TIDES, RINGS, SATELLITES, Saturn et al...

Aurélien CRIDA

TIDAL FORCES

Take two spheres of radius *a*, orbiting together a body of mass *M*, at a distance *r*. They both feel the gravity force from the planet, and the centrifugal force. As they are at *r*+/-*a*, the balance is not exactly zero for each of the spheres.

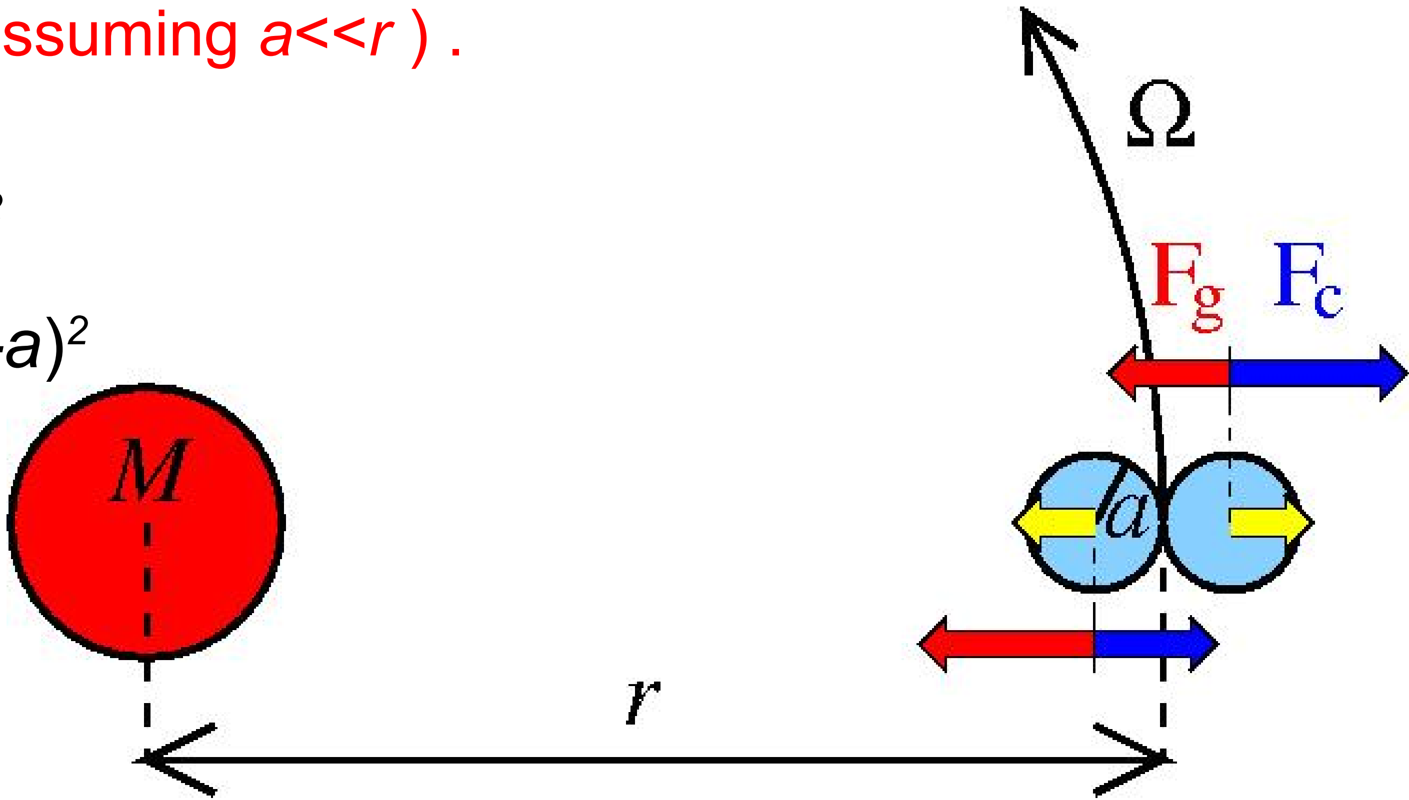
Exercice : Compute the resulting specific force (assuming $a \ll r$) .

Solution :

$\Omega = (GM/r^3)^{1/2}$

$F_g = GM / (r \pm a)^2$

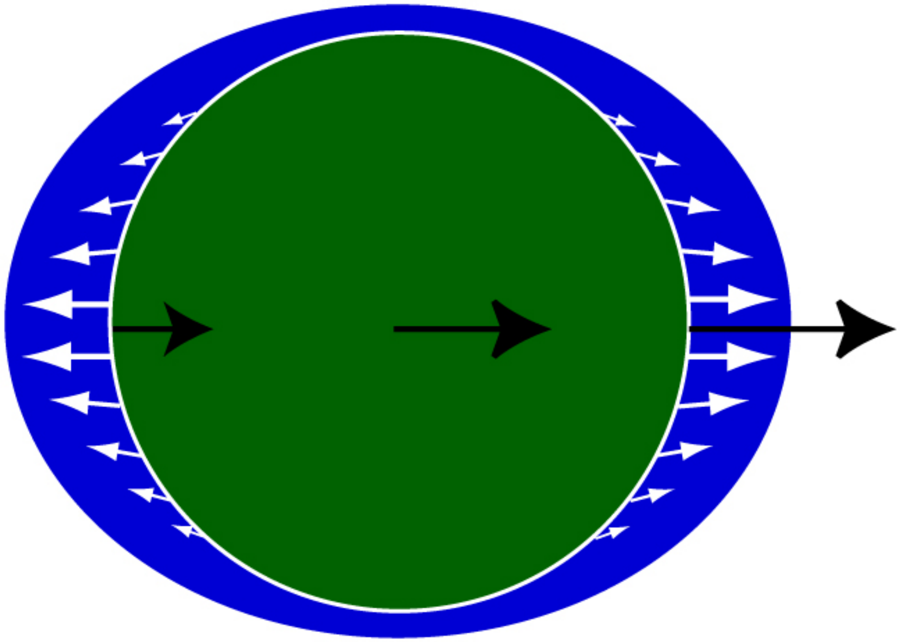
$F_c = \Omega^2 (r \pm a)$



$F_m = 3\Omega^2 a$

APPLICATION 1 : OCEANIC TIDES

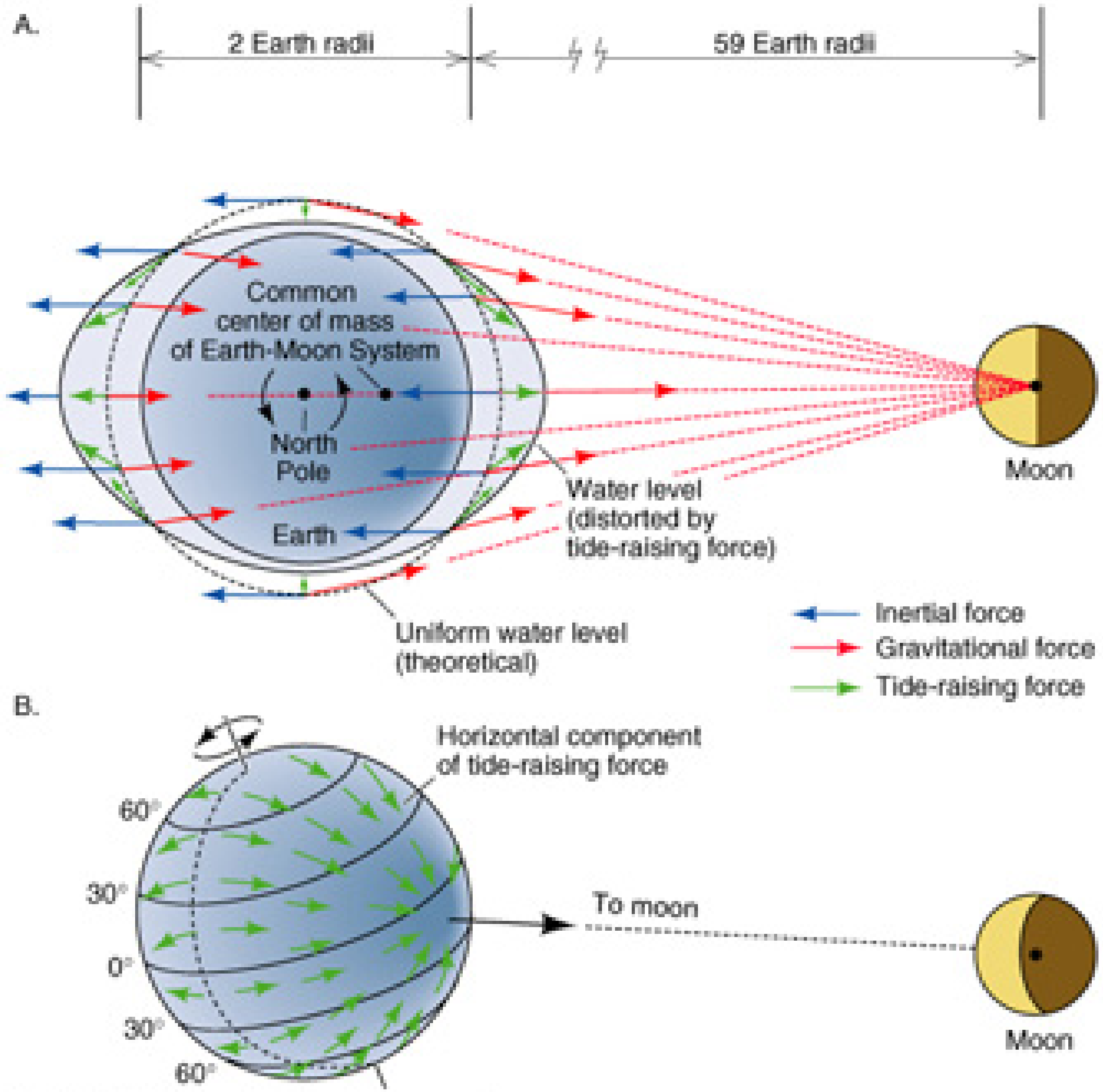
The Earth and the Moon are tidaly elongating each other, into the shape of a rugby ball, pointed toward the other body.



Black arrows: Gravitational force due to Moon.
White arrows: Net differential force relative to centre of the Earth - the tide-raising force.

The liquid oceans (blue) are more easily deformed than the solid Earth (green), so that the sea level increases twice a day.

APPLICATION 1 : OCEANIC TIDES



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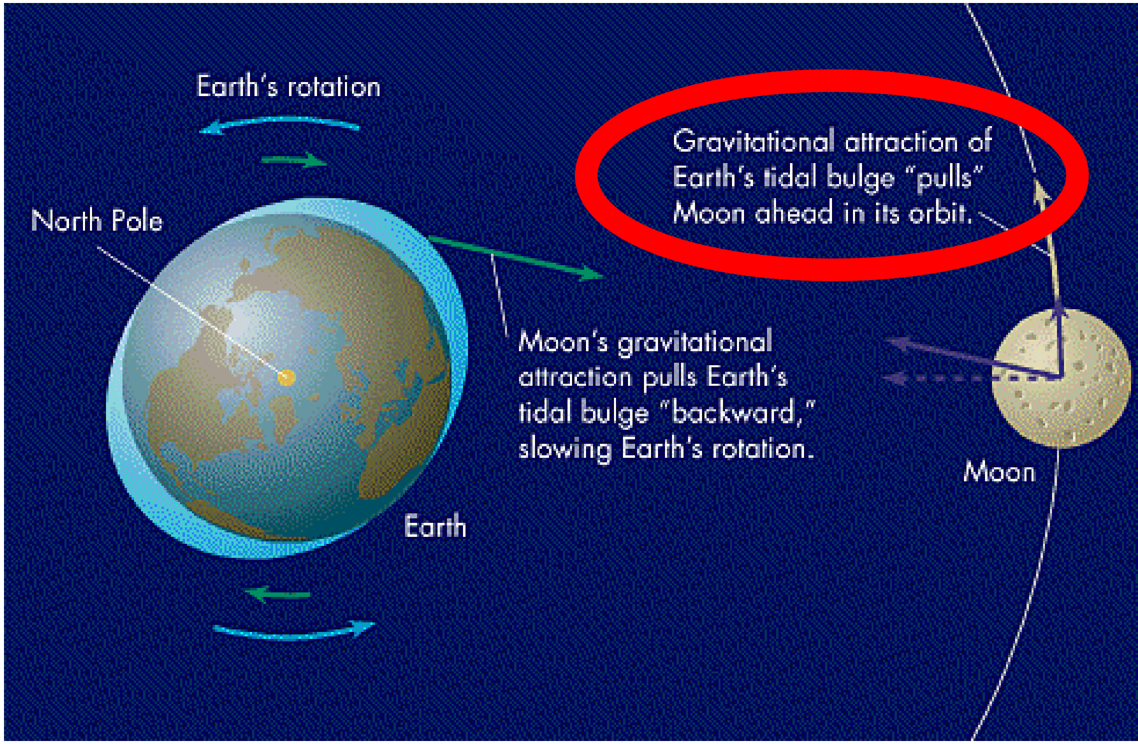
APPLICATION 2 : ORBITAL EVOLUTION

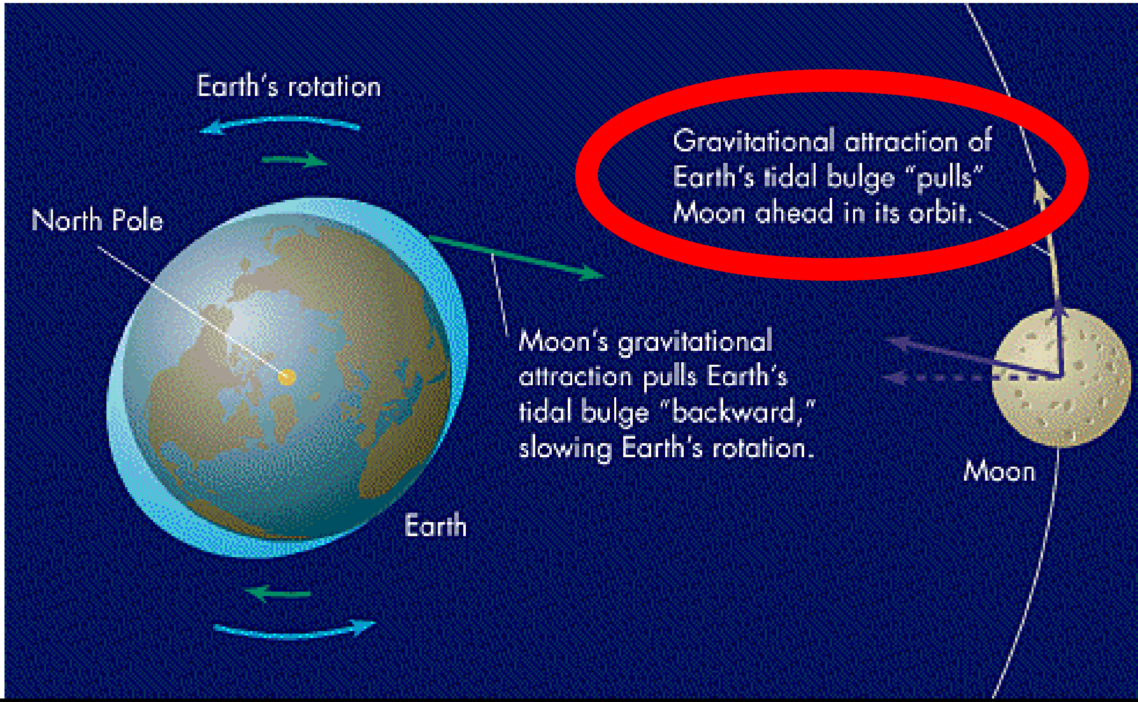
If the central body spins **faster** than the satellite rotates around it, and does not immedialtely respond to the tidal potential (dissipation), the tidal bulge is carried by the rotation, and leads in front of the satellite. As a consequence, the satellite feels a positive torque, and its orbital radius increases.

(and conversly, the central body's rotation is slowed down, for angular momentum conservation.)



APPLICATION 2 : ORBITAL EVOLUTION

In the case of the Earth-Moon system, it is measured at O.C.A. that the Moon goes further from the Earth at a rate of about 3.8 cm/year.

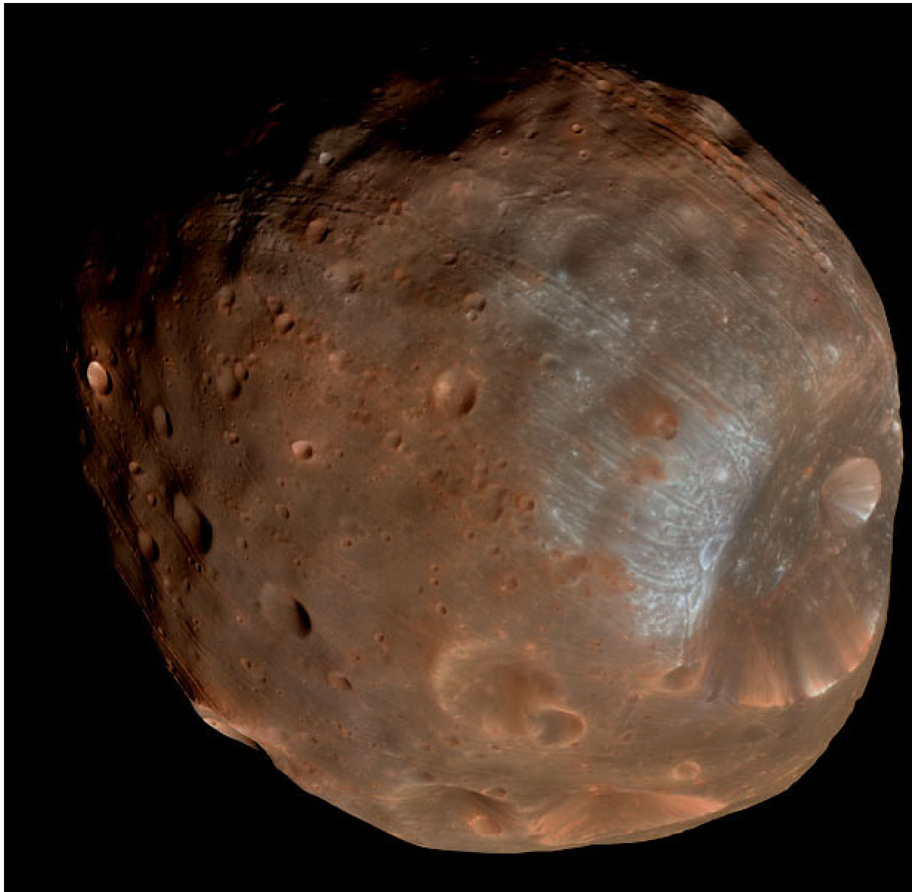


APPLICATION 2 : ORBITAL EVOLUTION

If the central body spins **slower** than the satellite rotates around it, and does not immedialtely respond to the tidal potential (dissipation), the tidal bulge stays behind the satellite, which feels a negative torque, and its orbital radius shrinks.

Phobos rotates around Mars with a period of 7h 39min, (r=9400 km) while Mars has a spin period of 24.63 hours.

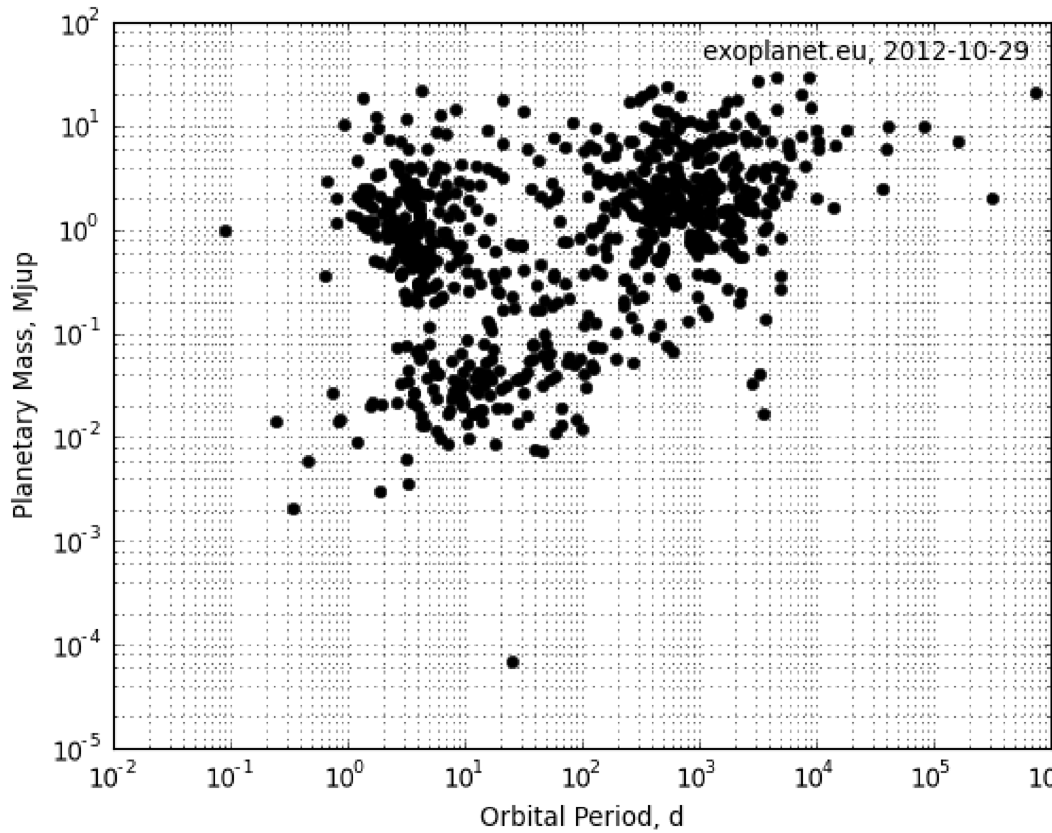
As a consequence, it recedes towards Mars of 1.8 cm/year, and will crash on the planet in ~11 Myrs (depending on the models).



