much: up to ~30%) of the angular momentum in the Earth-Moon system

Produce a proto-lunar disk that is more massive than the Moon (but not much: up to ~80%)

The mass of iron in the proto-lunar disk is <3% of the mass of the disk

Moon Forming Impact

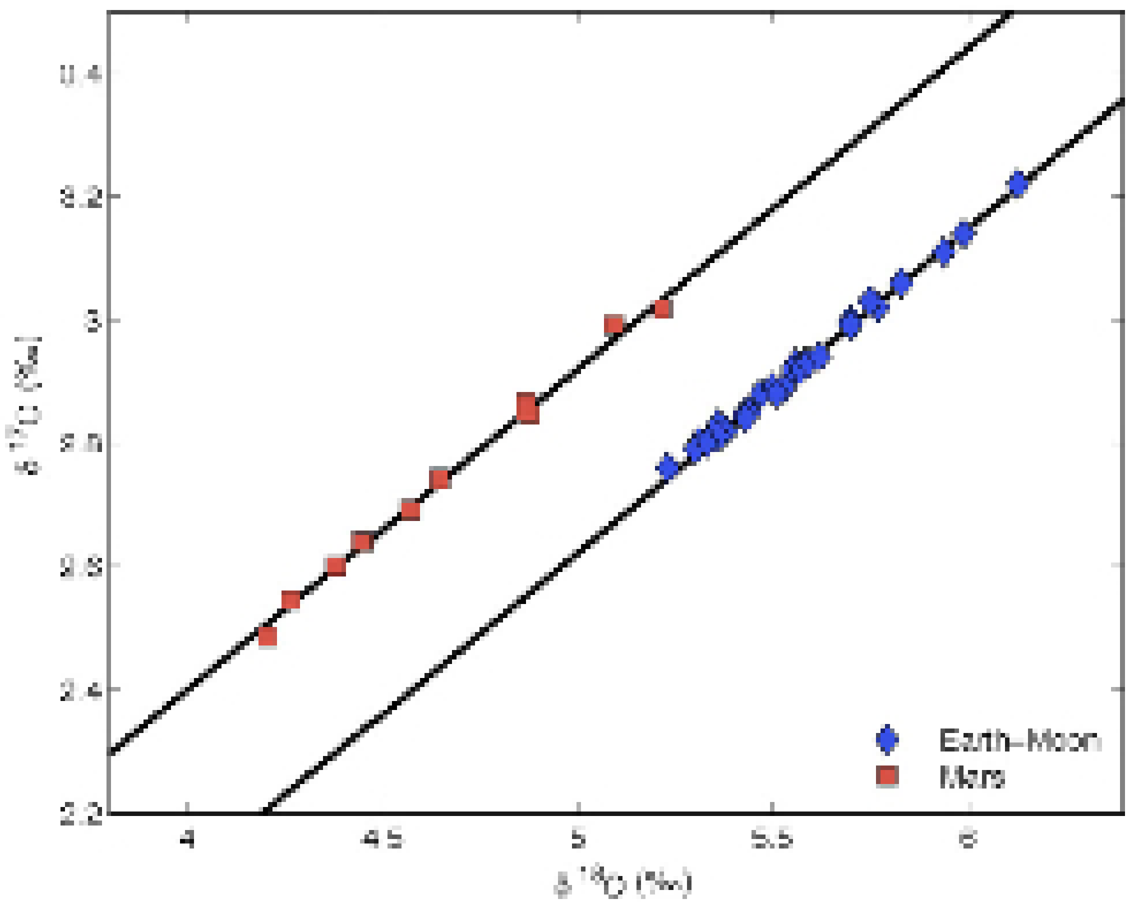
The Moon forming impact may well have been the last giant impact in the history of Earth accretion.

A late formation helps explaining the paucity of metal in the Moon:

the flux of projectiles on the Moon is 3 - 7% that on Earth.

Assume that the Earth accreted the rest of its mass from planetesimals with a terrestrial abundance of iron: then it could have accreted at most 5% of the Earth mass before that the Moon accreted more iron than observed.

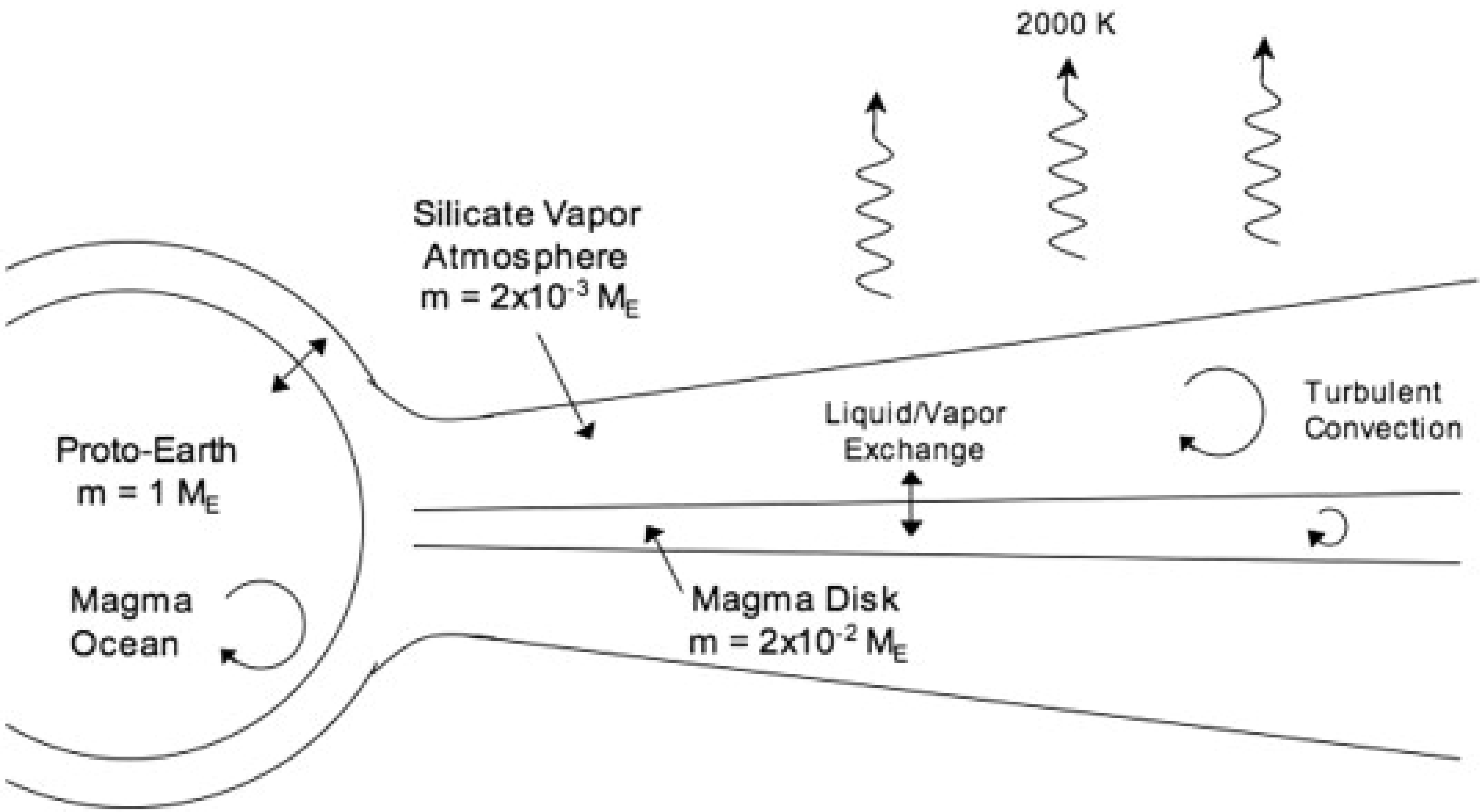
ISOTOPIC COMPOSITION



Need for an equilibration mechanism between the Moon and the Earth !

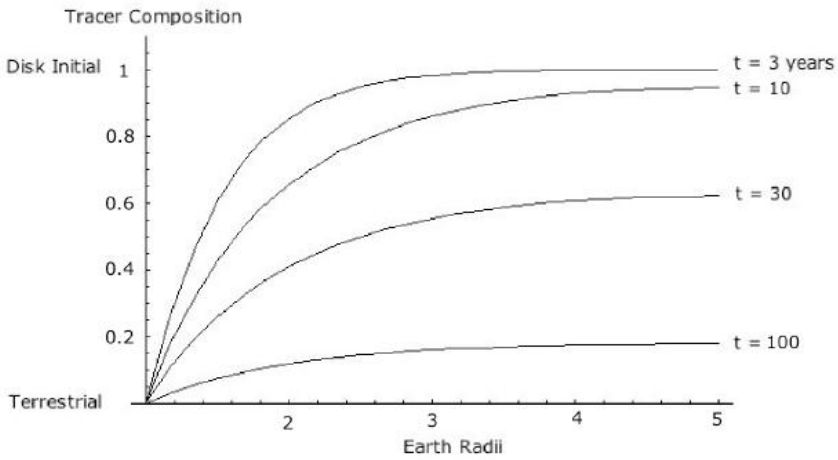
EQUILIBRATION

Pahlevan & Stevenson (2007) suggest that the proto-lunar disk was molten, with a gaseous silicate atmosphere. The Earth was also a magma ocean, with a gaseous silicate atmosphere. In the atmosphere, the isotopic compositions.



MOON FORMATION

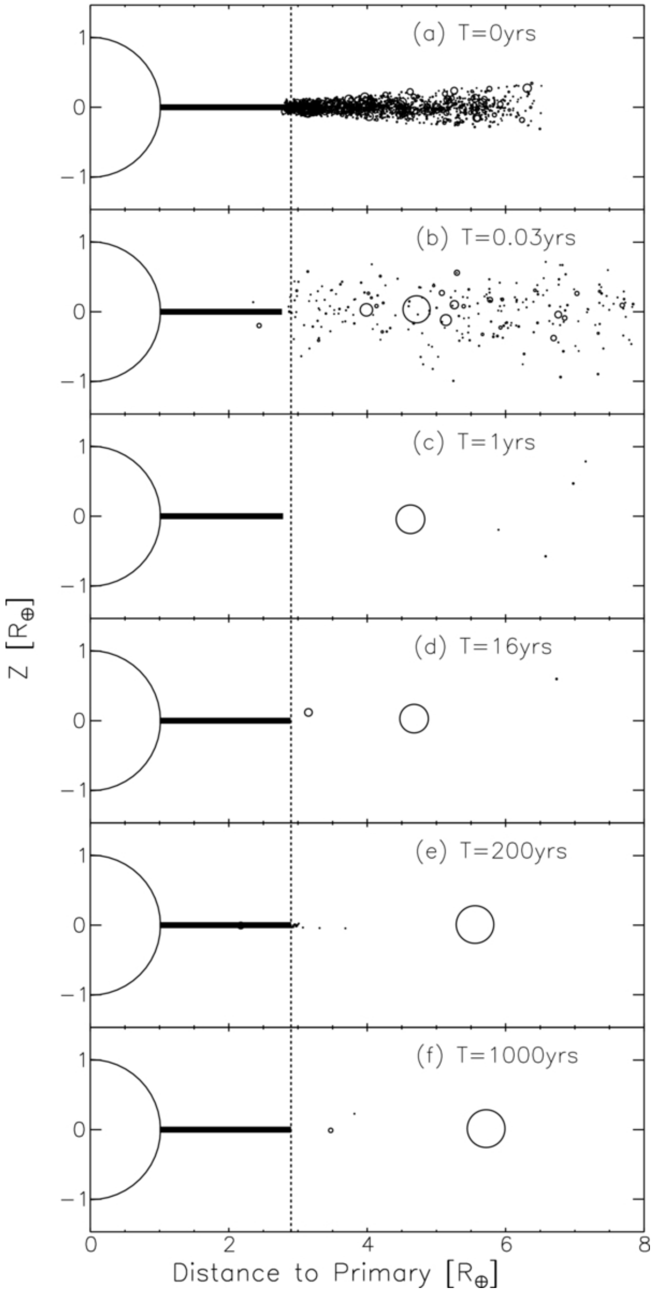
Problem : radial mixing is slow.



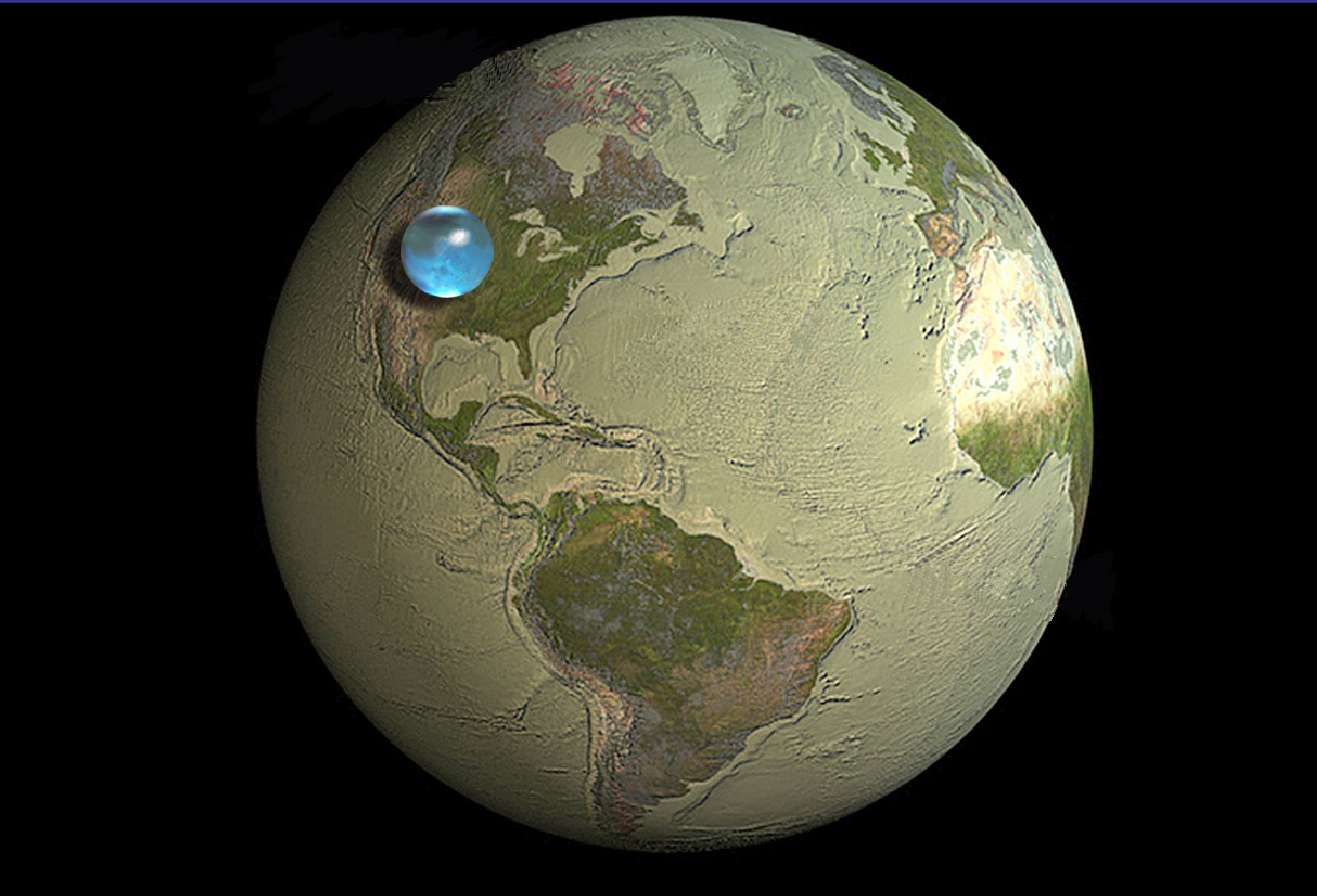
Salmon & Canup (2012) :

Moon accretion stalls for ~100 years because the growing Moon repels the disc, and prevents its spreading.

Work in progress, stay tuned...



L'EAU / WATER



L'EAU / WATER

Definition : **snowline**.

Distance to the Sun from which H_20 is in the solid form instead of vapour.

In the Proto-Planetary Disk, T(r) decreases.
$(H/r = h_0(r/r_0)^{2/7}$ => $T=T_0(r/r_0)^{???}$

Typically, the snowline is at 3 to 5 AU.