APPLICATION 2 : ORBITAL EVOLUTION

If the central body spins **slower** than the satellite rotates around it, and does not immedialtely respond to the tidal potential (dissipation), the tidal bulge stays behind the satellite, which feels a negative torque, and its orbital radius shrinks.

Many (exo)planets have an orbital period of just a few days, while the spin period of a star is generally a few weeks : they are expected to be eventually swallowed by their parent star.



APPLICATION 2 : ORBITAL EVOLUTION

Expression of the migration rate :

$$\frac{dr}{dt} = \frac{3 k_{2\,planet} \sqrt{G} R^5_{planet}}{Q_{planet} \sqrt{M_{planet}}} \frac{M_{satellite}}{r^{11/2}}$$

depends on the central body

depends on the orbiting body

where k_2 is the Love number,

Q is the dissipation factor,

and $k_2/Q = 2 \times 10^{-4}$ for Saturn (Lainey et al. 2012)

$k_2/Q = 0.025$ for the Earth.

APPLICATION 2 : ORBITAL EVOLUTION

Definition and importance of the synchronous orbit :

The synchronous orbit is the distance r_{sync} from a body where the orbital period is equal to the spin period of the body.

Beyond r_{sync}, $dr/dt>0$. Inside r_{sync}, $dr/dt<0$.

Exercice: Compute r_{sync} for the Earth.

Solution :

$r_{sync}^3 = (GM_{Earth}/4\pi^2)\, P^2$, where P = 23h 56min 4s = 86164 s.

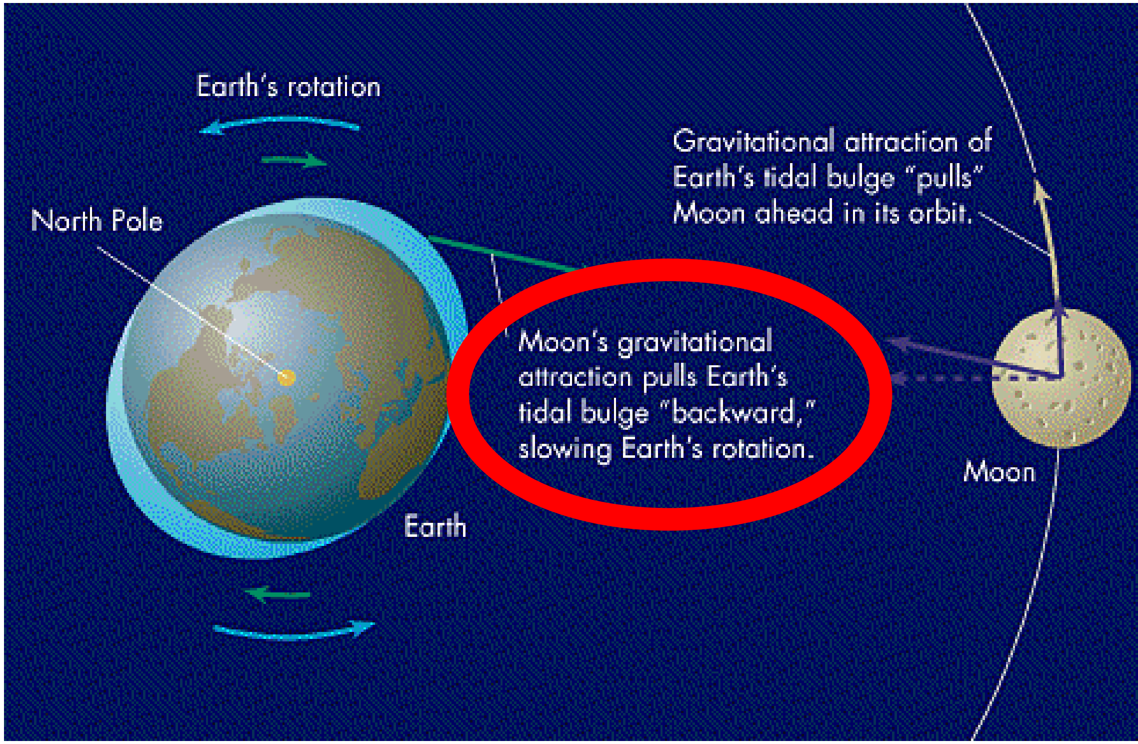
r_{sync} = 42 200 km.

Altitude of geostationary satellites : 42200-6400=35800 km.

APPLICATION 3 : SYNCHRONOUS ROTATION

When a body spins faster or slower than it rotates around the other body, it feels a torque pulling its spin backwards.

It slows down. The tides period has been measured to be smaller in coral fossiles, ~360 Myrs ago, hence the spin period of the the Earth (and the rotation period of the Moon).



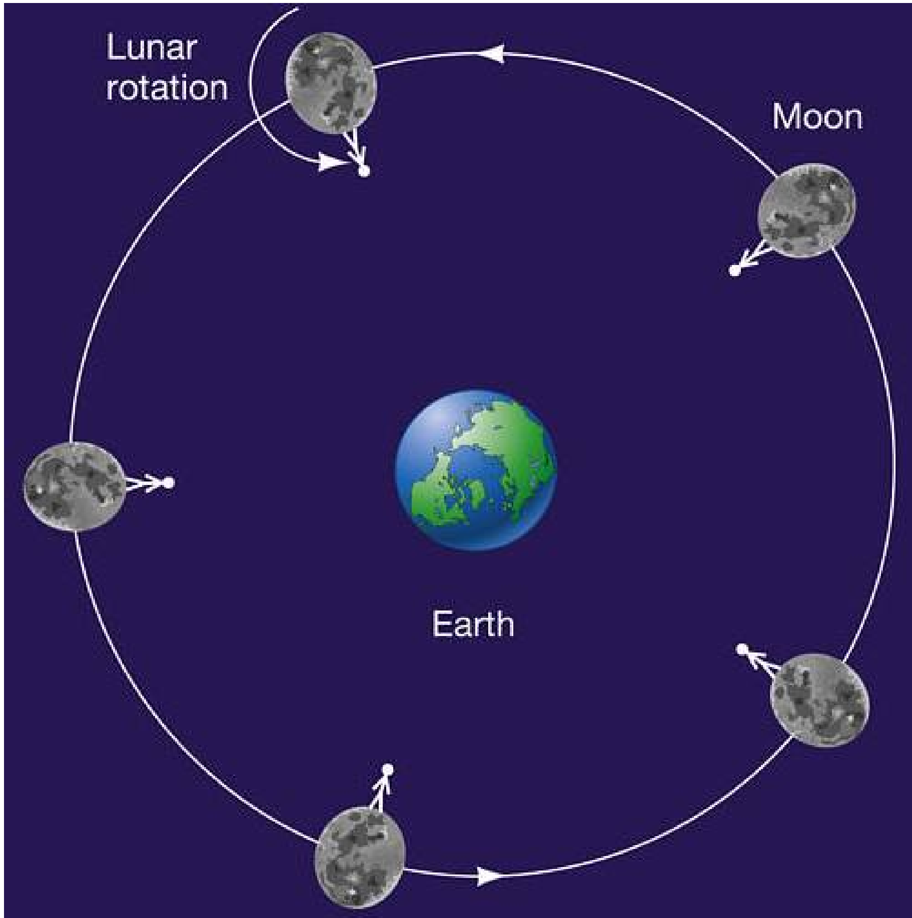
APPLICATION 3 : SYNCHRONOUS ROTATION

When a body spins faster or slower than it rotates around the other body, it feels a torque pulling its spin backwards.

It tends to a final state where the spin rate matches the rotation rate, called : ***synchronous rotation***.

It is the case of the Moon around the Earth, and of almost all satellites in the Solar System.

Most hot Jupiters are also supposed to be tidally locked, synchronsied.



EXERCICE : THE EARTH – MOON SYSTEM

1) Using this equation → compute k_2/Q for the Earth

$$\frac{dr}{dt} = \frac{3\, k_{2\,planet} \sqrt{G}\, R^5_{planet}}{Q_{planet} \sqrt{M_{planet}}} \frac{M_{satellite}}{r^{11/2}}$$

$M_{Earth} = 6 \times 10^{24}$ kg , $R_{Earth} = 6400$ km

$M_{Moon} = 7.4 \times 10^{22}$ kg , $R_{Moon} = 1700$ km , $P_{Moon} = 27$d 7h 43m

$G = 6{,}67 \times 10^{-11}$ S.I. , $r = 384000$ km, $dr/dt = 3{,}8$ cm/yr

2) Compute the orbital angular momentum of the Moon, and the spin angular momenta of the Earth and the Moon, and the total angular momentum of the system, L_{tot} .

3) Express the Earth's spin rate ω_E as a function of L_{tot} and r.

4) Deduce $d\omega_E/dt$ now.

5) Solve the equation of $r(t)$.
Where was the Moon 4.5 Gyr ago ? Where will it be in 5 Gyr ?

6) Plot Ω_{Moon} and ω_E as a function of r on the same graph.

SOLUTION : THE EARTH – MOON SYSTEM

1) **k_2/Q = 0.025** for the Earth (no dimension)

2) $L_{M\ orbital} = M_M (GM_E r)^{1/2} = 2.9 \times 10^{34}$ kg.m^2.s^{-1}
$L_{E\ spin} = 0.4\ M_E R_E^2 \omega_E = 7.17 \times 10^{33}$ kg.m^2.s^{-1}
$L_{M\ spin}$ is negligible. **$L_{tot} = 3.62 \times 10^{34}$ kg.m^2.s^{-1}**.

3) **$\omega_E = [\ L_{tot} - M_M (GM_E r)^{1/2}\ ] / (0.4\ M_E R_E^2)$**

4)
$$\frac{d w_E}{dt} = \frac{-5 M_M}{4 R_E^2} \sqrt{\frac{G}{M_E}} \frac{dr}{dt} \frac{1}{\sqrt{r}}$$

$d\omega_E/dt = -\ 4.6 \times 10^{-22}$ s^{-2} = -1.5×10^{-14} rad.s^{-1}/year

NB: $dP_E/dt = (-2\pi/\omega_E^2)(d\omega_E/dt) = 5.4\times10^{-13}$ = 1.7 ms/century.

5) $r^{11/2}\,dr = K\,dt$ -> **$r(t) = [13K/2\ (t+t_0)\ ]^{2/13}$** , where $K=2.0\times10^{38}$SI

Taking t=0 now when r=384000 km gives $t_0=5\times10^{16}$s=1.5 Gyr ?

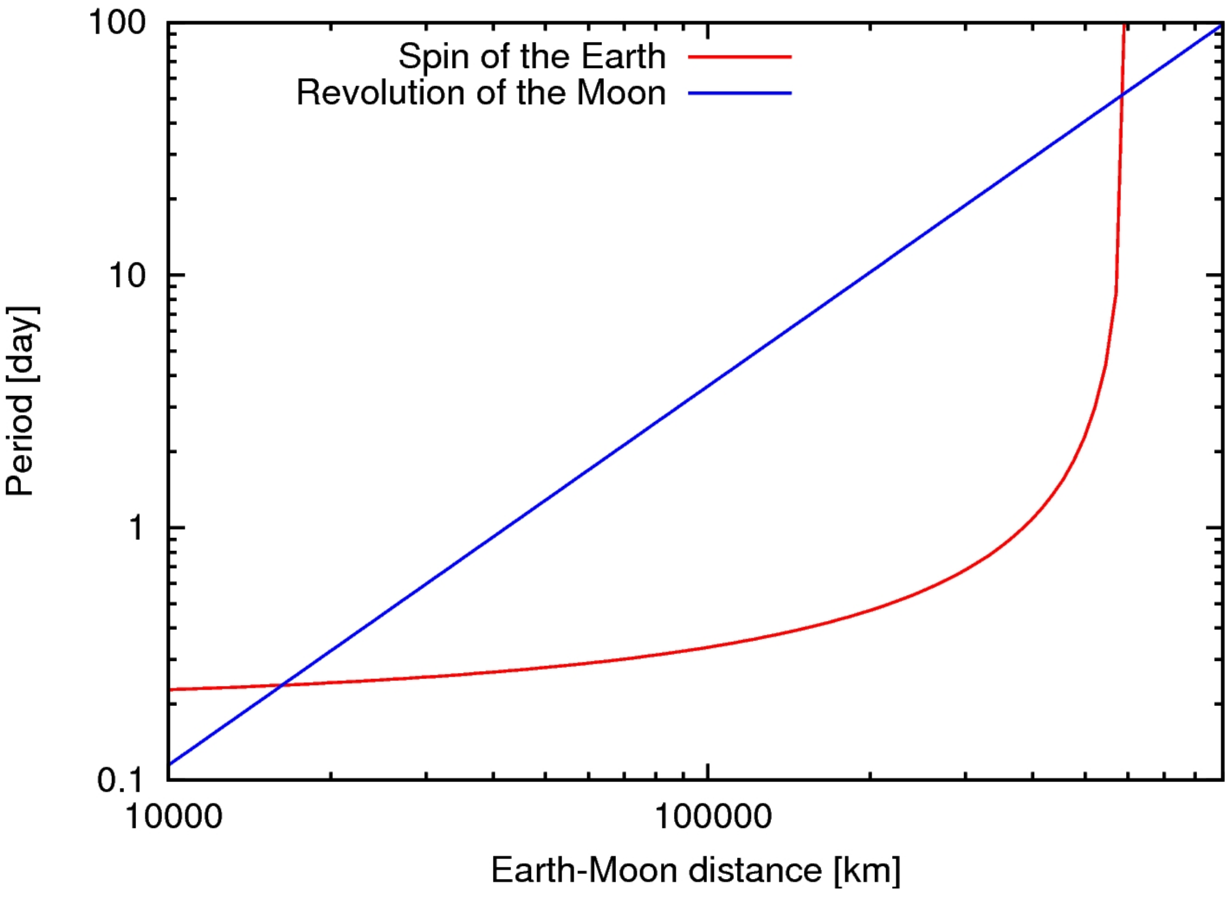
SOLUTION : THE EARTH – MOON SYSTEM

6) We see that there are two double synchronous states, where the spin period of the Earth is equal to the rotation period of the Moon. The short period one is unstable, while the long period one is stable.

There is no way out
16230 < r < 594338

In particular, the Moon was never in contact with the Earth's surface.

In the final state,
r = ~ 600 000 km,
P = ~52 days.



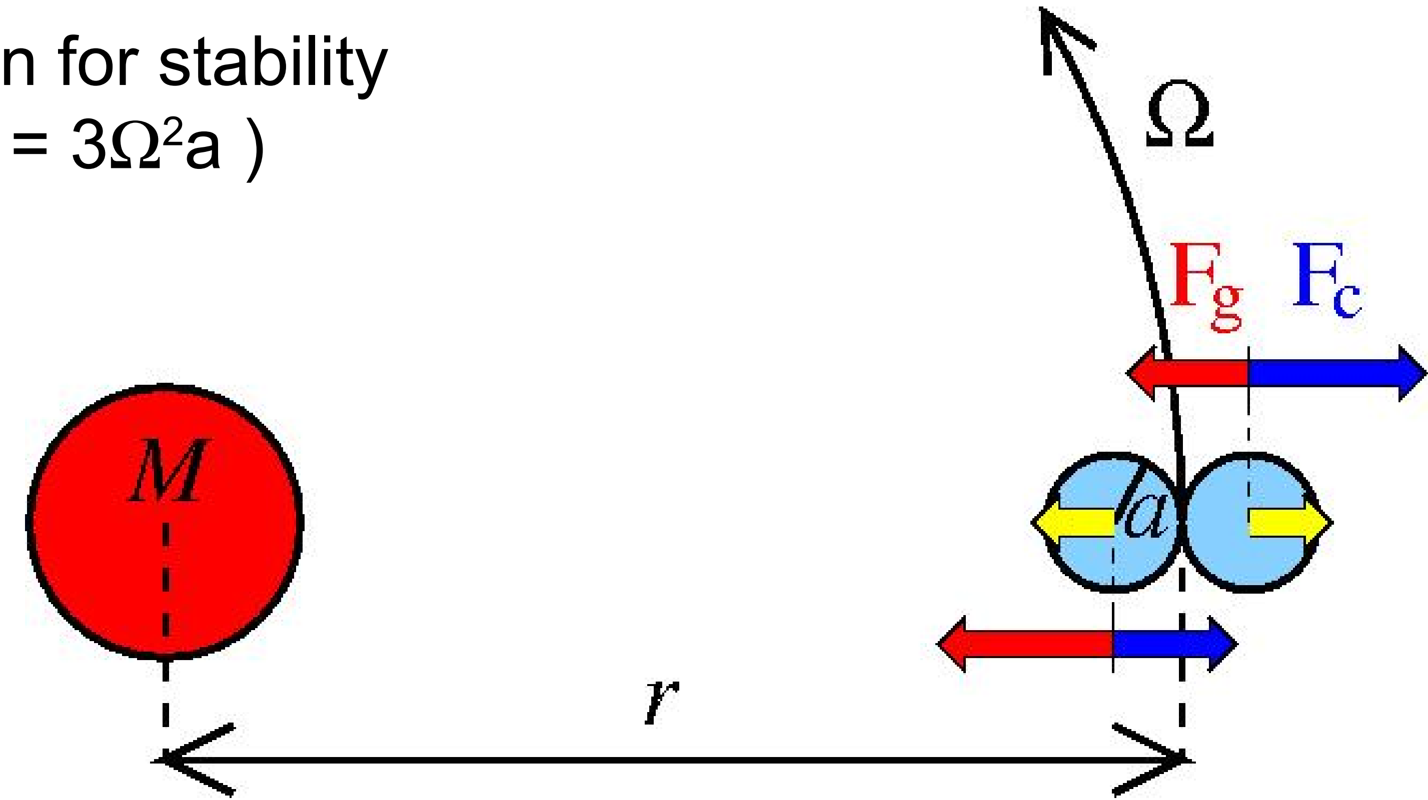
APPLICATION 4: STABILITY OF AGGREGATES

Self-gravity force of the two bodies (per mass unit) :

$F_{sg} = G^*(4/3)\pi\rho a^3 / (2a)^2$

Condition for stability of the aggregate : $F_{sg} > F_m$,

Find a criterion for stability
(reminder: $F_m = 3\Omega^2 a$)



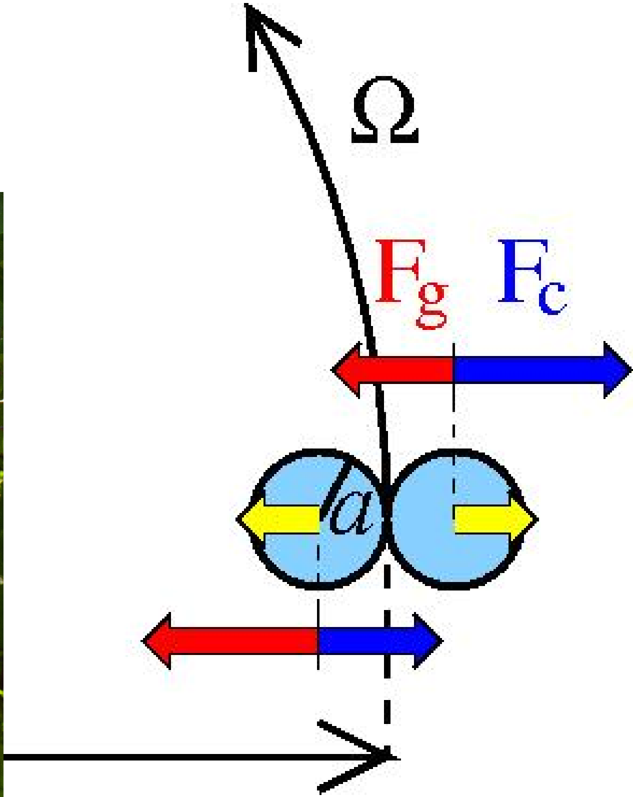
THE ROCHE RADIUS

Self-gravity force of the two bodies (per mass unit) :

$F_{sg} = G^*(4/3)\pi\rho a^3 / (2a)^2$

Condition for stability of the aggregate : $F_{sg} > F_m$,

or : $r > (9M/\pi\rho)^{1/3} = r_{Roche}$



THE ROCHE RADIUS

Application:

$M = M_{Saturn}$,

$\rho = 600$ kg.m^{-3}

$r_{Roche} = 1,4\ 10^8$ m = 140 000 km

Saturn's rings are inside their Roche radius.