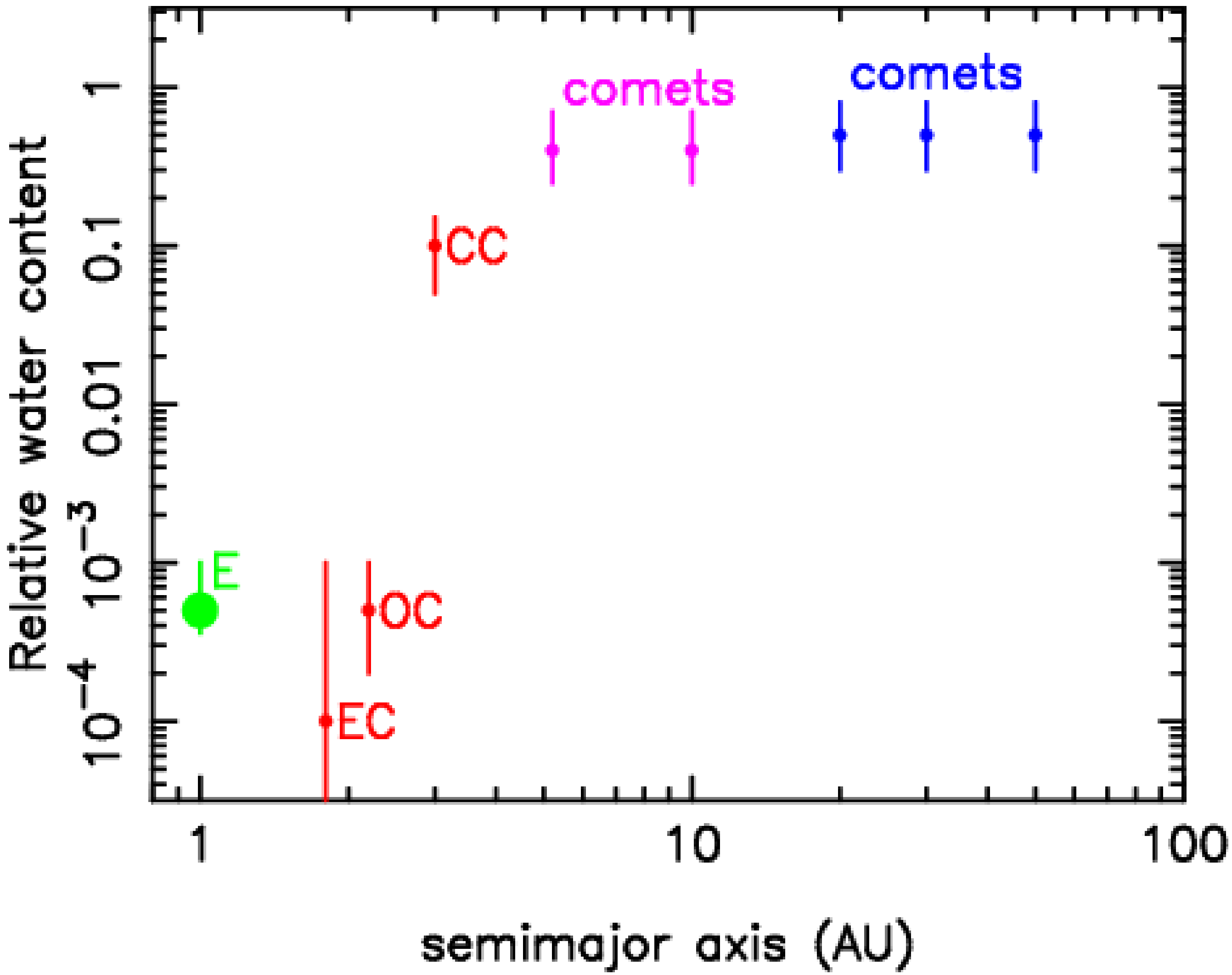
Asteroids are rocky, comets are snowy → OK.

BUT : all the planetesimals formed inside 3 AU should be completely dry, containing no H_20 at all !

Where does water come from on Earth for instance ?

L'EAU / WATER

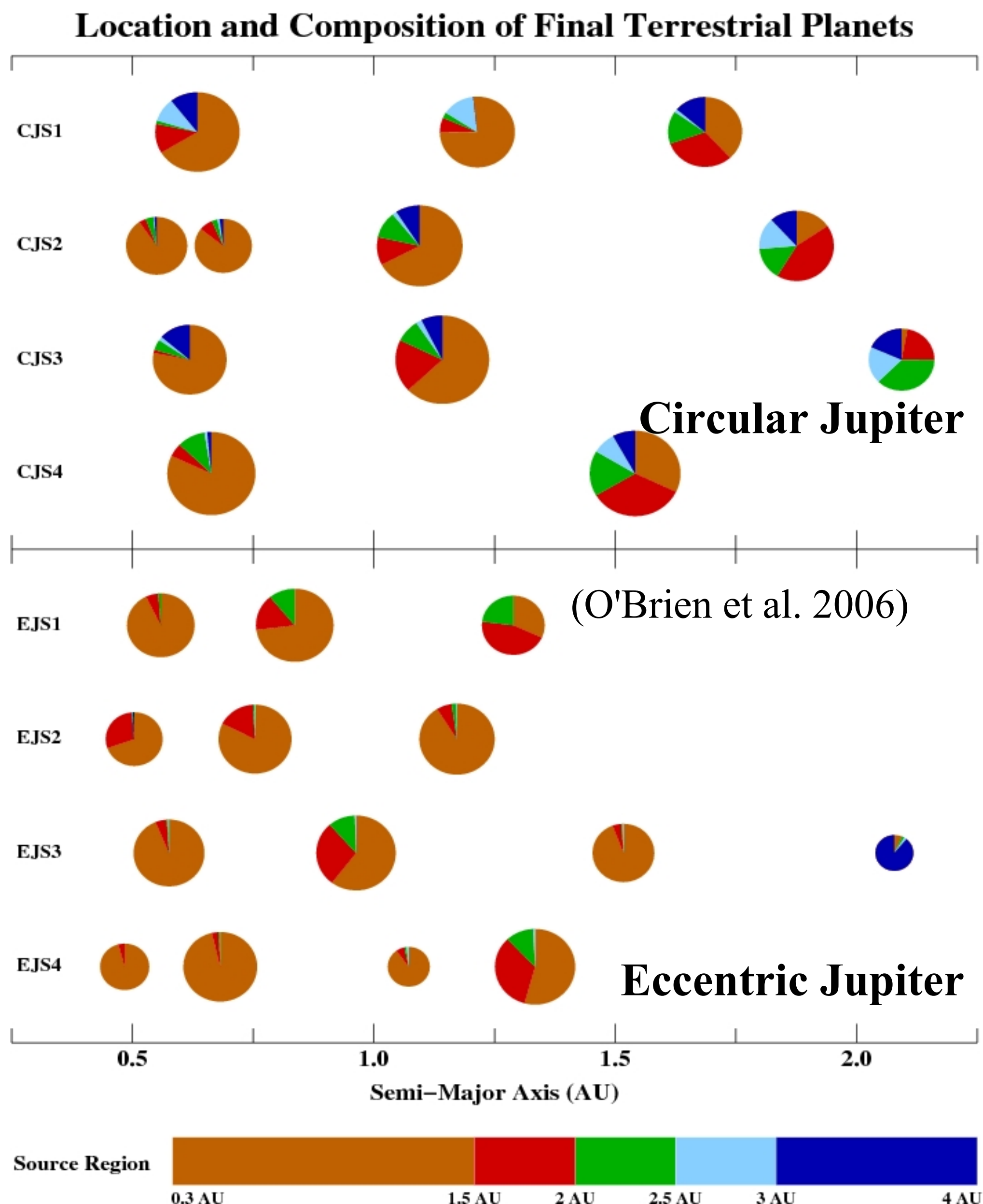
The evident gradient of relative water content with heliocentric distance implies that water should have been accreted from distant material.



L'EAU / WATER

The eccentricity of Jupiter plays a crucial role in determining from which source regions the planets accrete material.

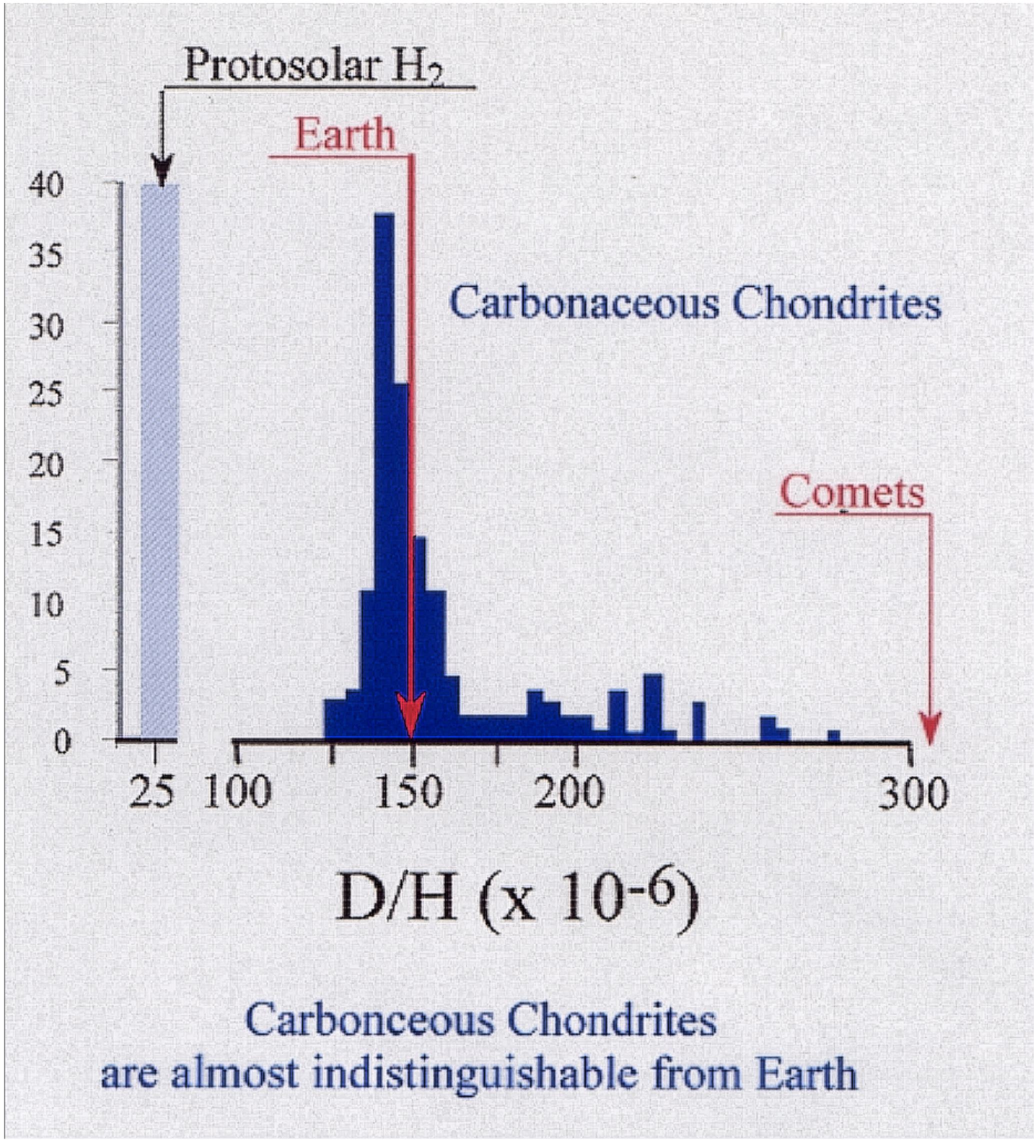
If Jupiter was originally on a quasi-circular orbit, some 15% of the terrestrial planet material should have come from water-rich carbonaceous chondrites from the outer asteroid belt.



L'EAU / WATER

The D/H ratio constraint:

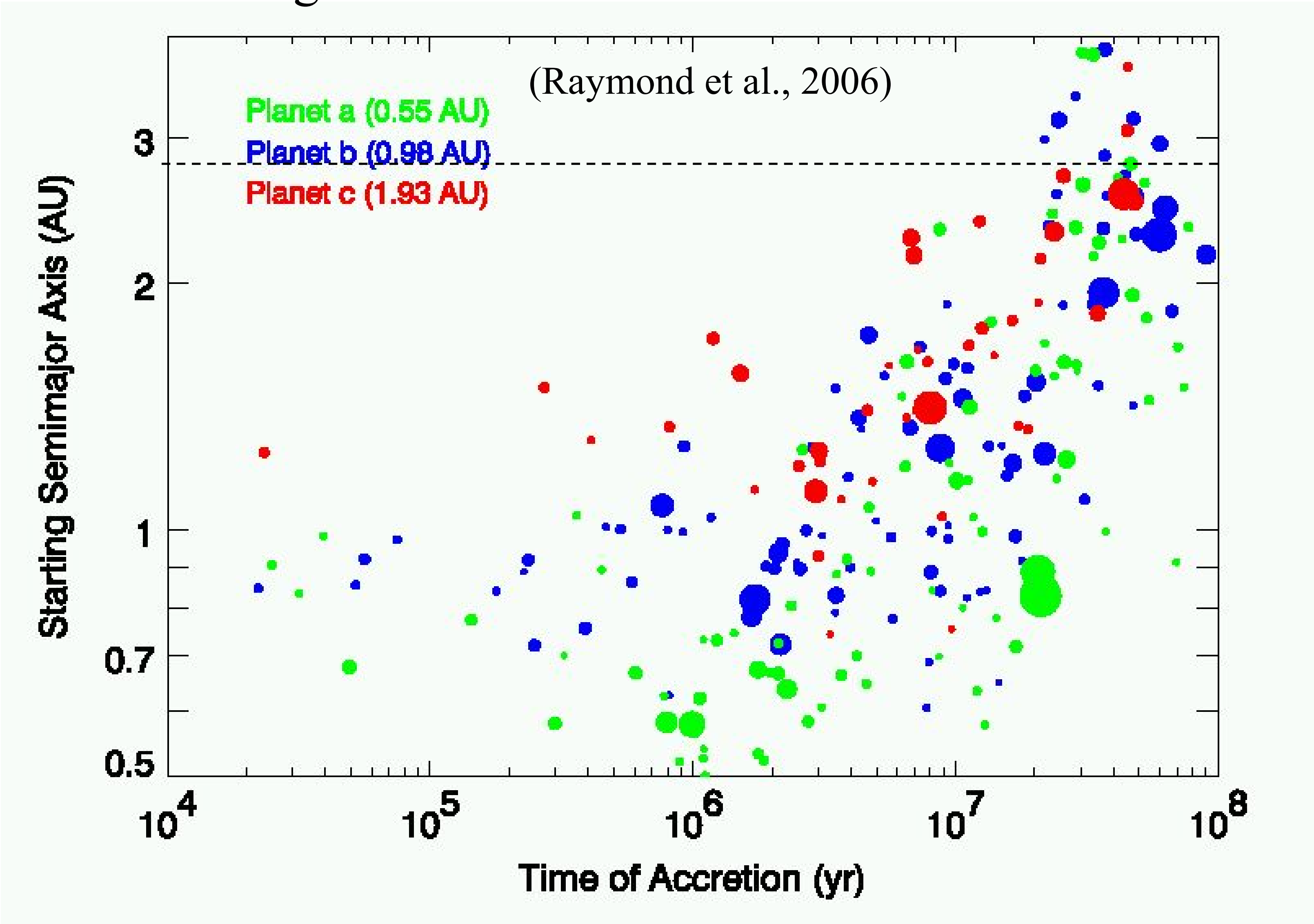
Numerical integrations also show that comets could have contributed at most 10% of the current water on Earth Levison et al. (2000); Morbidelli et al. (2000)