This is why the boulders don't collapse, aggregate, and eventually form one large satellite.

Movie by Hanno REIN,
in a shearing box,
with period boundary conditions at top and bottom.

THE ROCHE RADIUS

But what happens when the rings spread beyond the Roche radius ?

The tidal forces are weaker than the self gravity, and the boulders aggregate to form new satellites...

And what happens if a satellite migrates inside the Roche radius ?

The tidal forces are so strong that every stone at its surface is taken off, and the satellite is destroyed into small pieces...

TIDES, RINGS, SATELLITES, Saturn et al...

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1) <u>Origin of the rings</u>

Canup 2010, Nature.
(+ News and Views Crida & Charnoz)

2) <u>Evolution of the rings</u>

Salmon, Charnoz, Crida, Brahic 2010, Icarus.

3) <u>Satellites children of the rings</u>

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Charnoz, Crida, et al., 2011, Icarus.
Crida & Charnoz, 2012, Science.

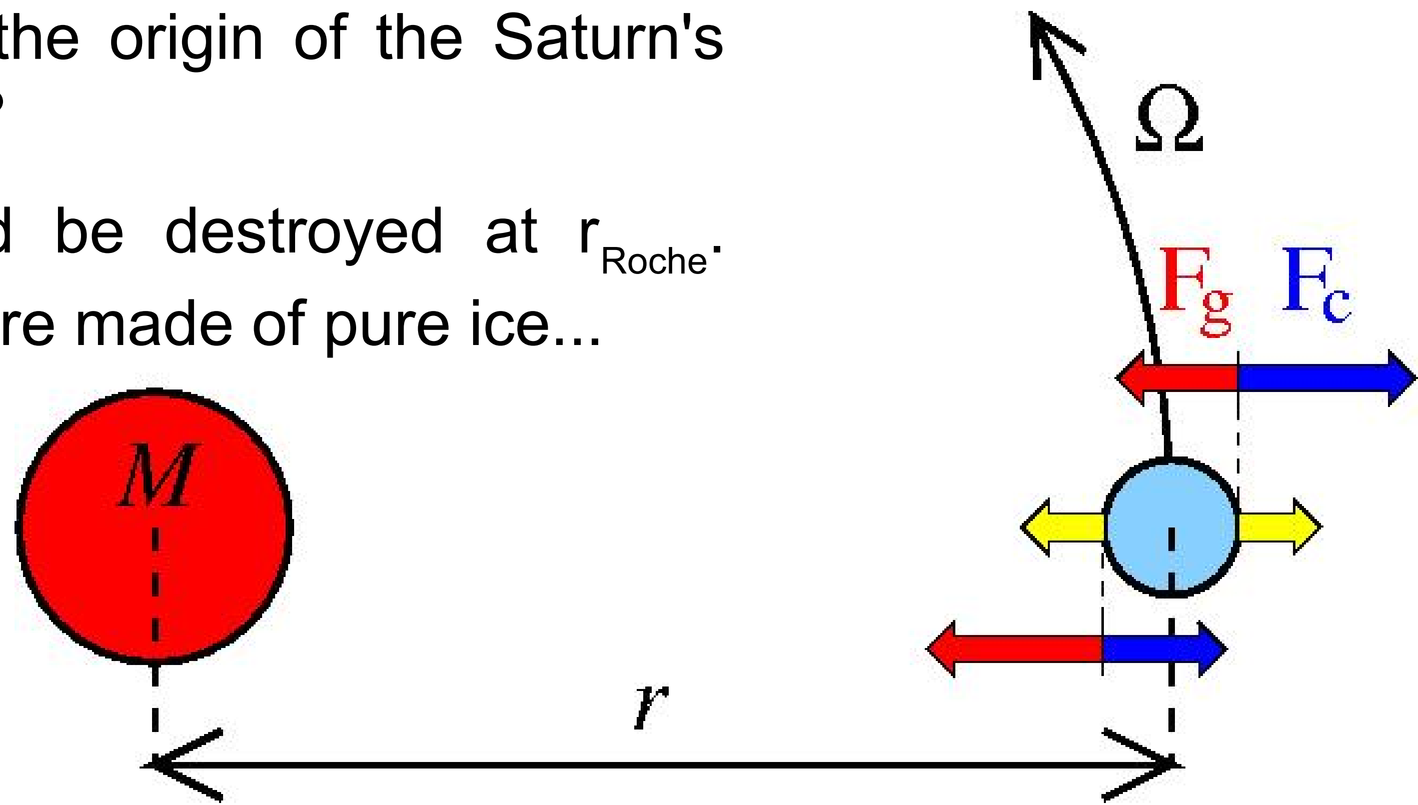
RINGS CHILDREN OF A SATELLITE

What happens to a satellite input inside the Roche radius ?

If homogeneous of density ρ such that $r < r_{Roche}(\rho)$, the satellite is dislocated. It should be destroyed into small chunks that are bound by internal stress forces.

Could this be the origin of the Saturn's rings material ?

Wait, it should be destroyed at r_{Roche}. And the rings are made of pure ice...



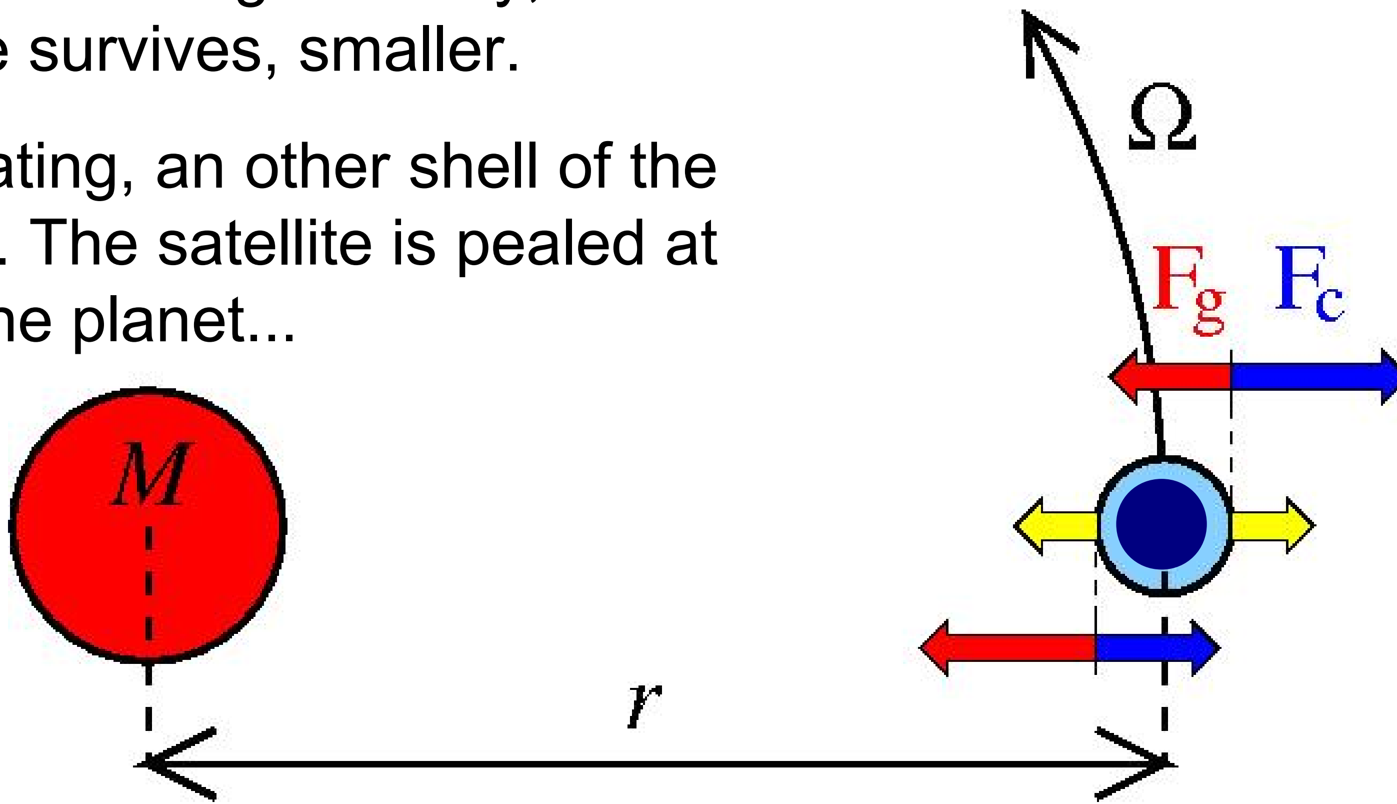
RINGS CHILDREN OF A SATELLITE

What happens to a **differentiated** satellite inside r_{Roche} ?

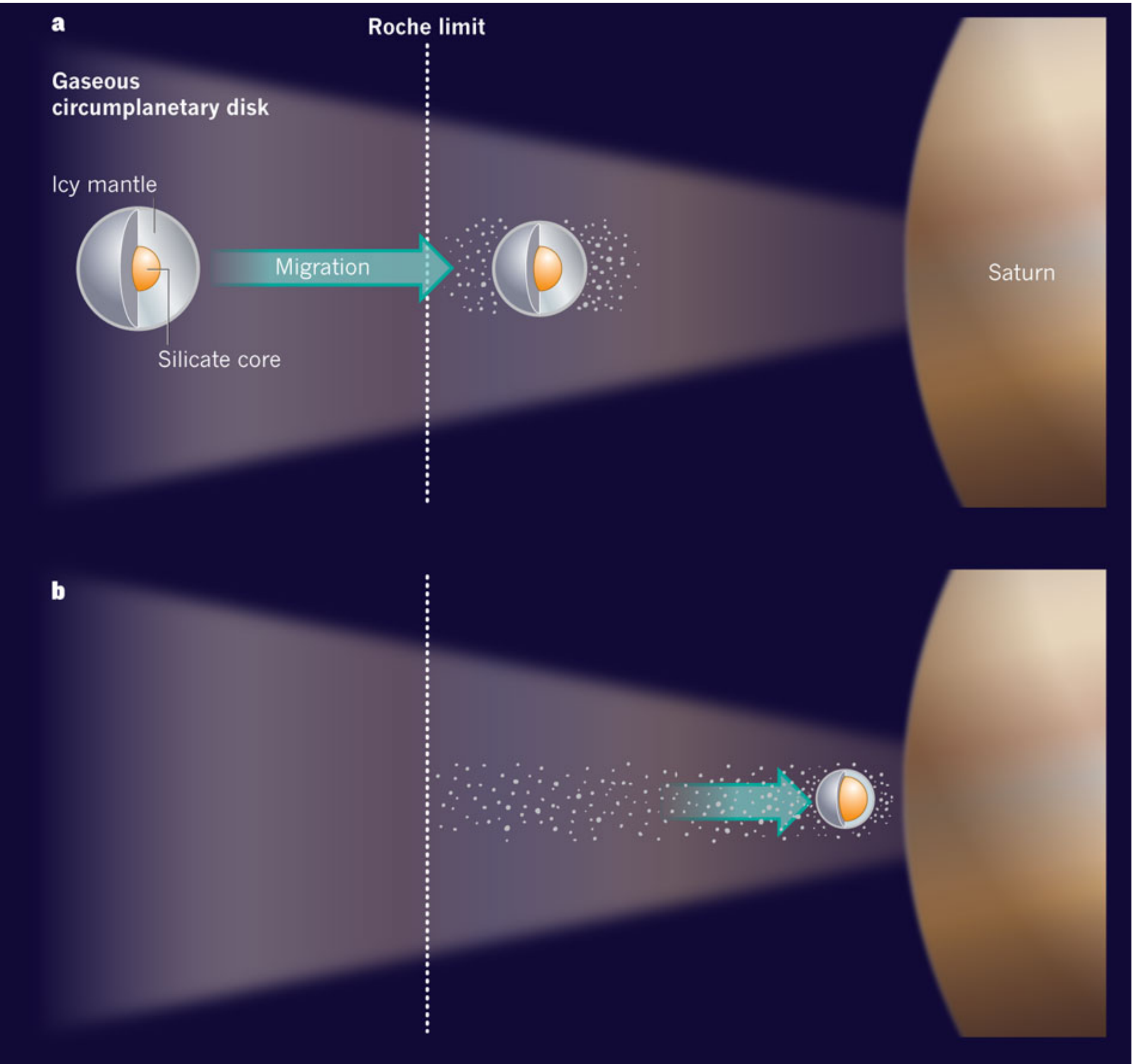
Its underdense mantle is first stripped off by the tidal forces.

This increases its average density, and the satellite survives, smaller.

If it keeps migrating, an other shell of the mantle is taken. The satellite is pealed at it approaches the planet...



RINGS CHILDREN OF A SATELLITE



Idea : a massive, differenciated satellite with

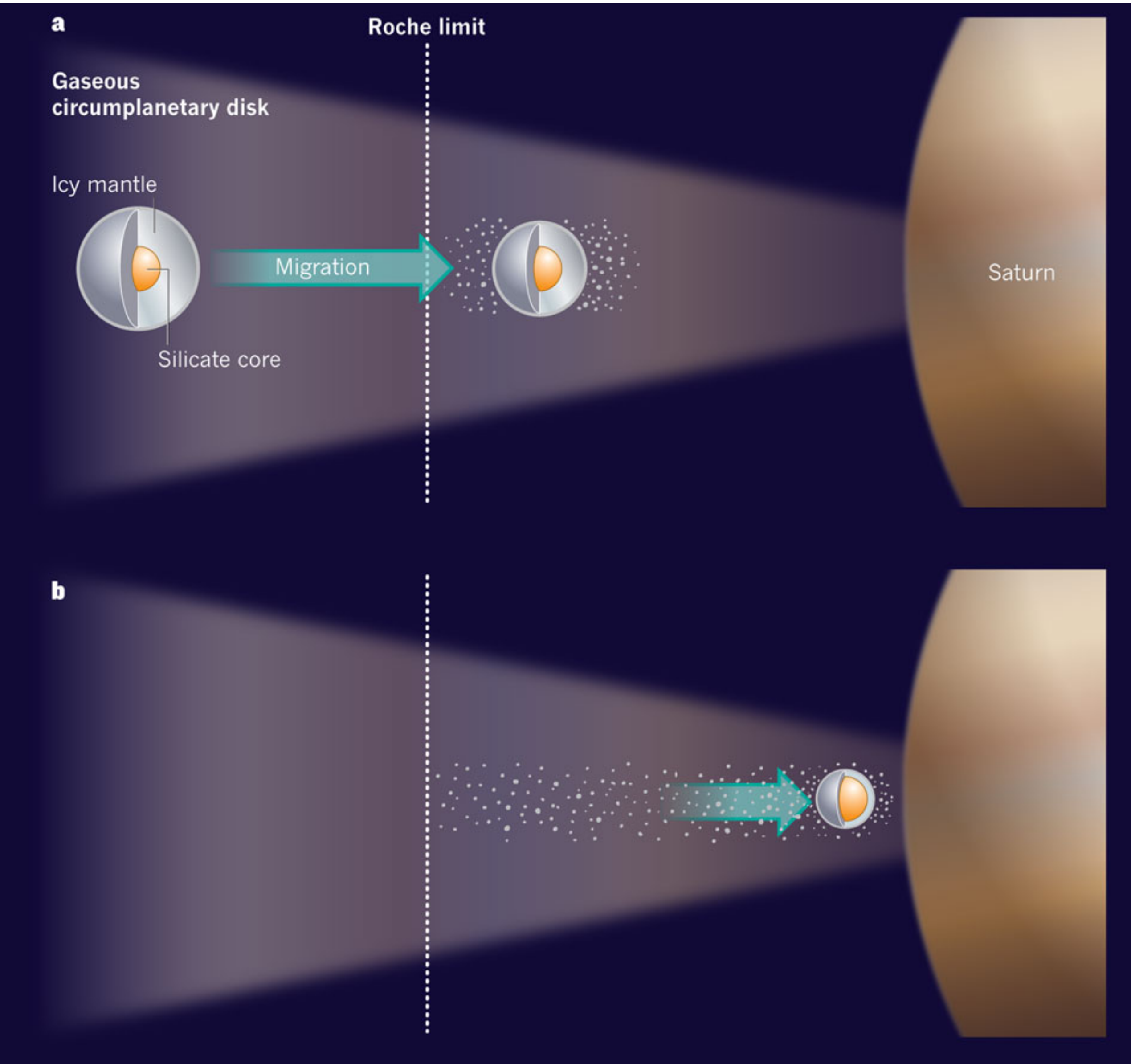
- icy mantle
- silicates core

migrates toward Saturn,
crosses the Roche limit,

loses progressively its mantle,

and the core finally falls into Saturn.

RINGS CHILDREN OF A SATELLITE



We are left with a pure ice ring, because

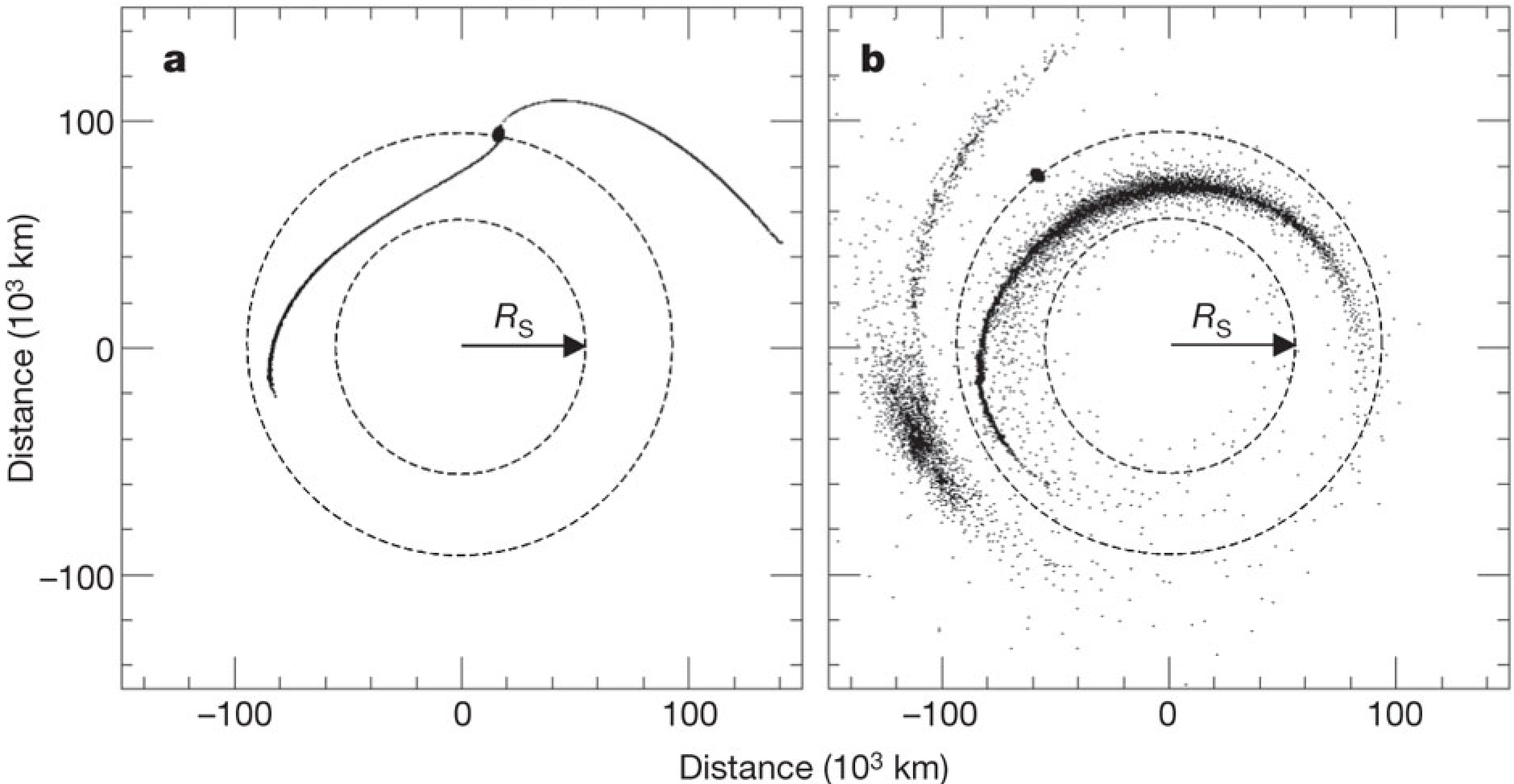
$r_{Roche}(\rho(\text{silicate})) < R_{saturn}$ at time of Saturn's formation :

the core disappears before its destruction by tides.

RINGS CHILDREN OF A SATELLITE

Canup (2010, Nature) : SPH simulations of the process.

+ test of the survival of the ice debris over gas drag and temperature.



RINGS CHILDREN OF A SATELLITE

Key points of this model :

- **differenciated** satellite => pure water ice

- **migration** => progressive pealing off
  + get rid of the core

- **last** big satellite lost => the debris stay there.

In the end :

- a **very massive** ring (~Titan's mantle),

- formed **4.5 10^9 years ago**.

- no satellite left between the rings and Titan

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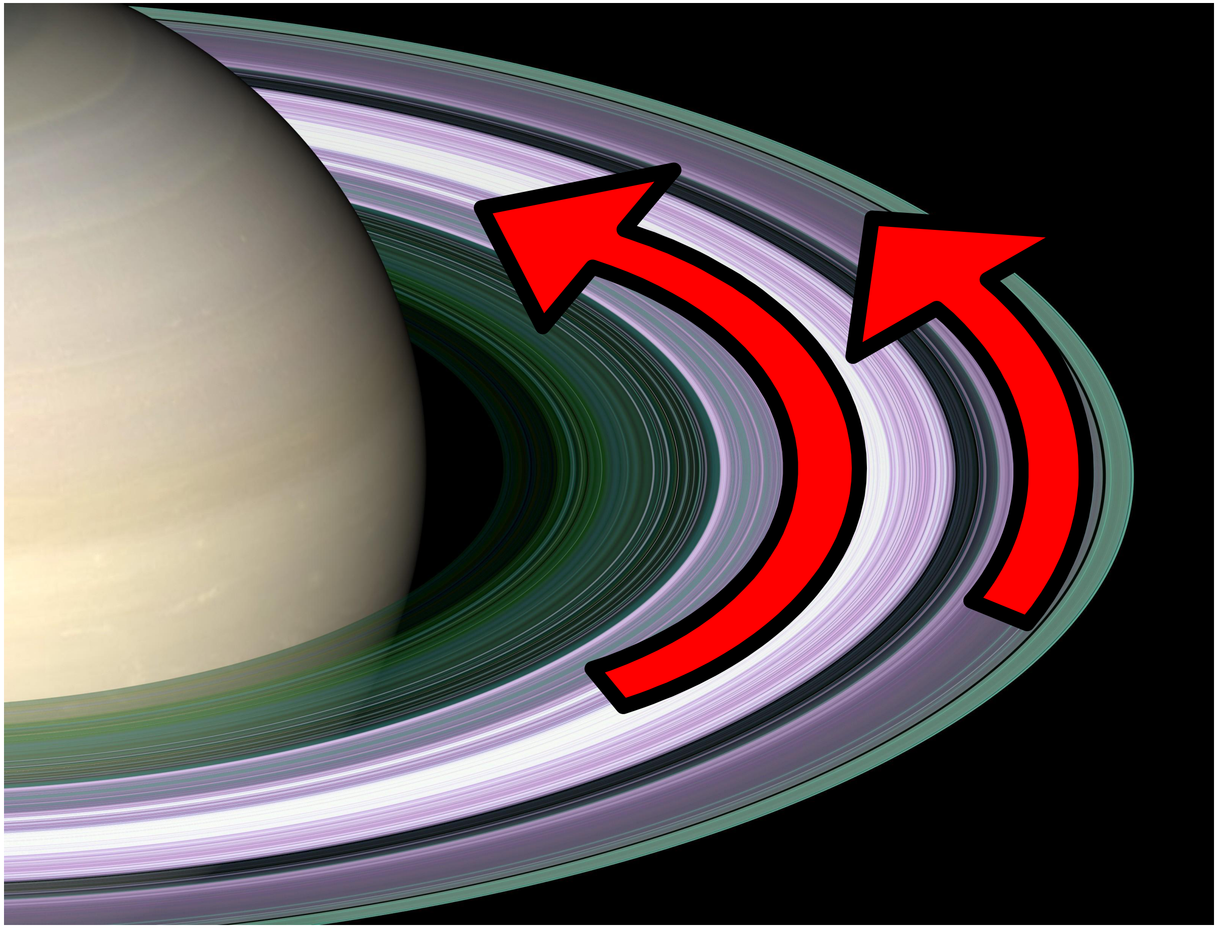
Crida & Charnoz, 2012, Science.

EVOLUTION OF THE RINGS

Any astrophysical disk in Keplerian rotation spreads by viscous friction (eg. Lynden-Bell & Pringle 1974).



The inside rotates faster than the outside,

so friction accelerates the outside (thus going further),

and slows down the inside (thus falling).

Total: spreading.

EVOLUTION OF THE RINGS

Mass conservation :

$$r\frac{\partial\,\Sigma}{\partial\,t}+\frac{\partial}{\partial\,r}\left(r\,\Sigma\,v_r\right)=0$$

Angular momentum conservation :

$$\frac{\partial}{\partial\,t}\left(\Sigma\,r^2\,\Omega\right)+\frac{1}{r}\frac{\partial}{\partial\,r}\left(\nu\,\Sigma\,r^3\,\frac{\partial\,\Omega}{\partial\,r}\right)=0$$

Thus density evolution :

$$\frac{\partial\Sigma}{\partial\,t}=\frac{3}{r}\,\frac{\partial}{\partial\,r}\left[\sqrt{r}\,\frac{\partial}{\partial\,r}\left(\nu\,\Sigma\sqrt{r}\right)\right]$$

EVOLUTION OF THE RINGS

Viscosity in the rings (Daisaka et al. 2001) :

$\nu = \nu_{coll} + \nu_{trans} + \nu_{grav}$.

Parametre Q of Toomre : $Q = \Omega\, \sigma_r / (\, 3{,}36\; G\, \Sigma\,)$.

where σ_r = radial velocity dispersion of the particles.

ν_{coll} independent of Q.

$\nu_{grav} = 0$ if $Q>2$, $\nu_{grav} = \nu_{trans}$ if $Q<2$.

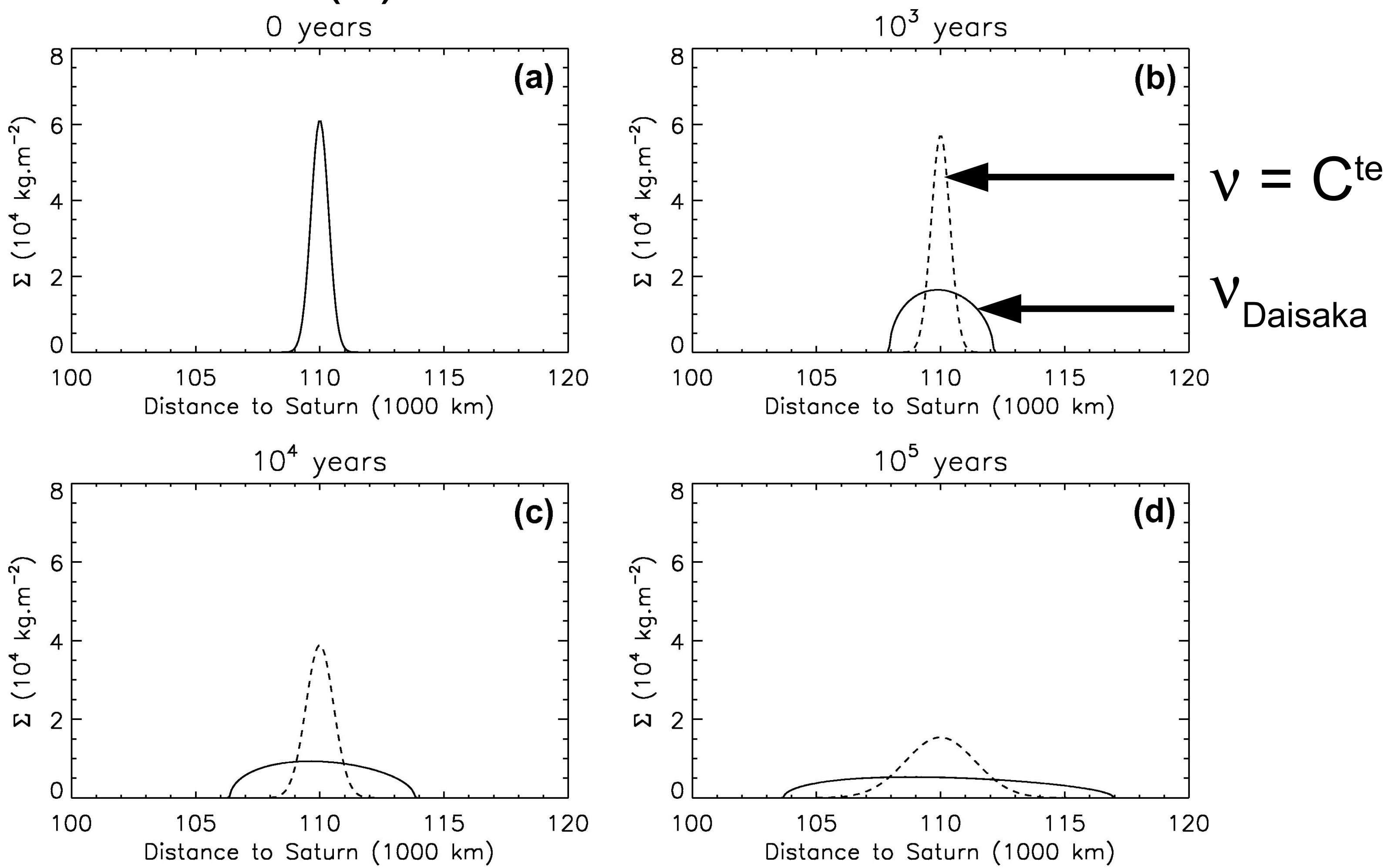
$\nu_{grav} = 26\, r_H^*\, G^2\, \Sigma^2 / \Omega^3$ $(r_H^* = r_H/d)$

EVOLUTION OF THE RINGS

Implicit 1D code solving :

$$\frac{\partial \Sigma}{\partial t} = \frac{3}{r}\frac{\partial}{\partial r}\left[\sqrt{r}\frac{\partial}{\partial r}\left(\nu \Sigma \sqrt{r}\right)\right]$$

with or without $\nu(\Sigma)$...



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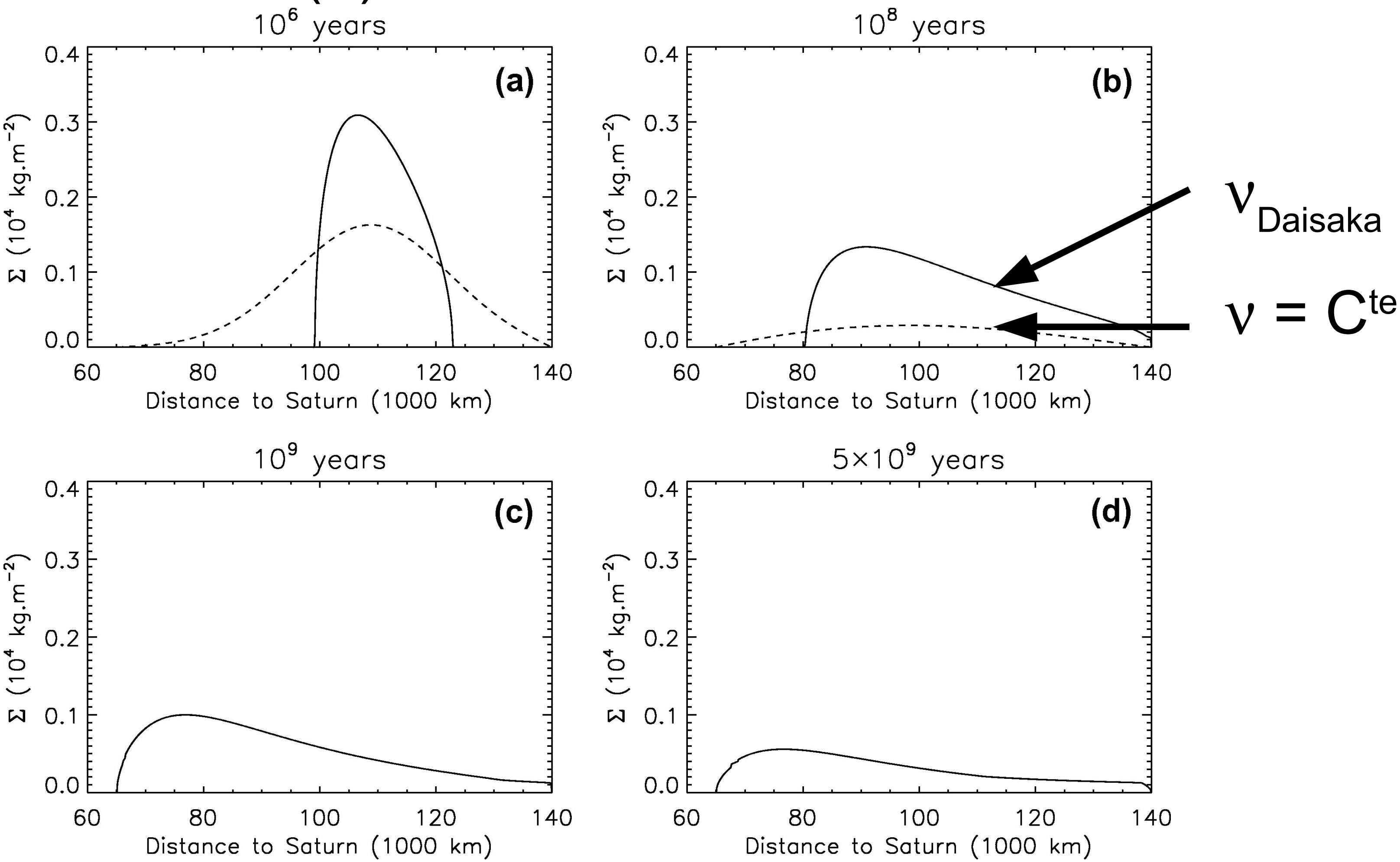
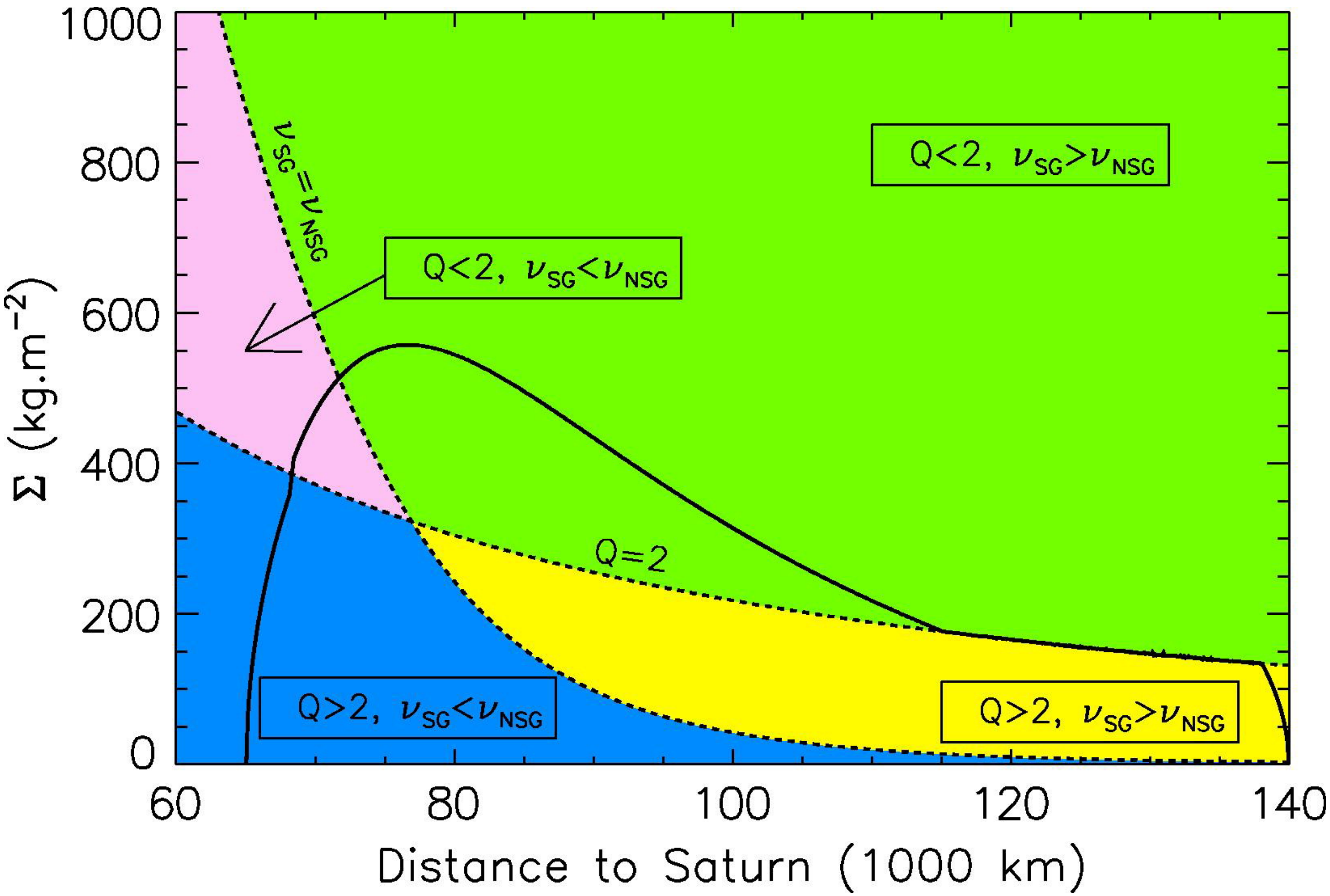


EVOLUTION OF THE RINGS

The spreading slows down as soon as Q>2 .



Convergence towards the density profile such that Q=2.

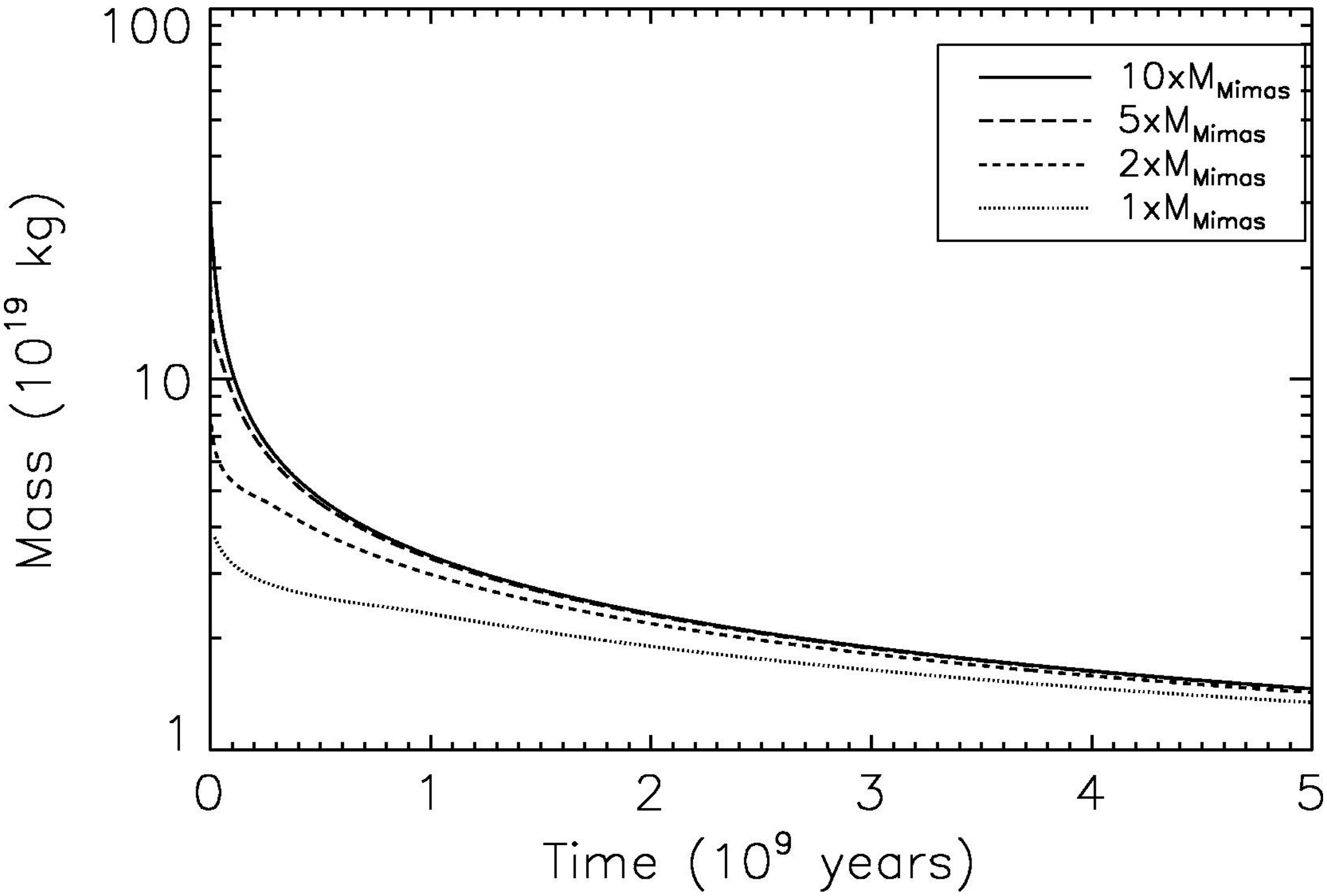
EVOLUTION OF THE RINGS

$$Q>2 \Rightarrow \nu_{grav} = \sim 46\, G^2\, \Sigma^2 / \Omega^3 \quad \text{at } r_R.$$

$$t_\nu = r_R^{\,2} / \nu = -\, M_{rings} / (dM_{rings}/dt)$$

Exercice : Find $M_{rings}(t)$. (NB: $M_{rings} = \pi\, r_R^{\,2}\, \Sigma$)

EVOLUTION OF THE RINGS



Whatever the initial mass of the rings, after 4.5 10^9 years, the final mass is about the present mass.