Courtesy of F. Robert

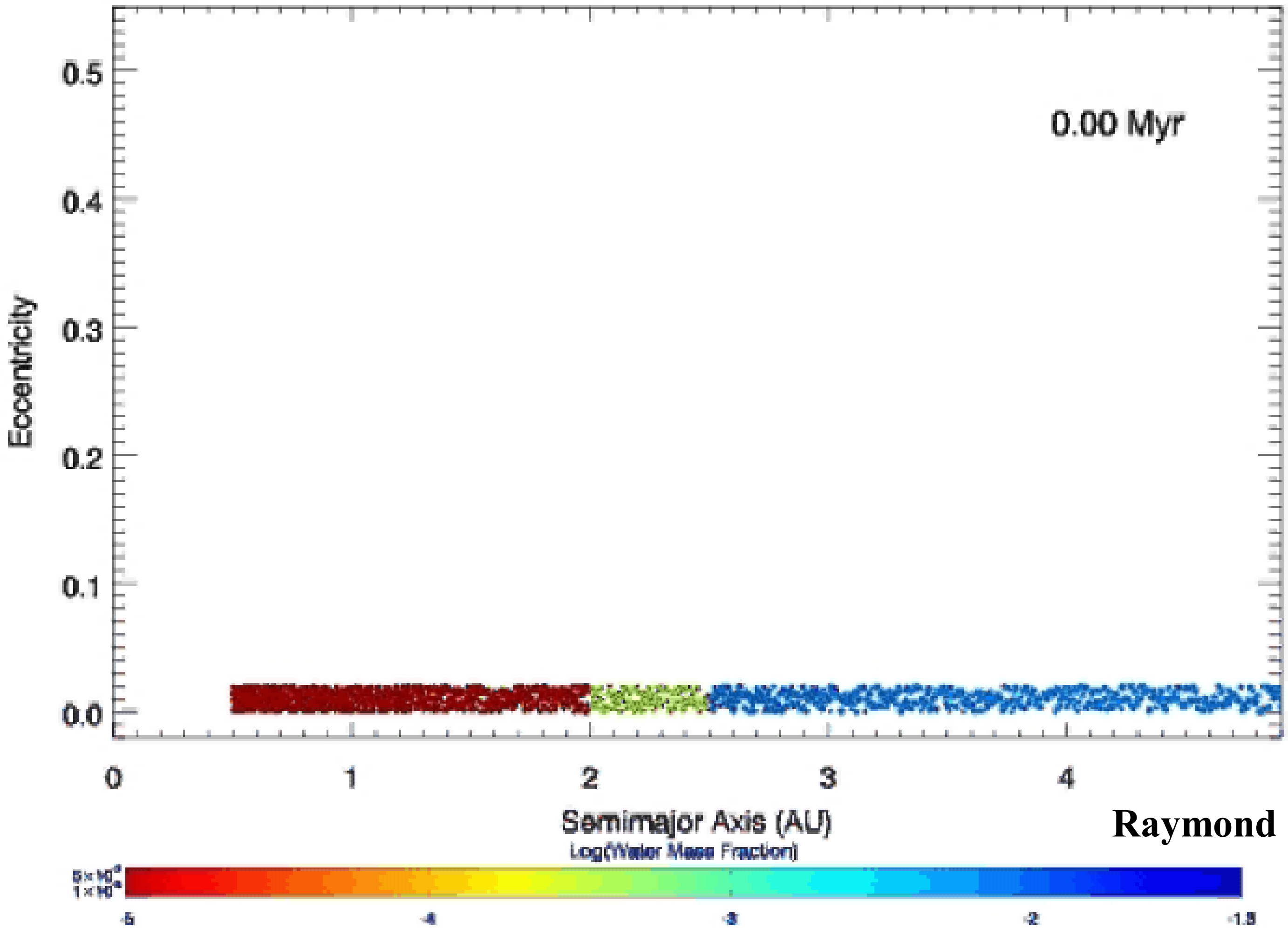
L'EAU / WATER

In the circular-Jupiter case, the material from the outer asteroid belt is accreted among the latest.



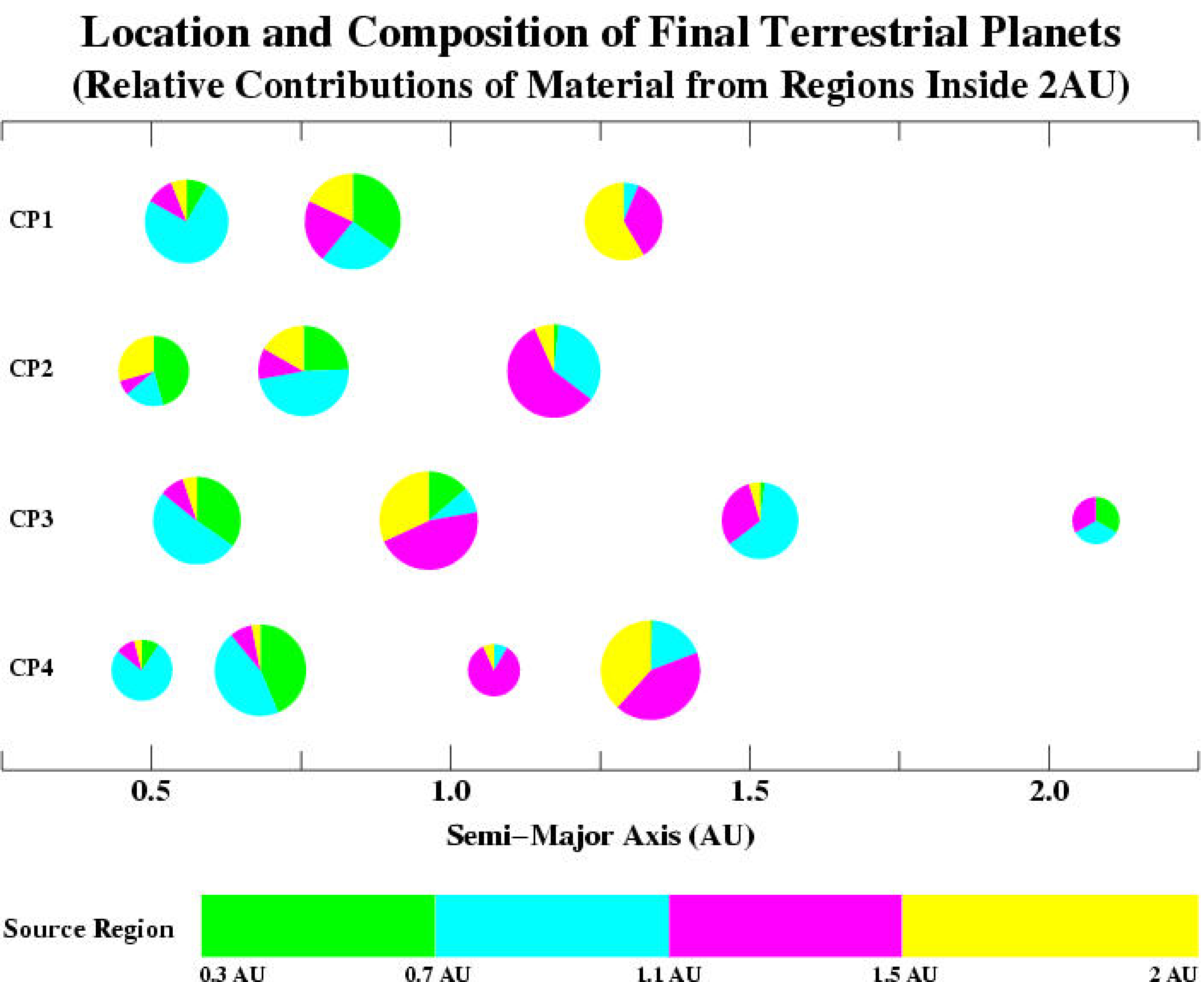
L'EAU / WATER

The Earth acquires the water relatively late, but nevertheless during its own accretion process.



Raymond et al., 2006

COMPOSITION



Even if planets had accreted NO material from the asteroid belt, it is unlikely that the proto-Earth and the proto-Lunar impactor could have the same composition even if they formed very close to each-other.

L'EAU / WATER

If the Earth acquired its water from giant impacts of embryos formed beyond the snowline,

if the moon forming impact is the last one,

if the Earth and the Moon equilibrate,

THEN

Why is the Moon so dry, much dryer than the Earth ?