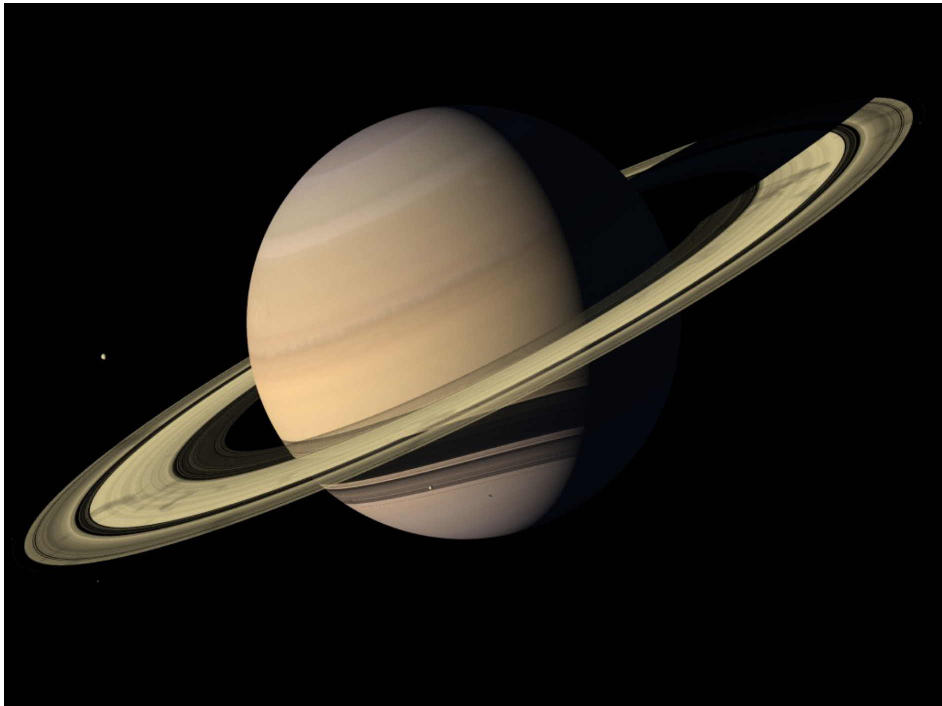
EVOLUTION OF THE RINGS

Conclusion :

The rings spread, their mass decreases with time.

But Daisaka et al. (2001)'s viscosity enable them to survive over the age of the Solar System, which was not possible with a constant viscosity.

Conversly, it is possible that the rings were much more massive in the past...



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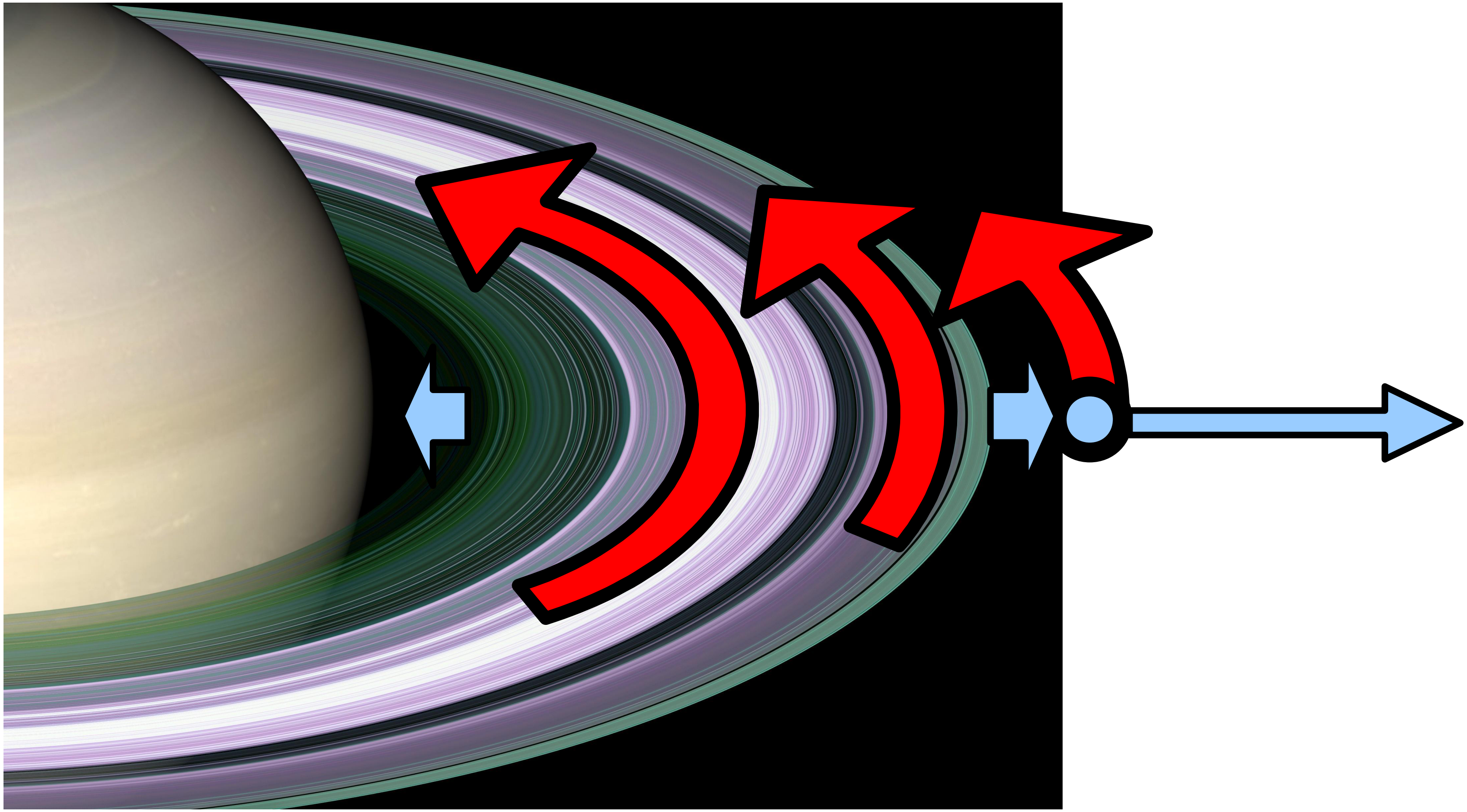
SATELLITES CHILDREN OF THE RINGS



The new satellites have a smaller angular velocity than the rings particles. Therefore, they are accelerated and repeled outwards...

SATELLITES CHILDREN OF THE RINGS

Total torque : $$\Gamma = \frac{8}{27}\left(\frac{M_{satellite}}{M_{Saturne}}\right)^2 \Sigma\, r^7 \Omega^2 \Delta^{-3}$$ Eq.(1)

proportionnal to $M_{\text{satellite}}^{2}$ and to Δ^{-3}.

The bigger satellites migrate outwards faster, the further you are, the more slowly you move.

(Lin & Papaloizou 1979)

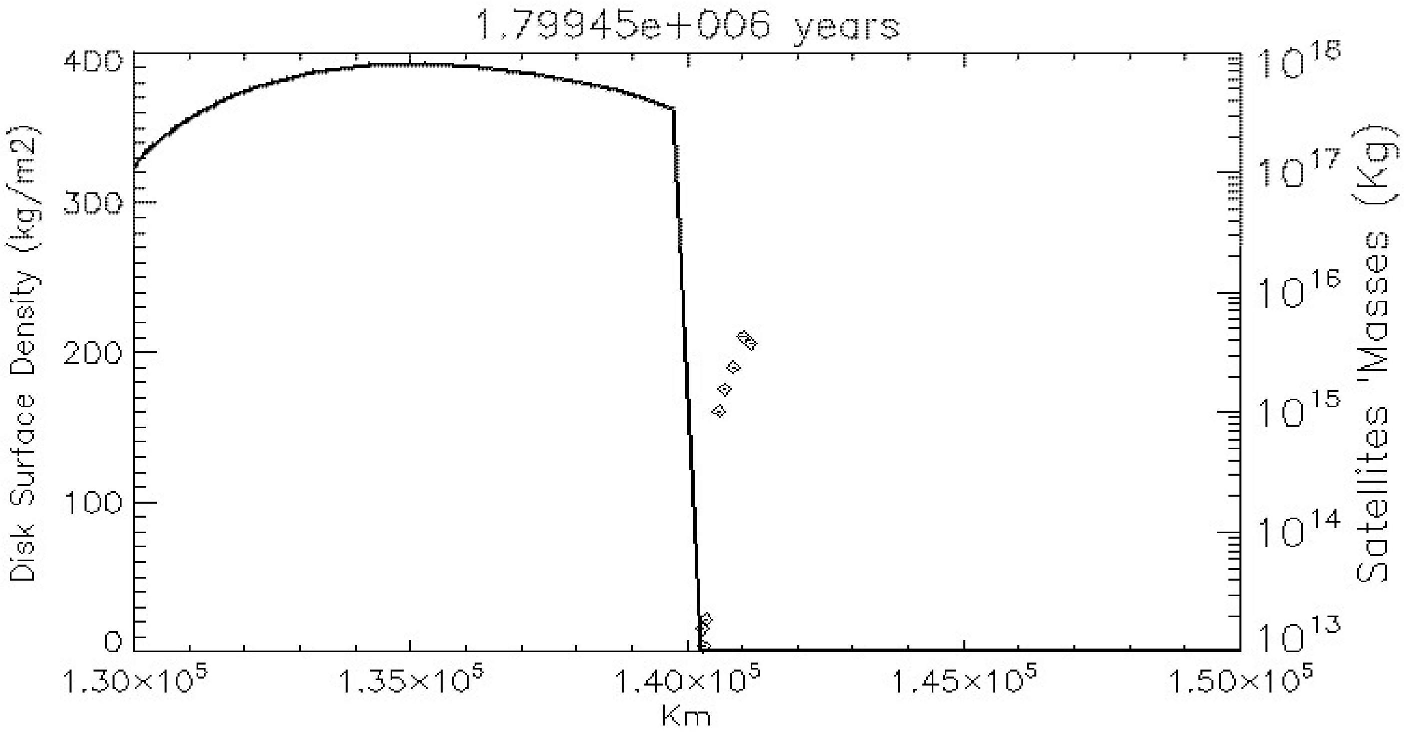
Numerical application :

~450 x 10^6 years ago, Janus was in the rings !

Janus is *dynamically young* .

SATELLITES CHILDREN OF THE RINGS

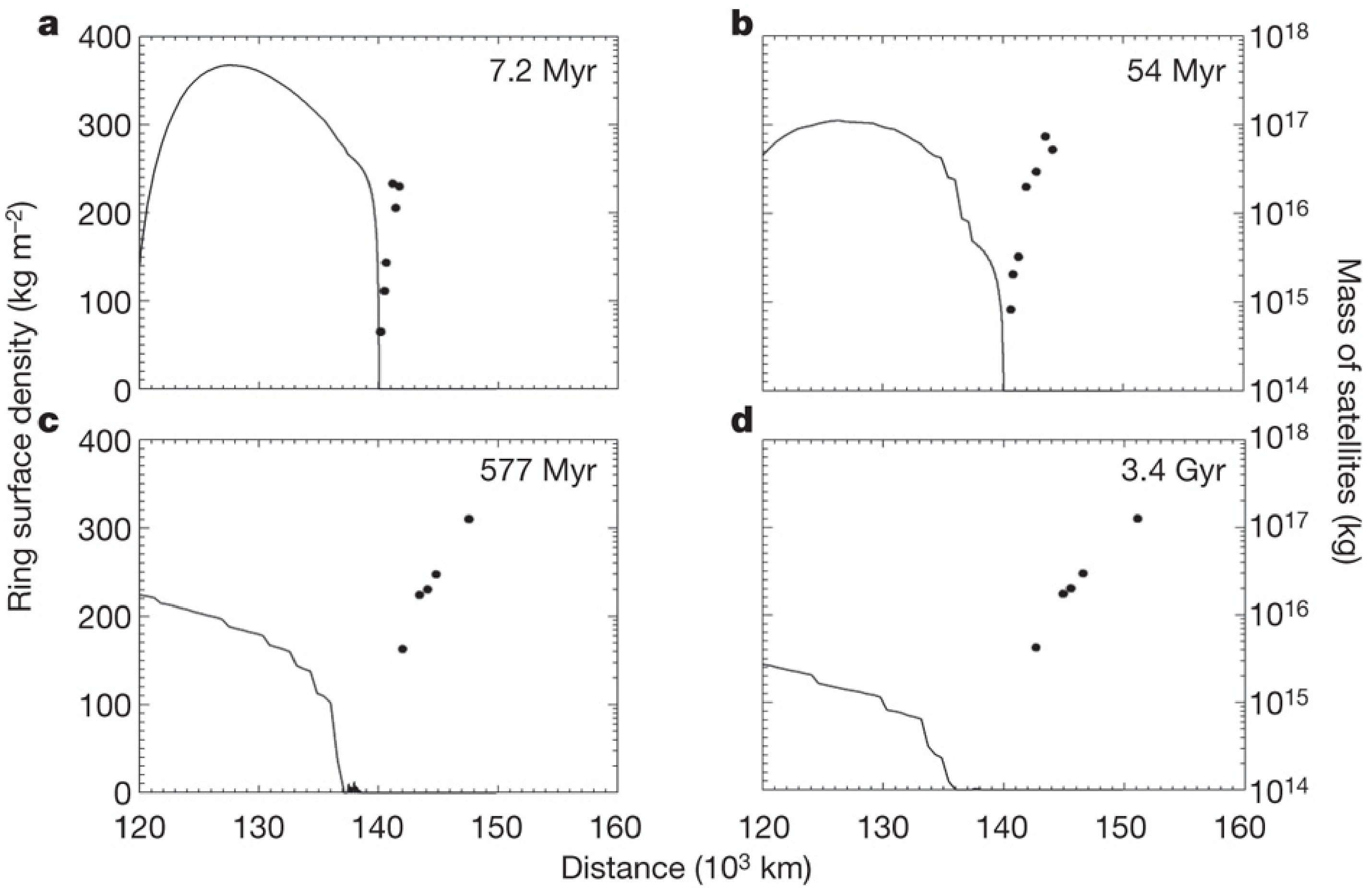
Numerical simulation with satellite formation beyond r_{Roche} :
(movie by S. Charnoz)



SATELLITES CHILDREN OF THE RINGS

Numerical simulation with satellite formation beyond r_{Roche} :

Formation of Prometheus, Pandora, Epimetheus, Janus.

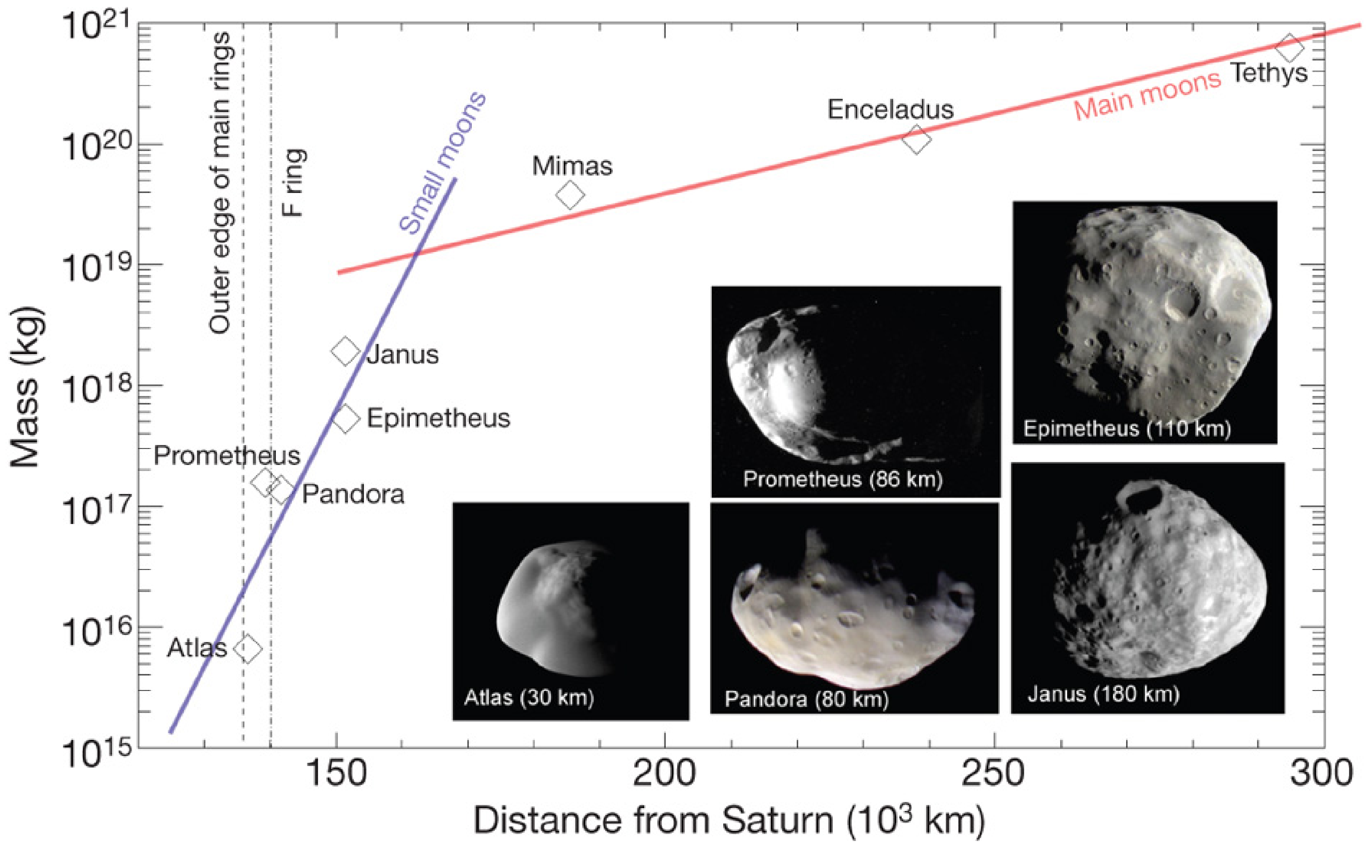


Charnoz, Salmon, & Crida (2010)

SATELLITES CHILDREN OF THE RINGS

This model explains that Janus, Epimetheus, Pandora, Prometheus, and Atlas are:

- underdense (~600 kg.m-3)
- same spectrum as the rings
- dynamically young
- young surfaces

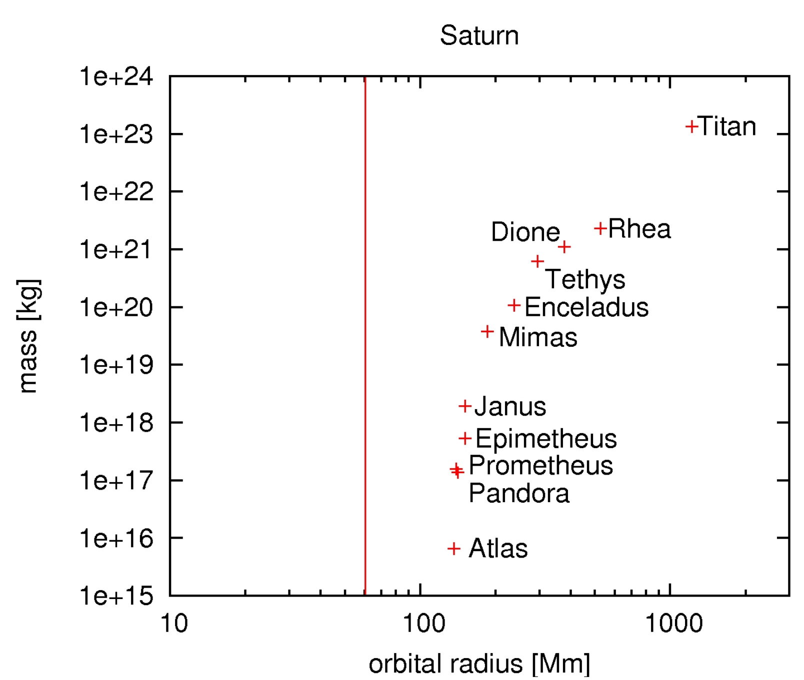


Charnoz, Salmon, & Crida (2010)