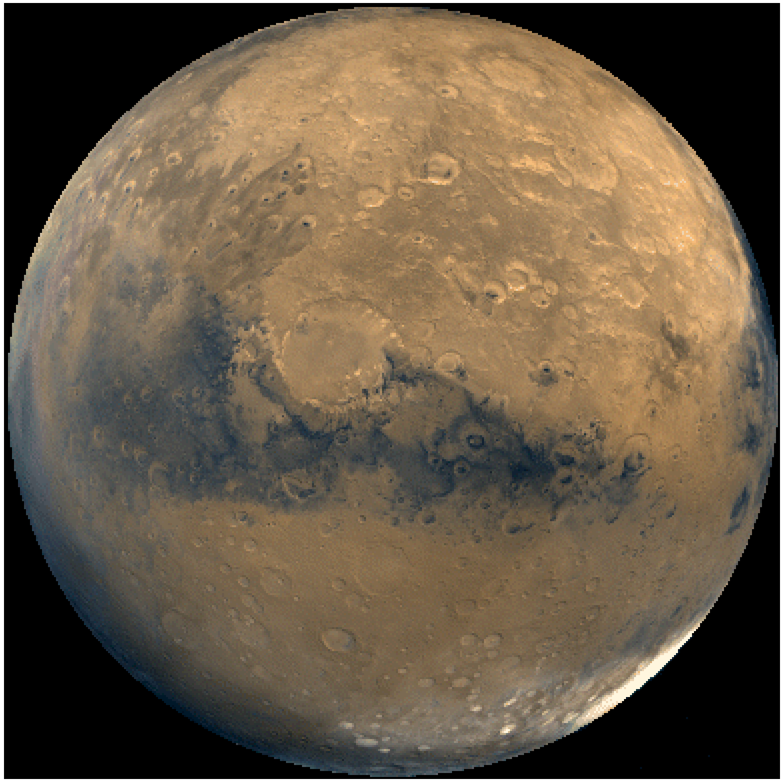
late veneer ?

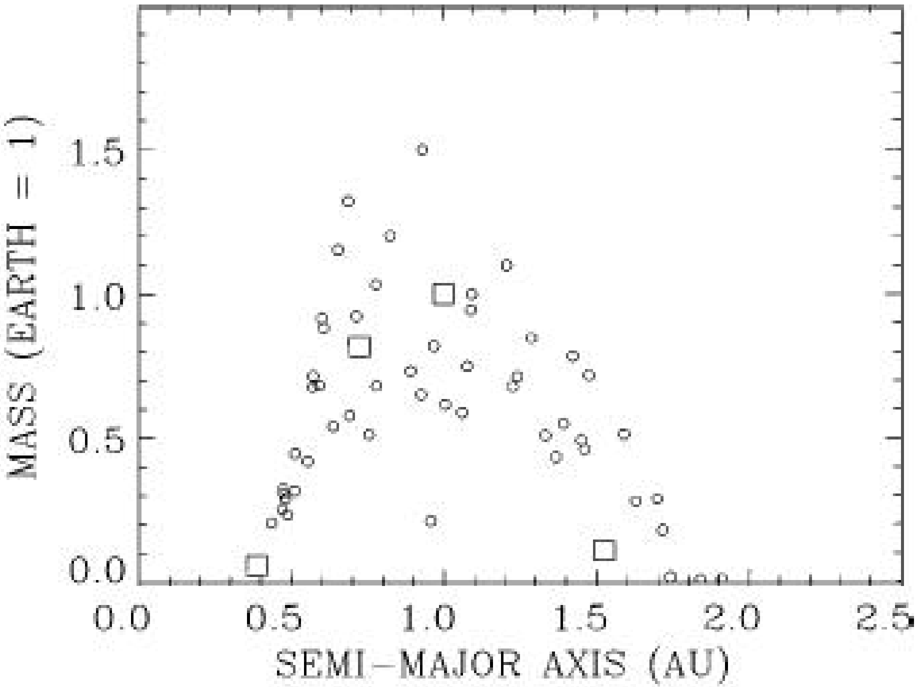
Still an open issue.

THE MASS OF MARS PROBLEM

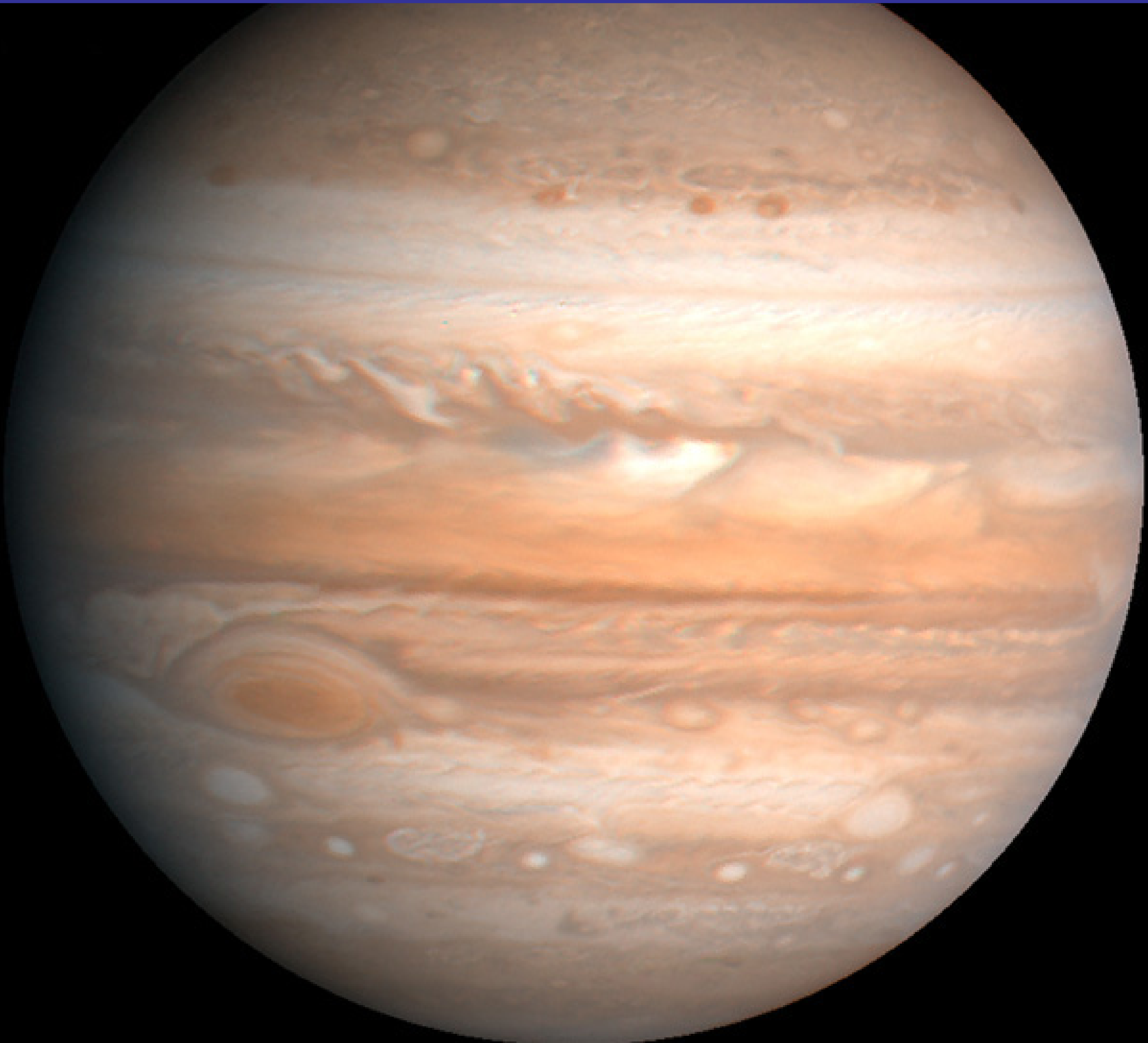
In the simulations, the planet formed in the zone of Mars is always too massive (even more than the Earth). The only case in which the mass of Mars is correct is when the planetesimals and embryos disk is initially truncated at 1 AU (Hansen 2007).

Stay tuned...





GIANT PLANETS



GIANT PLANETS

a/ Retention of the gas.

The escape velocity at the surfce of a body of mass M_p and radius R_p is : $v_{esc} = \sqrt{(2GM_p/R_p)}$.

The average velocity of molecules of gas is the speed of sound $c_s \sim T(r)$.

Capture of gas possible if $c_s < v_{esc}$.

At 1 AU, with a density of (~4000 kg/m^3), it gives $M_p > 0{,}01M_{\oplus}$.
Threshold ≈ $0{,}3M_{\oplus}$ à 10UA.

Bondi radius : r_B such that $\sqrt{(2GM_p/r_B)}=c_s\sqrt{2}$: $r_B = GM/c_s^2$.

GIANT PLANETS

b/ Structure of the gaseous envelope.

Hydrostatic equilibrium : $dP / dr = - (GM(r) / r^2) \rho(r)$,
where M(r) = mass inside the sphere of radius r : $dM(r) / dr = 4 \pi r^2 \rho(r)$.

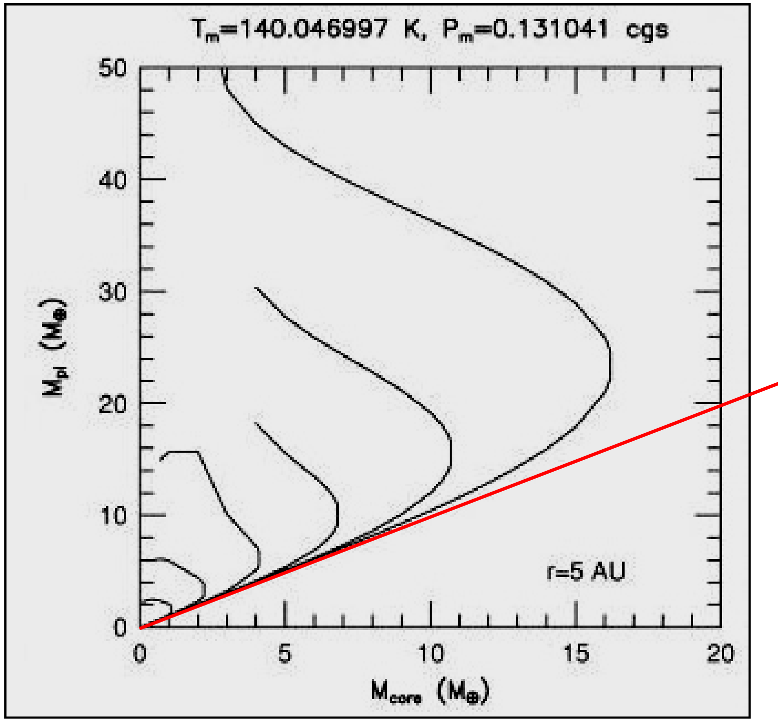
Energy : Lots of potential energy to lose in order to accrete.
Rate of energy (luminosity) brought by accretion of solids onto the core:
$L_{core} = (GM_{core} / r_{core}) (dM_{core} /dt)$.
It must be radiated away by the atmosphere. Convection or Radiative transfert → T(r), M(r), et P(r).

Figure (Papaloizou & Terquem, 1999) : M_p as a function of M_{core}, at 5 UA, for $dM_{core} /dt = 10^{-11, 10, 9, 8, 7, 6}\ M_\oplus$/an.

Red line : $M_p = M_{core}$, no gas.

For all M_{core}, there are 2 solutions.

Starting from 0, → as the planet gets bigger, so does the atmosphere.

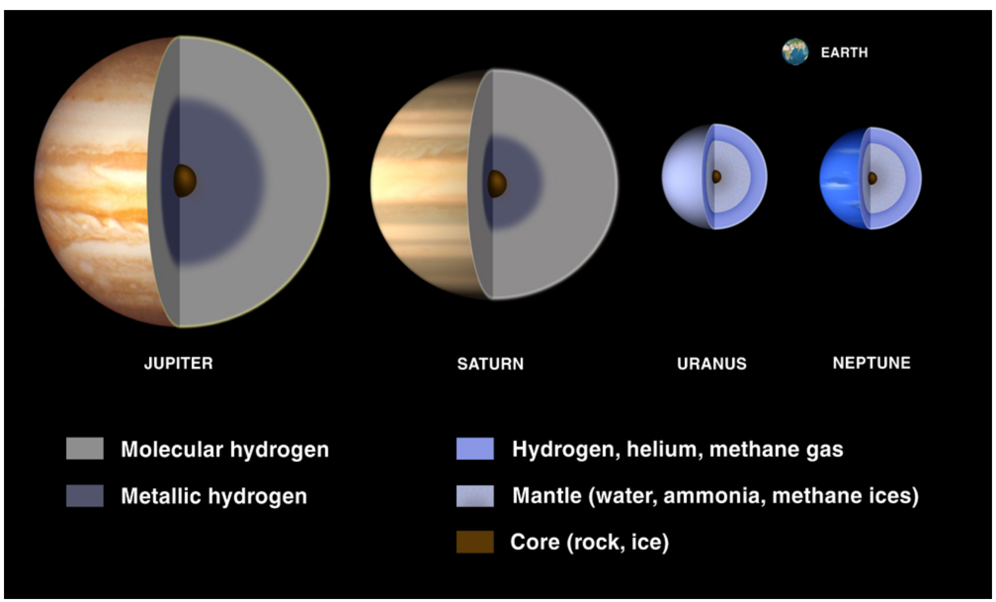


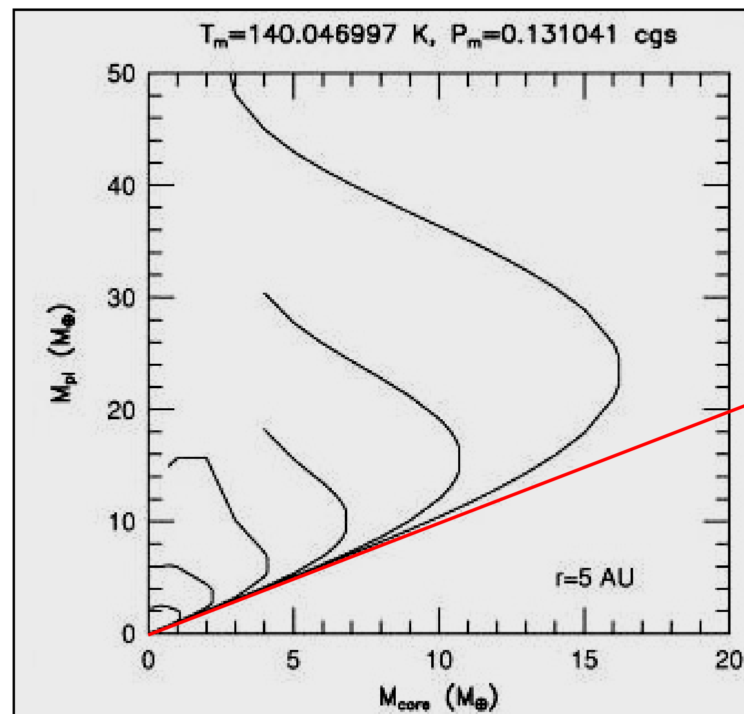
c/ Collapse.

For all accretion rate, there exists a maximum mass of core beyond which no hydrostatic equilibrium of the enveloppe is possible.

Thus, the planet collapses, and grows exponentially, as nothing prevents gas from infalling → formation of a giant planet, whose radius is small compared to its Hill radius.