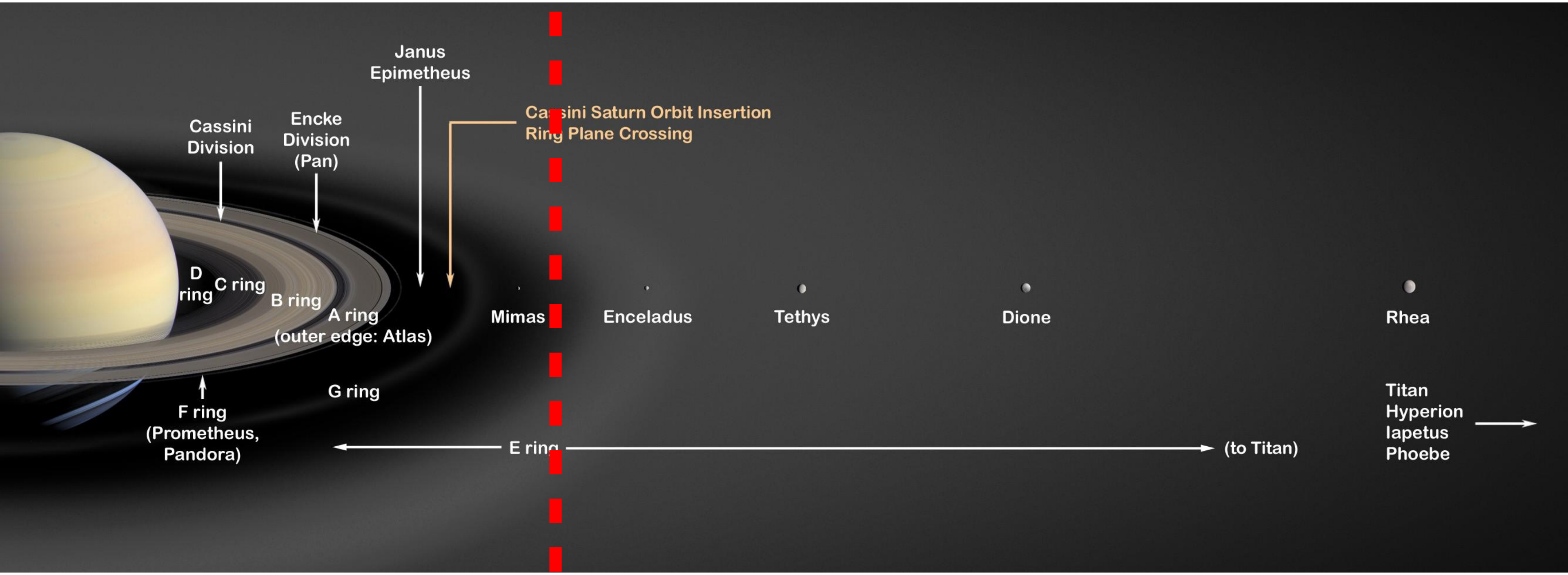
SPREADING OF MASSIVE RINGS

Total mass of the rings formed according to Canup (2010):
~ 10^{22} kg.

~ Mass of all satellites until and including Téthys !



SPREADING OF MASSIVE RINGS

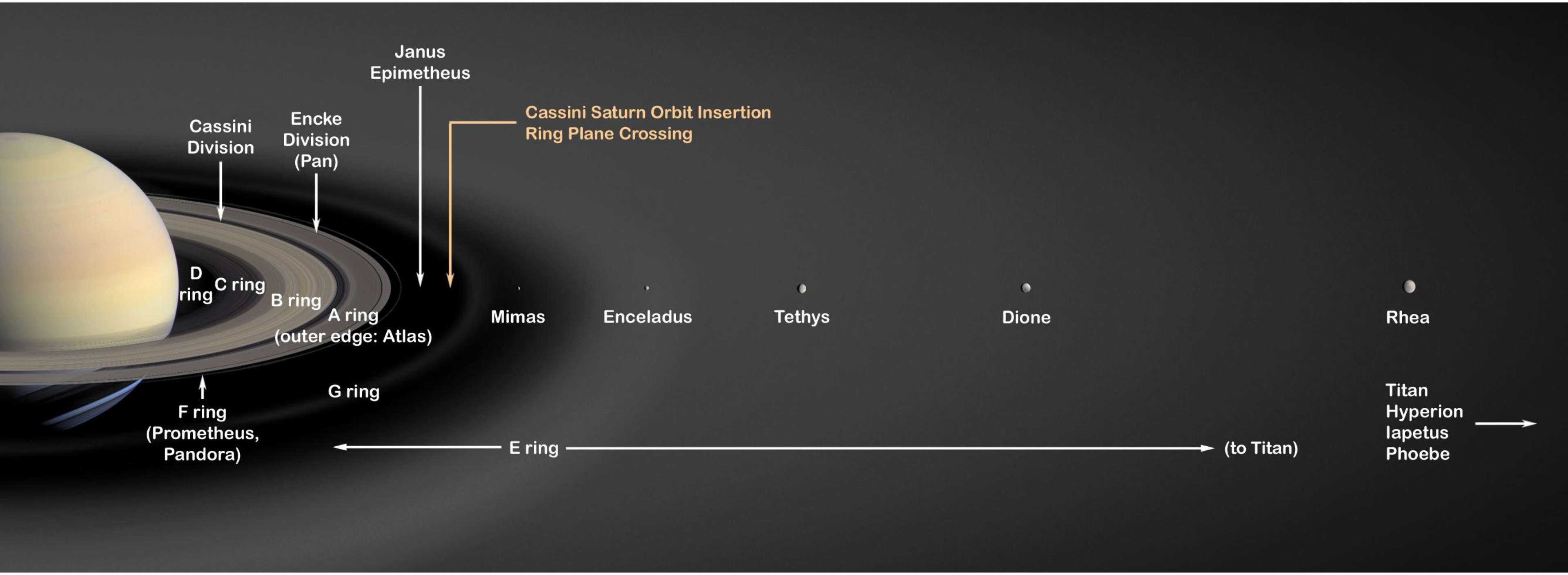


Problem :

Beyond 222 000 km, the interaction Eq(1) with the rings vanishes.

How to input Enceladus and Tethys at their present position ?

SPREADING OF MASSIVE RINGS



<u>Solution :</u>
tides from Saturne

$$\frac{dr}{dt} = \frac{3\, k_{2\,Saturn} \sqrt{G}\, R^5_{Saturn}}{Q_{Saturn} \sqrt{M_{Saturn}}} \frac{M_{satellite}}{r^{11/2}}$$

Too slow with $Q_{Saturn} \sim 10^4$.

But recent results suggest $Q_{saturn} \sim 1700$ (Lainey, et al., 2012).

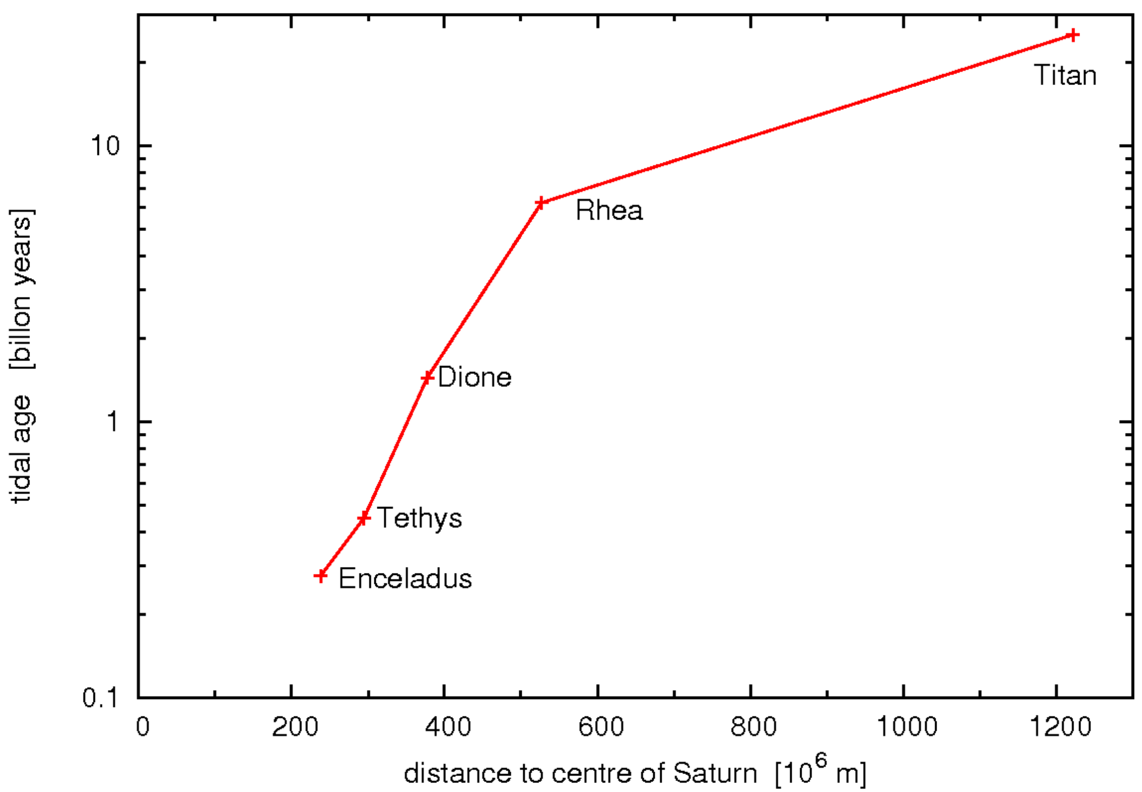
SATELLITES CHILDREN OF THE RINGS

Integrate back in time => **tidal age** of the satellites = time needed to reach their present position from 222 000 km.

- these ages are increasing with the distance to Saturn.

- Tethys is less than 10^9 years old.

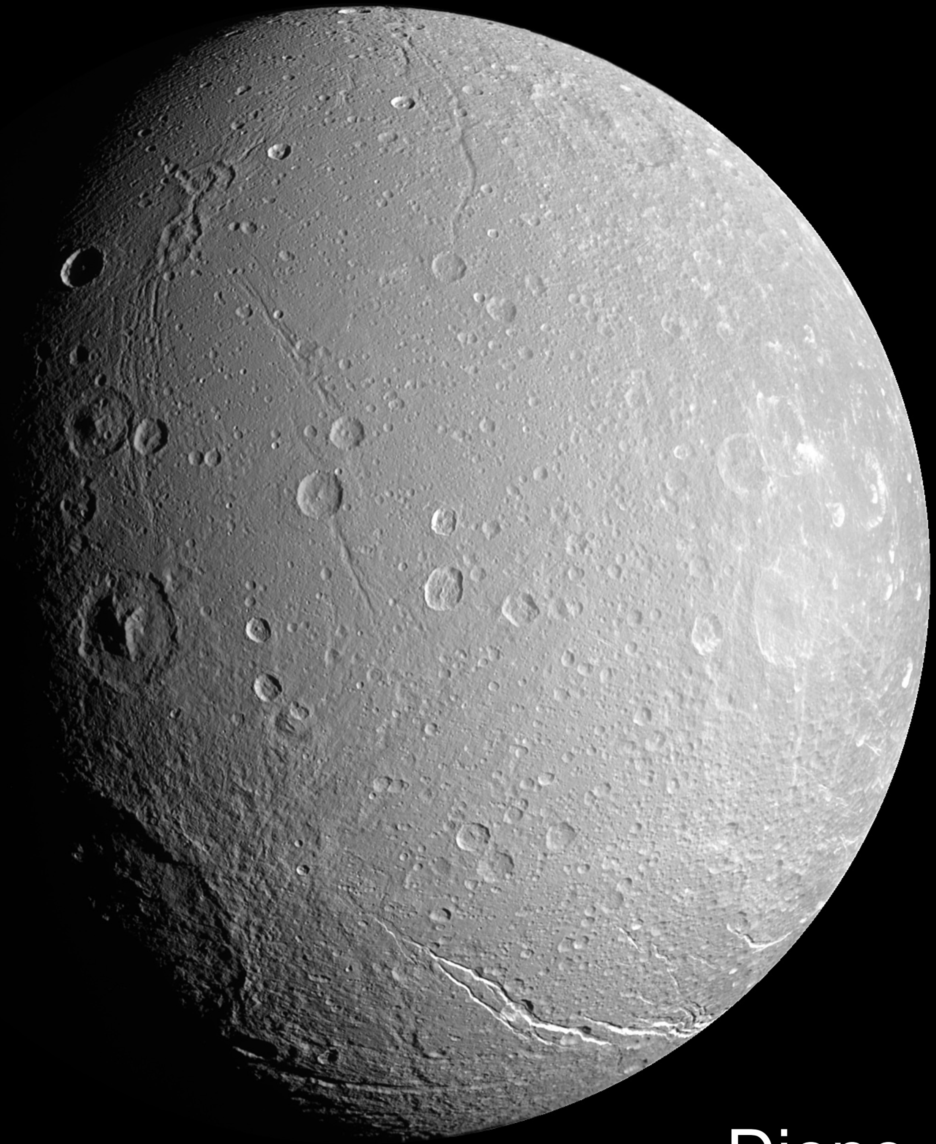
- even Rhea is younger than the Solar System.



SATELLITES CHILDREN OF THE RINGS

- These satellites are not much craterized

=> younger than the Solar System

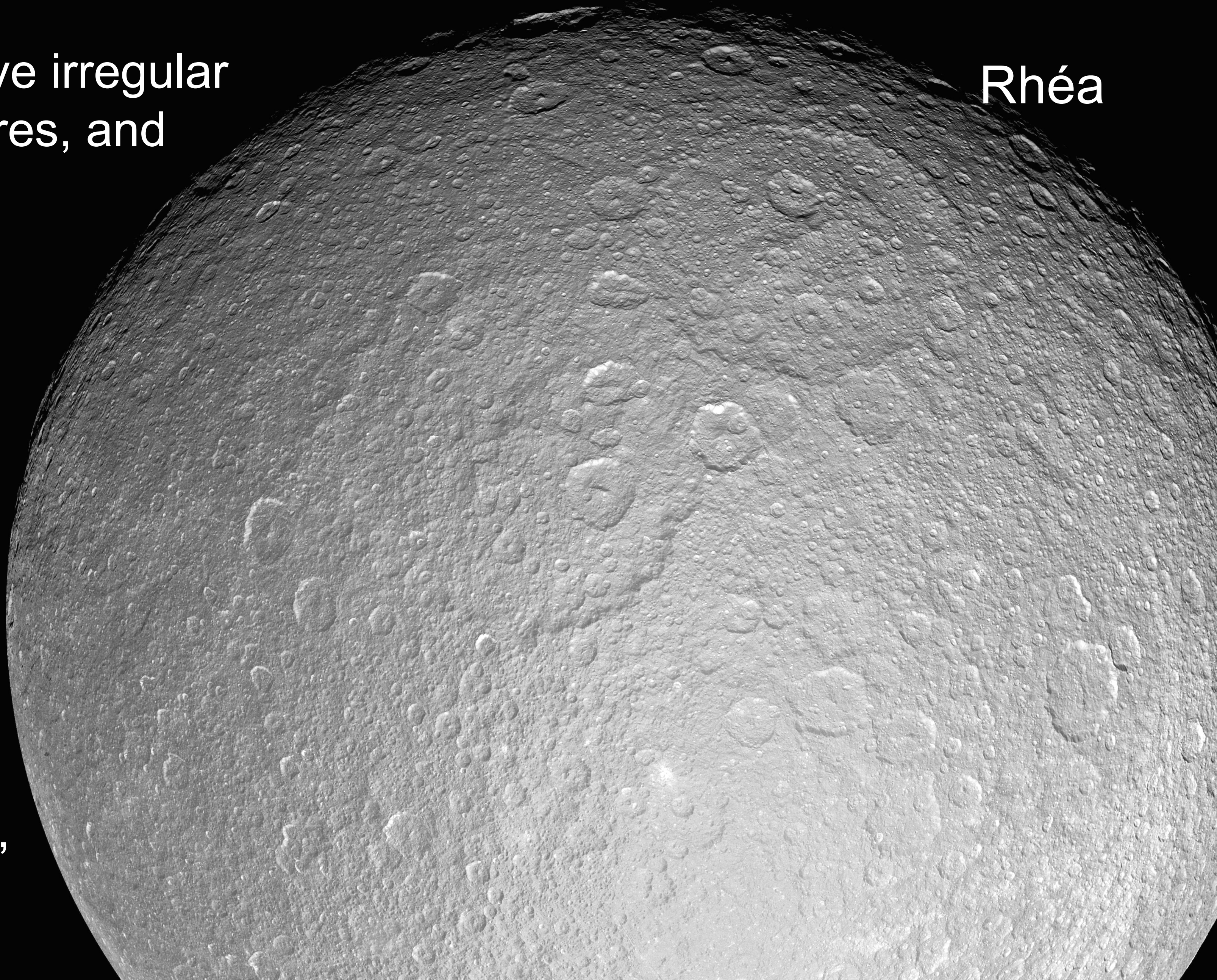


Dione

SATELLITES CHILDREN OF THE RINGS

They have irregular rocky cores, and irregular average compo- -sition :

Chunks of sili- cates initialy in the rings, coated with ice.



SATELLITES CHILDREN OF THE RINGS

All these facts favour this model.

Téthys

Charnoz, Crida, Castillo-Rogez, Lainey, et al, 2011, Icarus

Prix du magasine « La Recherche » 2012

SATELLITES CHILDREN OF THE RINGS

Charnoz, Crida, Castillo-Rogez, Lainey, et al, 2011, Icarus

Prix du magasine « La Recherche » 2012

CONCLUSION

All the satellites inside Titan are coming from the rings, that had to be initialy very massive.

The rings themselves may well be coming from a differentiated satellite that migrated inside the Roche radius.

The system is still evolving as the rings spread !

Can we model this analytically ?

TIDES, RINGS, SATELLITES, Saturn et al...

Formation of Regular Satellites from Ancient Massive Rings in the Solar System

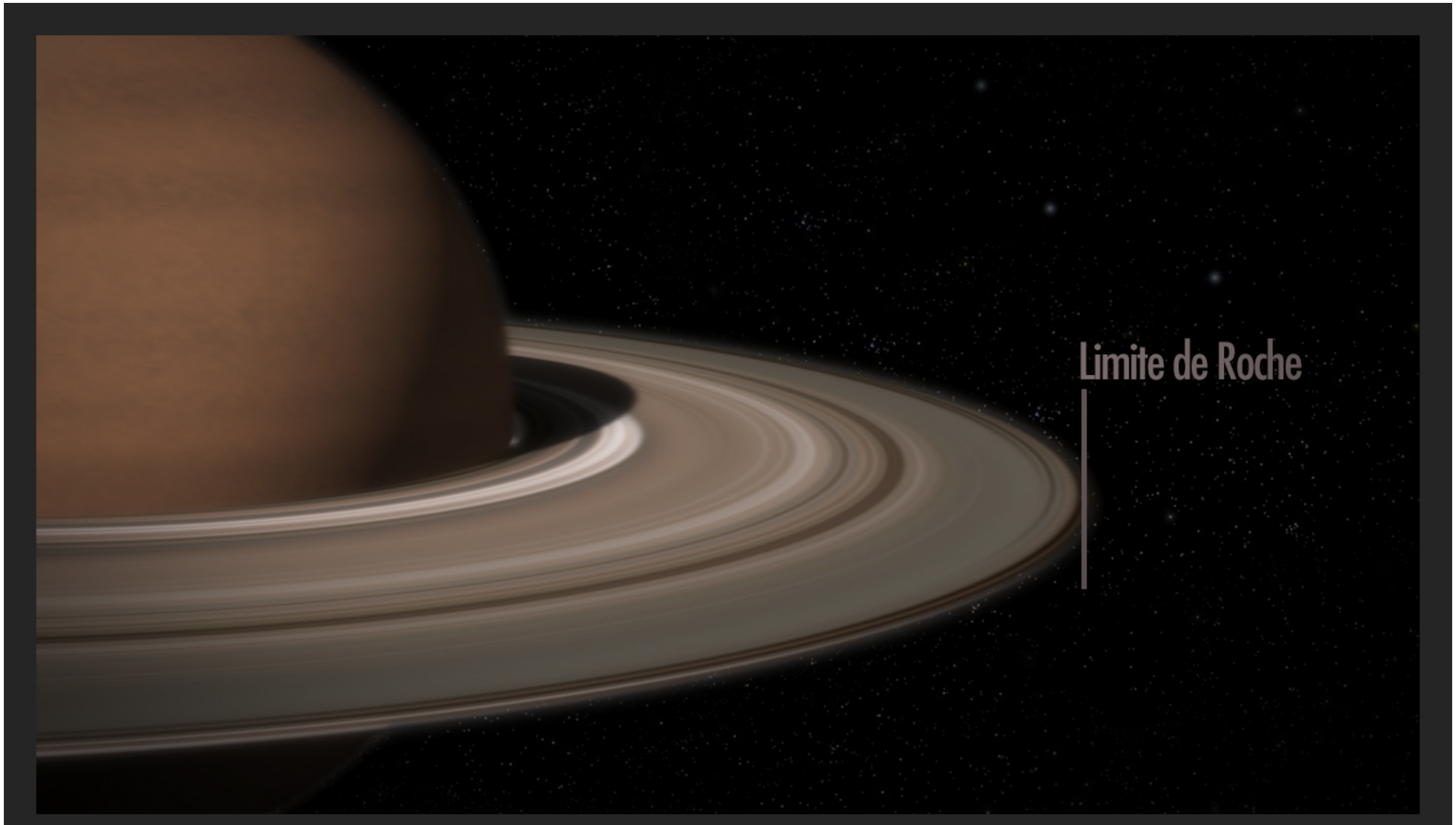
Crida & Charnoz (2012), *Science*, 338

http://arxiv.org/abs/1301.3808

Notations

Be T_R the orbital period at the Roche limit r_R ,
F the flow through r_R, and

$\tau_{disk} = M_{disk} / FT_R$, the normalized life-time of the disk.



Notations

Be T_R the orbital period at the Roche limit r_R ,
F the flow through r_R, and

$\tau_{disk} = M_{disk} / FT_R$, the normalized life-time of the disk.

The disk spreads with a viscous time $t_\nu = r_R^2/\nu$.

Using Daisaka et al. (2001)'s prescription for ν,
we find $\tau_{disk} = t_\nu / T_R = 0.0425\ D^{-2}$ where $D = M_{disk}/M_p$,

and $F = 23\ D^3\ M_p / T_R$.

Continuous regime

Say 1 satellite forms. Its mass is : $M = F\,t$ (2)

It feels a torque from the disk : $\Gamma = \frac{8}{27}\left(\frac{M}{M_p}\right)^2 \Sigma r^4 \Omega^2 \Delta^{-3}$ (1)

where $\Delta = (r-r_R)/r_R$.

→ Migration rate : $$\frac{d\Delta}{dt} = \frac{32}{27} q D T_R^{-1} \Delta^{-3}$$ (3)

where **q** = M / M_p .

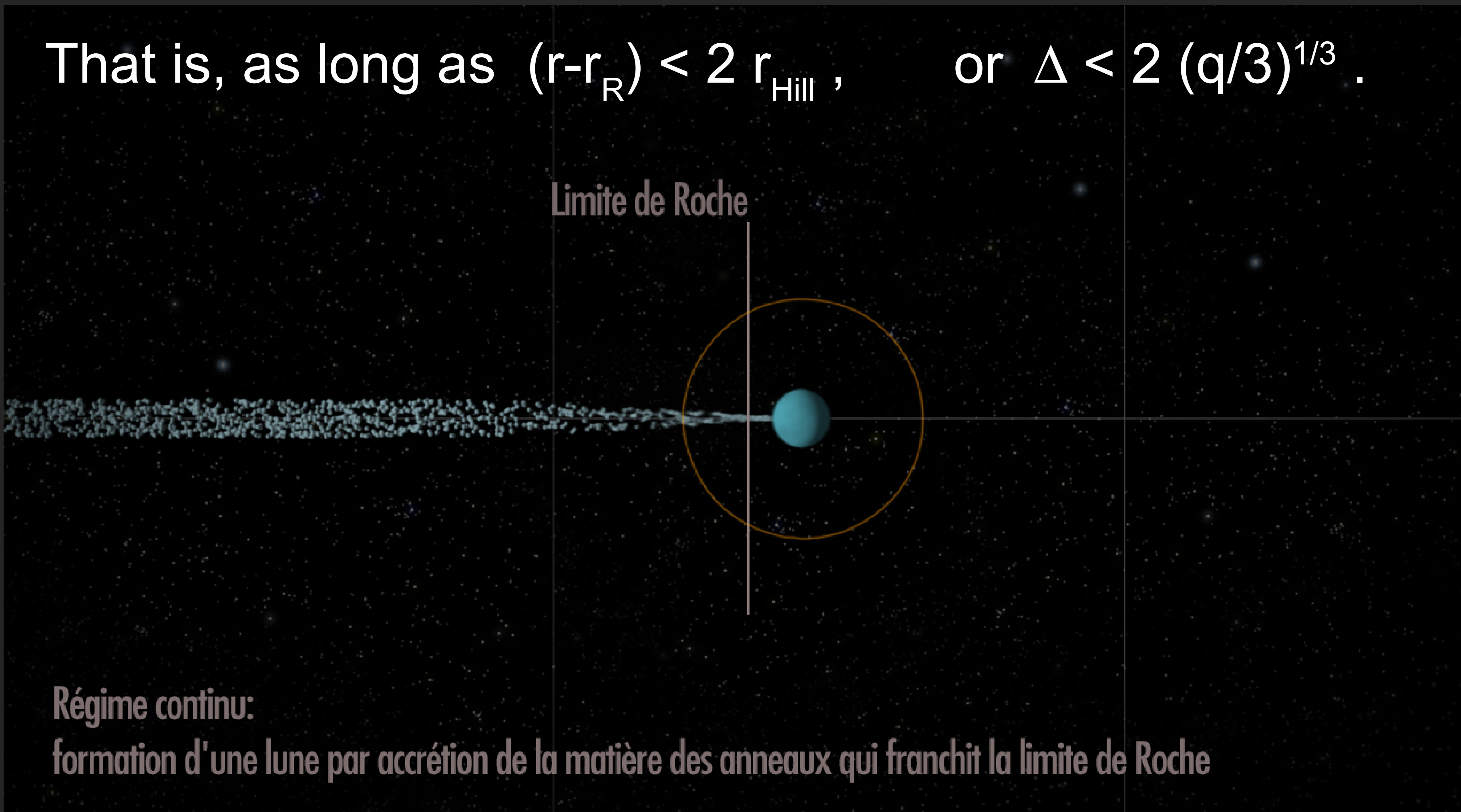
Solution of (2) & (3) : $$q = \left(\frac{\sqrt{3}}{2}\right)^3 \tau_{disk}^{-1/2} \Delta^2$$ (4)

We call this the *continuous regime* .

Continuous regime

This holds as long as the satellite captures immediately what comes through r_R.

That is, as long as $(r-r_R) < 2\, r_{Hill}$, or $\Delta < 2\,(q/3)^{1/3}$.



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Input into Eq.(4), this gives a condition of validity for the continuous regime :

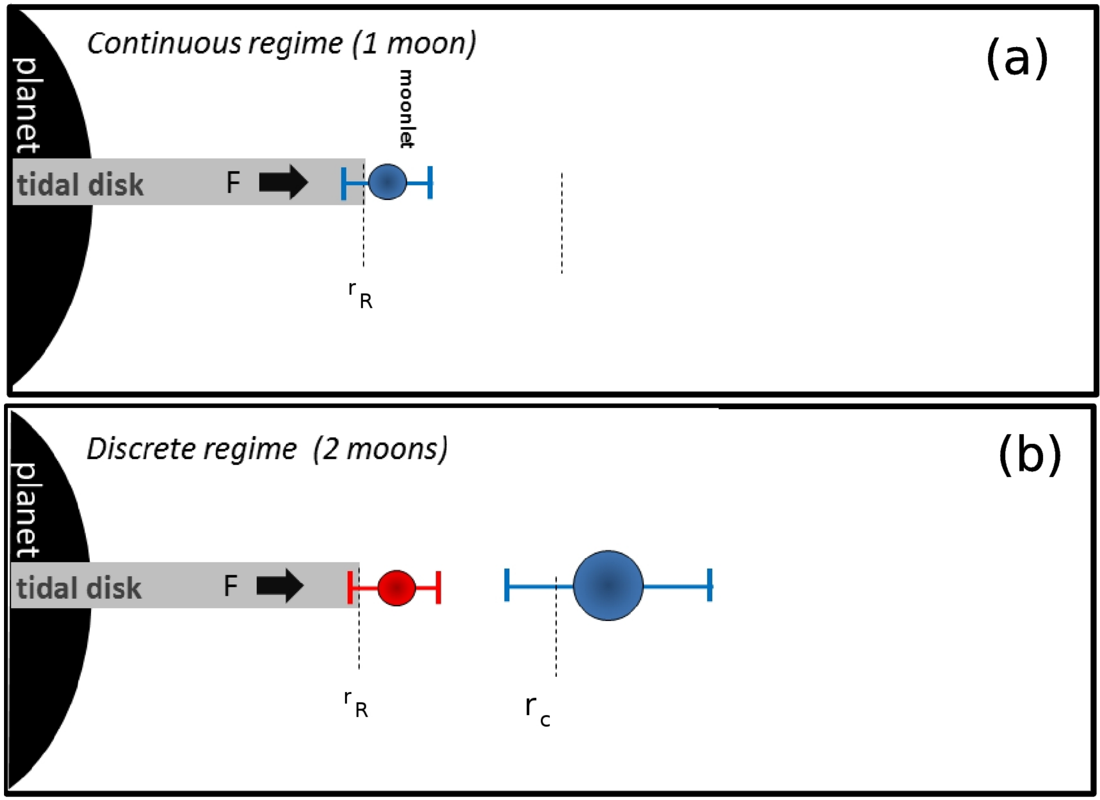
$$\Delta < \Delta_c = \sqrt{\frac{3}{\tau_{disk}}} = \sim 8.4\ D$$

$$q < q_c = \frac{3^{5/2}}{2^3} \tau_{disk}^{-3/2} = \sim 222\ D^3$$

Duration of the continuous regime: $10\ T_R$.

Discrete regime

When the satellite is beyond Δ_c (or q_c), the material flowing through r_R forms a new satellite at r_R.



This new satellite is immediately accreted by the first one.

And so on...

The first satellite still grows as $M=Ft$, but by steps : *discrete regime*.

Discrete regime

This holds as long as $\Delta < \Delta_c + 2(q/3)^{1/3}$.

It gives the condition :

$$\Delta < \Delta_d = 3.14\,\Delta_c \quad = \sim 26\ D$$

$$q < q_d = 9.9\,q_c \quad = \sim 2200\ D^3$$

The duration of the discrete regime is $\sim 100\ T_R$.

Discrete regime

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The duration of the discrete regime is ~100 T_R .

Application :

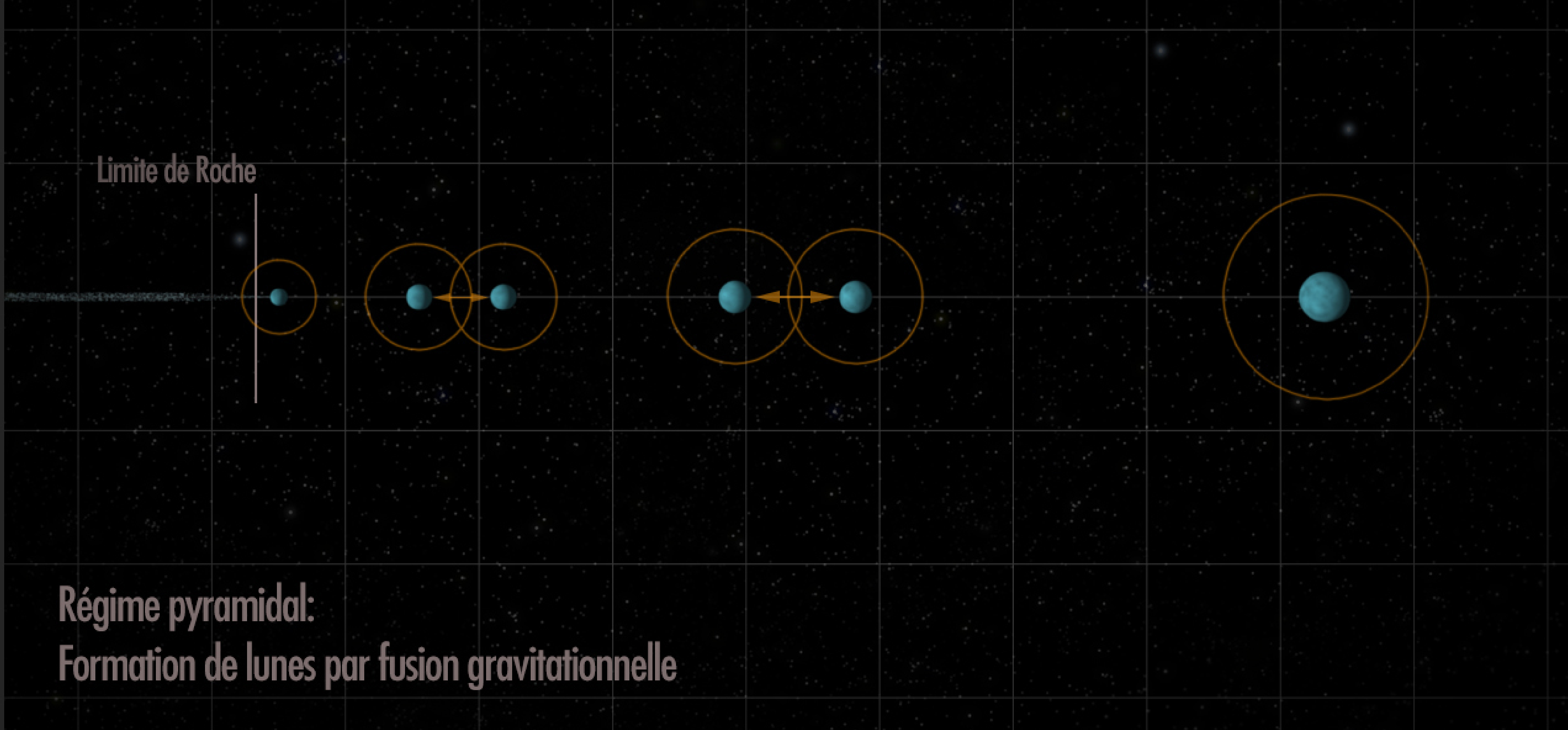
In Saturn's rings : $q_d = \sim 10^{-18}$

$M = \sim 10^9$ kg, ~100m sized bodies.

Pyramidal regime

Satellites of mass q_d are produced at Δ_d every q_d / F .

Then, many satellites of constant mass migrate outwards, at decreasing rates. They approach each other, and merge.



Pyramidal regime

Satellites of mass q_d are produced at Δ_d every q_d / F .

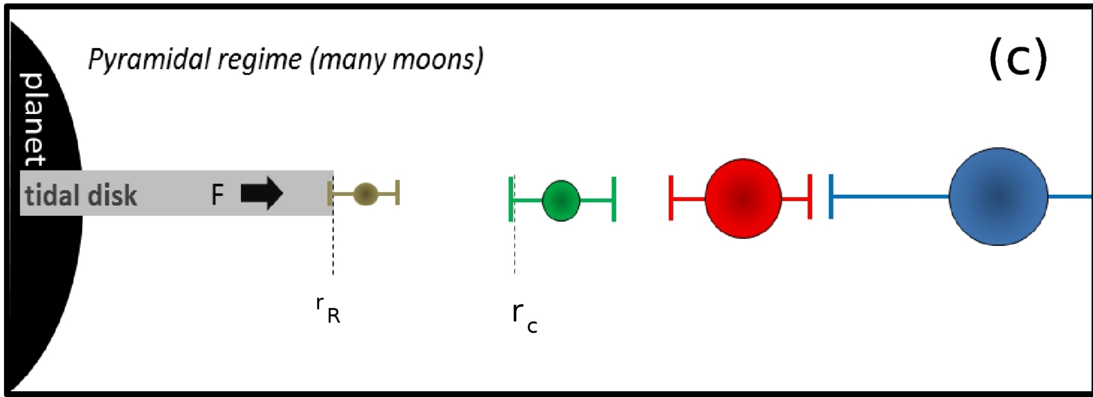
Then, many satellites of constant mass migrate outwards, at decreasing rates. They approach each other.

If their distance decreases below 2 mutual Hill radii, they merge.

=> Formation of satellites of masses $2q_d$, every $2\, q_d / F$, which migrate away and merge further...

And so on, hierachicaly...

We call this *the pyramidal regime*.



Pyramidal regime

- Using Eq.(3), we show that in the pyramidal regime, while the mass is doubled, Δ is multiplied by $2^{5/9}$.

Thus, $q \propto \Delta^{9/5}$.

In addition, the number density of satellites should be proportionnal to $1/\Delta$, explaining the pile-up.

Pyramidal regime

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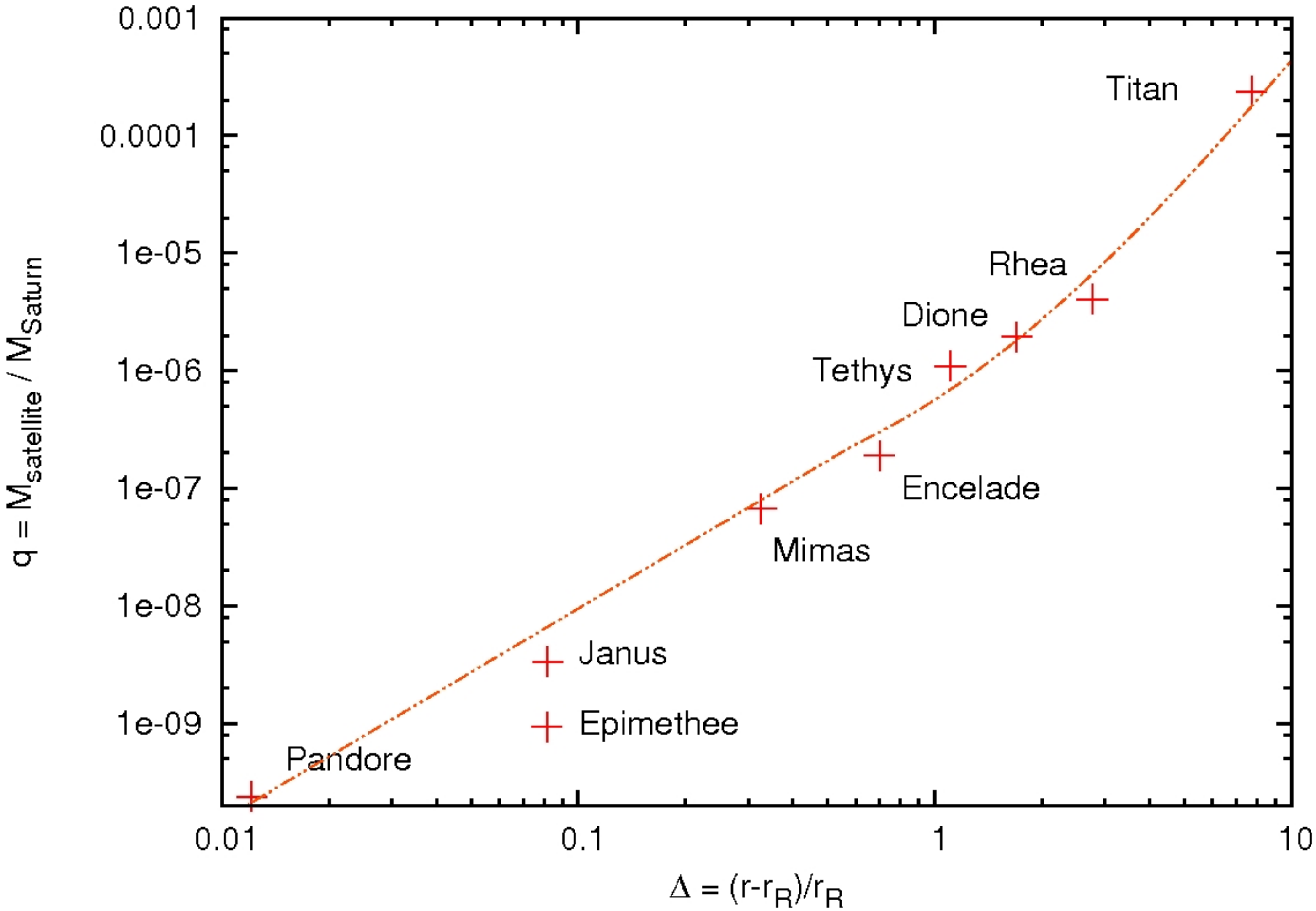
- Beyond the 2:1 Lindblad resonance with r_R (Δ=0.58), Eq.(3) doesn't apply. Migration is driven by planetary tides:

$$\frac{dr}{dt} = \frac{3 k_{2p} M \sqrt{G} R_p^5}{Q_p \sqrt{M_p} r^{11/2}} \quad (5)$$

Using Eq.(5), we find $q \propto r^{3.8}$.