Cores: Saturn ~ 15 $M_{\oplus}$. Jupiter < 10 $M_{\oplus}$





GIANT PLANETS

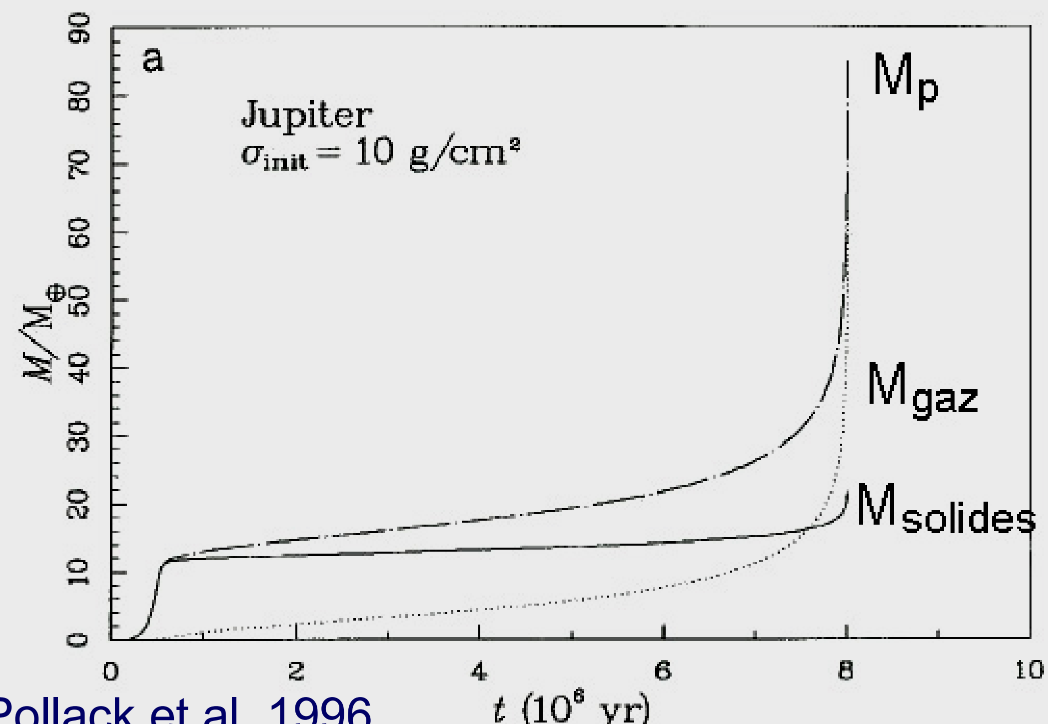
Summary : 3 phases

- Formation of a solid core.
- Enveloppe of gas. Slow growth by accretion of gases and solids.
- Collapse. Runaway growth (from ~20-30 $M_\oplus$, or when $M_{solids}=M_{gas}$).

Remark :

To obtain 20$M_\oplus$ of solids, better be beyons the snowline.

It is expected that giant planets form far from their stars.



Pollack et al, 1996

GIANT PLANETS

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- Formation of a solid core.
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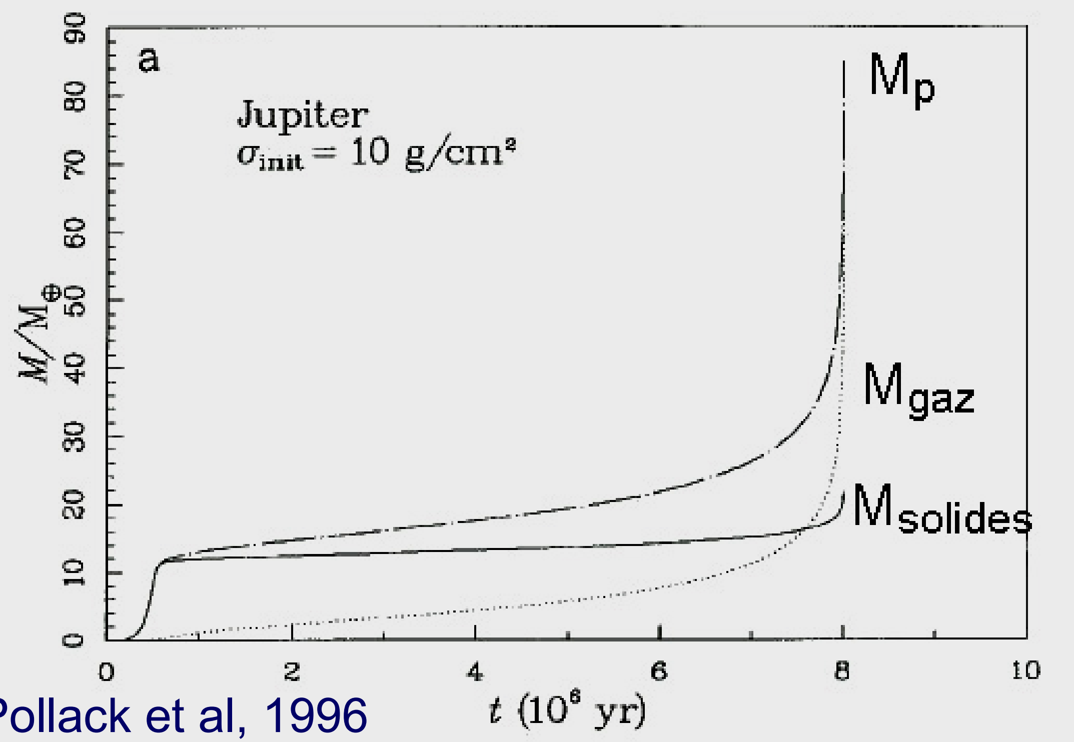
Problem :

The phase before the collapse is very slow, comparable to the disks' lifetime…

NB: dM_{core} / dt could be increased adding migration (see a futer lecture).



Pollack et al, 1996

GIANT PLANETS

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- Formation of a solid core.
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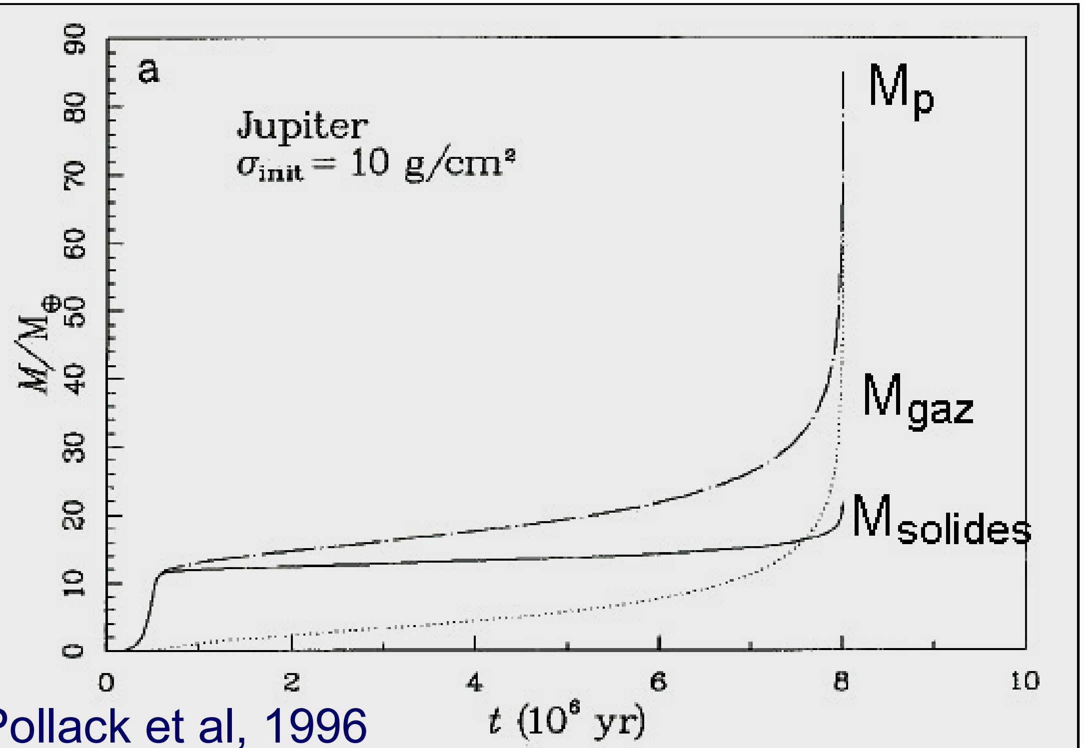
Problem 2 :

Nothing stops the collapse ! One should only have super giant planets (5 $M_{Jupiter}$!)

Solution : could a Circum-Planetary disk act like a bottleneck ?

Work in progress...



Pollack et al, 1996

GIANT PLANETS

Gravitational Instability :

Jeans instability : In a localized perturbation, the self-gravity is stronger than the thermal pressure, leading to the formation of a gravitationally bound object.

Toomre (1964)'s criterion : $Q = \Omega c_s / \pi G \Sigma > 1$ for stability.

Thus, the more massive the disk is (large Σ), and cold (small c_s), the smaller is Q and an instability is possible.

In favorable conditions, objects of jovian masses are easily formed.

$Q>1 \Leftrightarrow M_{disc} < (H/r) M_*$

$c_s = H\Omega$, so

$$Q = \Omega^2 H / \pi G \Sigma = G M_* H / \pi G \Sigma r^3 = (M_* / \pi \Sigma r^2) (H / r)$$

GIANT PLANETS

Numerical simulation :

Here, the temperature is constant → no compressionnal heating, no cooling problem of the formed object. Objects appear...

Source : http://hydro.astro.indiana.edu/, groupe of Pr. Durisen.

Other reference on gravitaional instability : Alan Boss (principal promotor of this theory).

Problems :

Are these condittions realistic ?

Can these objects survive and really become planets ?