Pyramidal regime

The result spectacularly matches the distribution of the Saturnian system !

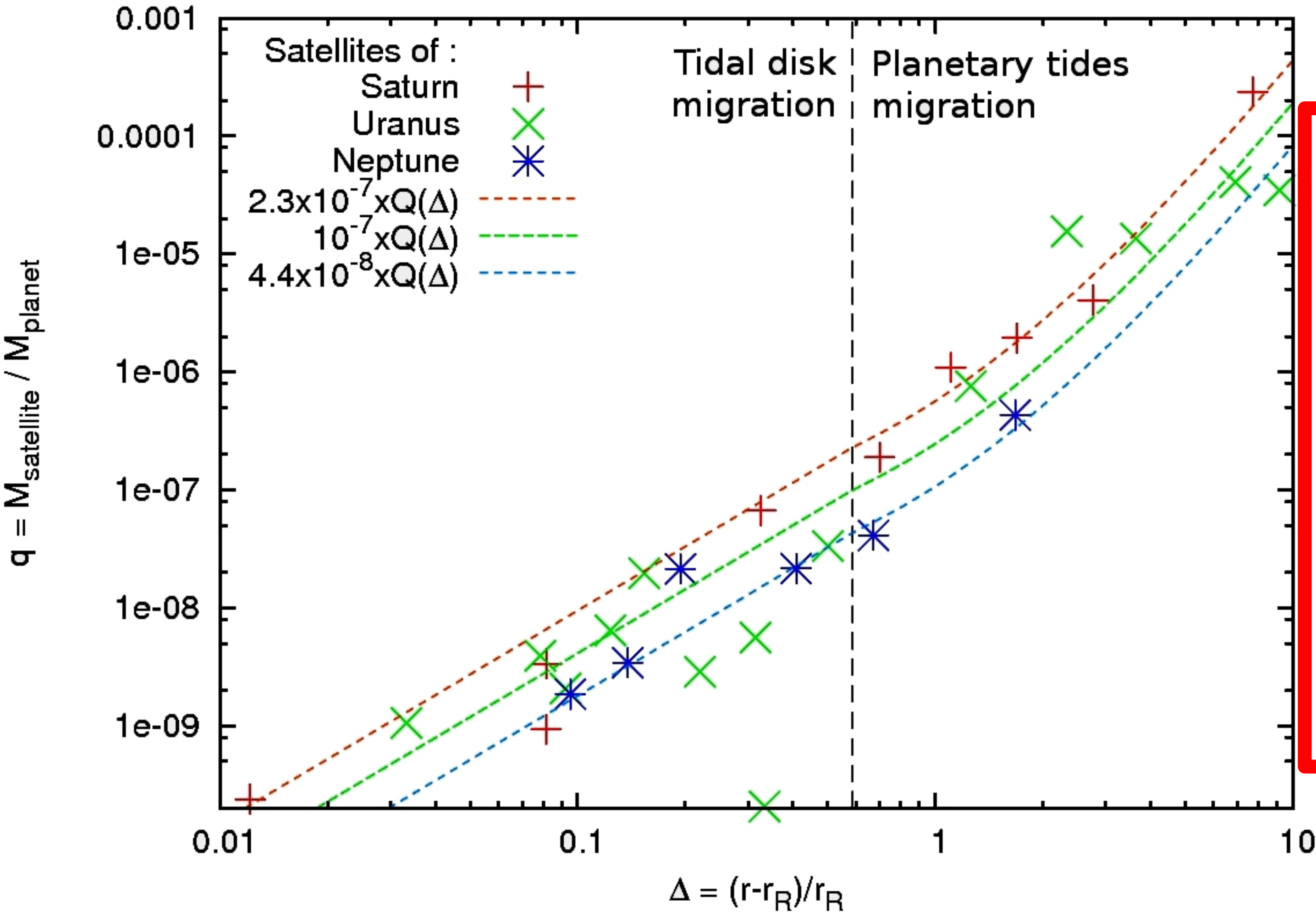


$\Delta<0.58$: $q \propto \Delta^{9/5}$

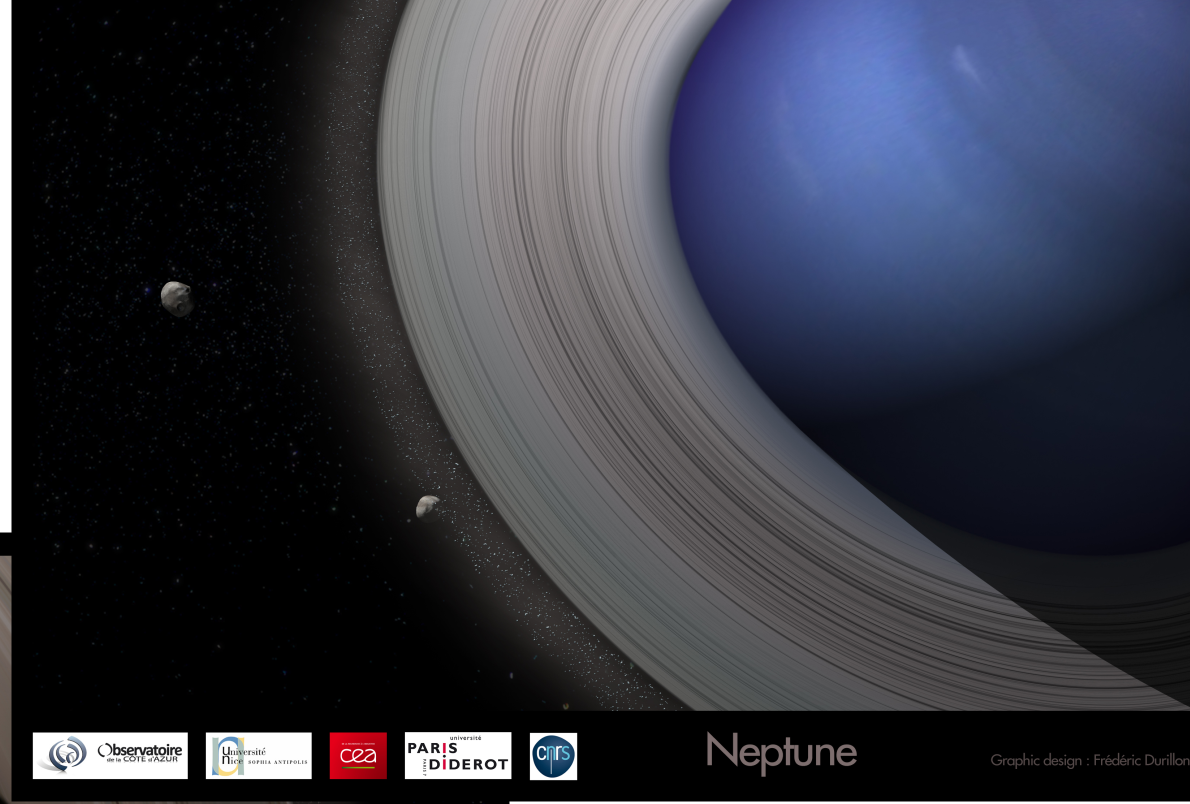
$\Delta>0.58$: $q \propto r^{3.8}$

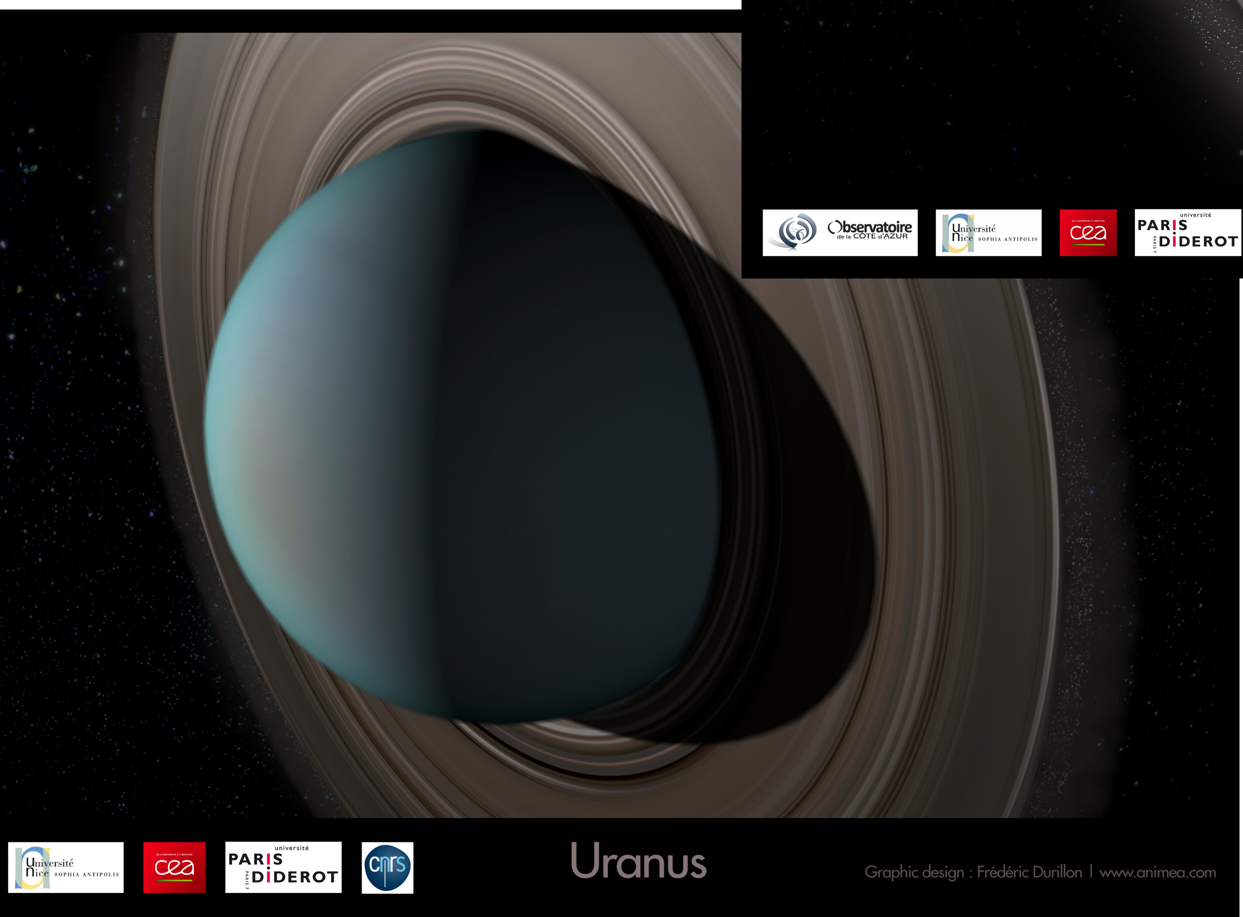
Pyramidal regime

The result spectacularly matches the distribution of the Saturnian, Uranian, and Neptunian systems !



I claim that Uranus and Neptune had massive rings, from which their regular satellites were born.





4 billion years ago ???

The return of the Discrete regime

The limit of the discrete regime is :

$$\Delta < \Delta_d = 3.14\,\Delta_c \quad = \sim 26\ D$$

$$q < q_d = 9.9\,q_c \quad = \sim 2200\ D^3$$

<u>Moon forming disk :</u> D=0.02, q_d=mass of the Moon =~D !

Only one satellite forms around the Earth, in agreement with observation. Possibly with a low mass companion crashing later => formation of the highlands.

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Only one satellite forms around the Earth, in agreement with observation. Possibly with a low mass companion crashing later => formation of the highlands.

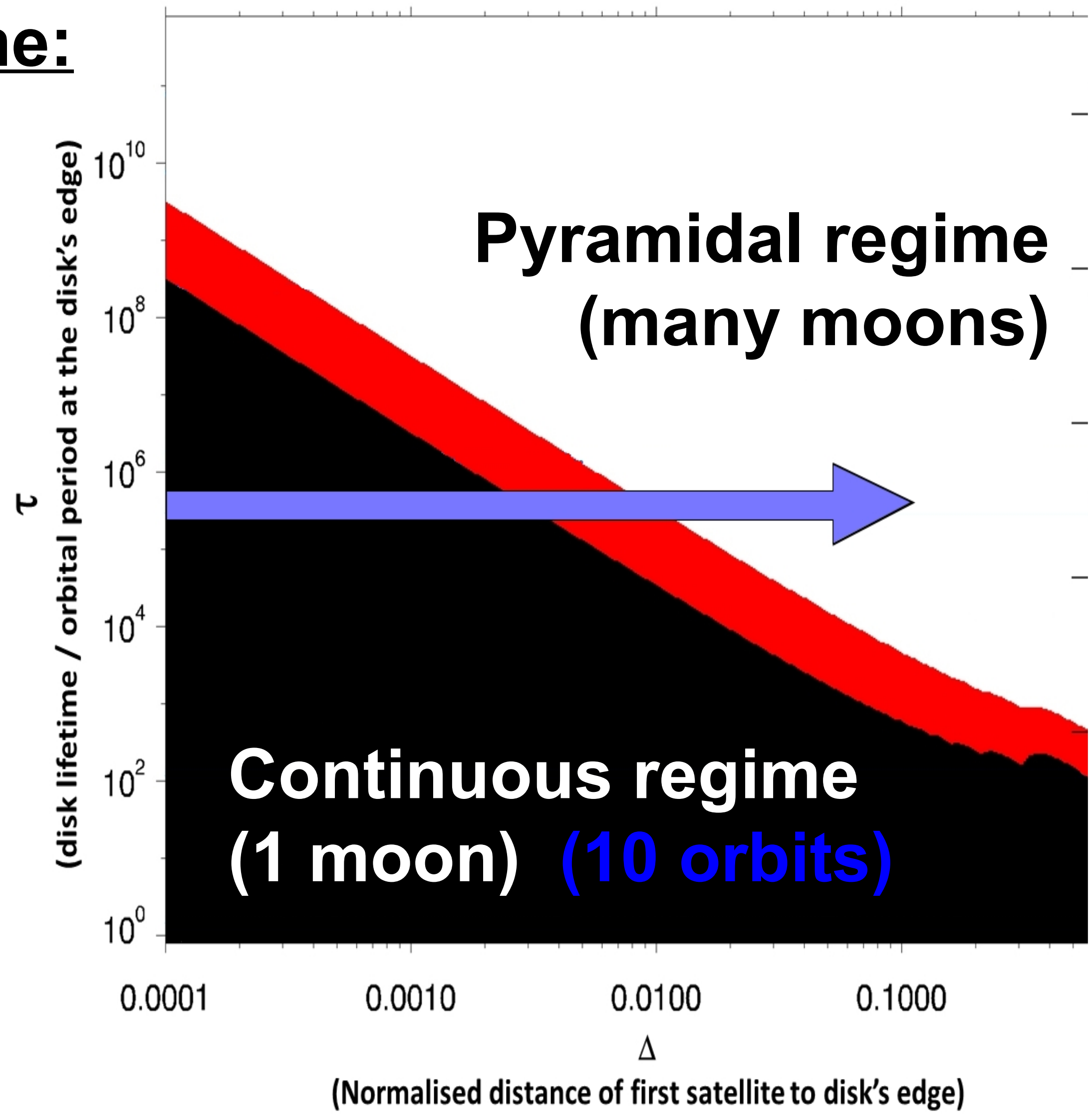
Charon forming disk : always in the continuous regime.

Summary

1) <u>Continuous regime:</u>
1 moon grows
$q \alpha \Delta^2$
until Δ_c or q_c.

2) <u>Discrete regime:</u>
2 moons,
growth by steps
until Δ_d or q_d .

3) Pyramidal regime:
Many moons in the system.
$q \alpha \Delta^{9/5}$ or $r^{3.8}$.

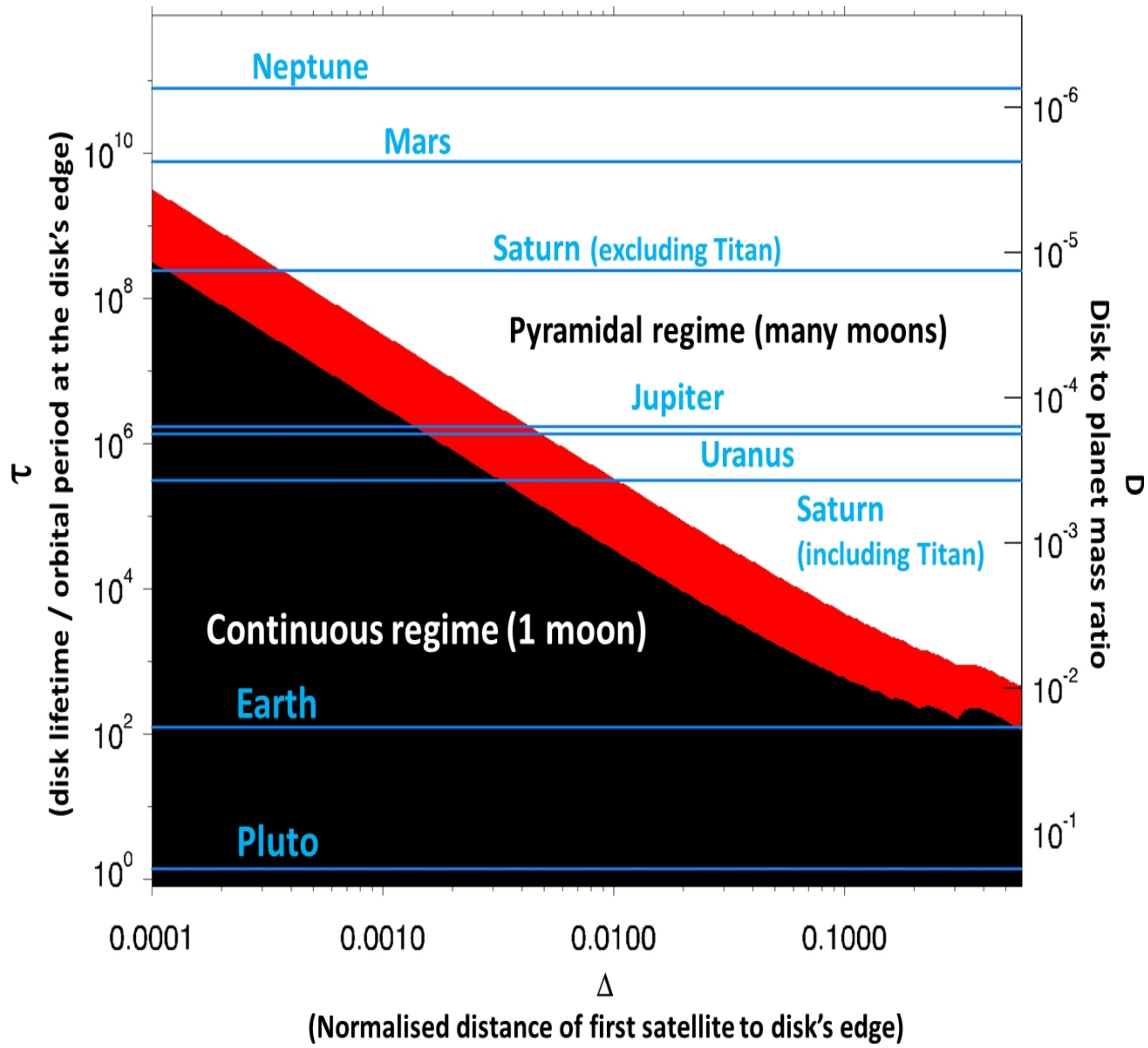


Summary

Take M_{disk} = 1.5 x the mass of the present satellite system.

Giant planets must be dominated by the pyramidal regime,

while we expect the Earth and Pluto to have 1 large satellite.



Summary

The spreading of a tidal disk beyond the Roche radius

- ✓ explains the mass-distance distribution of the regular satellites of the giant planets (observational signature of this process)
- ✓ unifies terrestrial and giant planets in the same paradigm.
- ✓ most Solar System regular satellites formed this way.

- ✗ Jupiter doesn't fit in this picture : probably formed in a circum-planetary disk (e.g. Canup & Ward 2002, 2006 ; Sasaki et al 2010)

- Titan fits very well in this picture, though its « tidal age » is too large... Coincidence ?

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avec Sébastien CHARNOZ

Université Nice SOPHIA ANTIPOLIS

LAGRANGE

Observatoire de la CÔTE d'AZUR