Radiation Terms

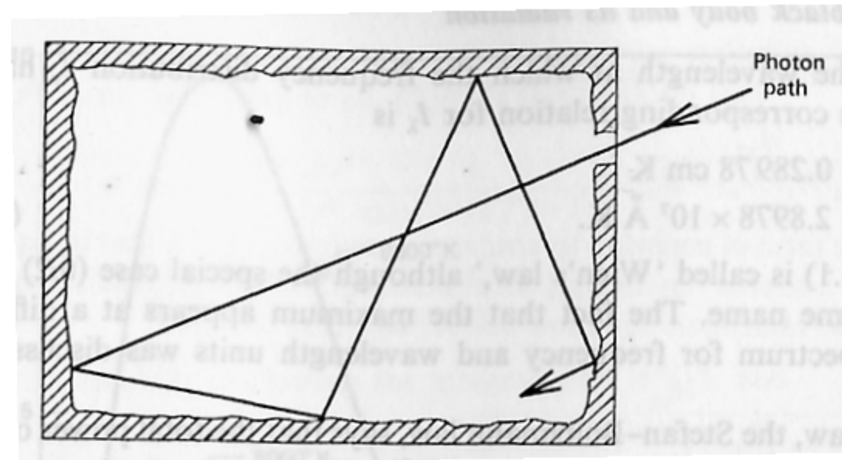
Black body radiation (Planck function)

Effective Temperature (Stefan-Boltzmann)

Specific and mean Intensity

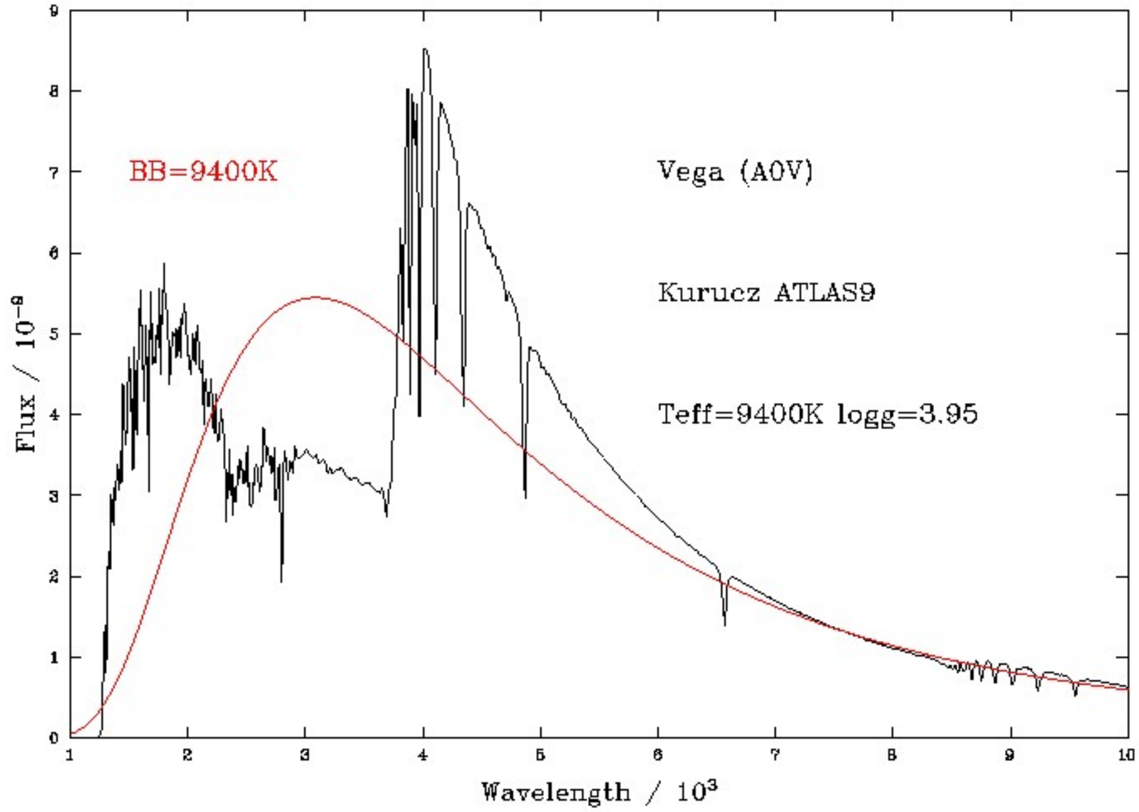
The Black Body

Imagine a box which is completely closed except for a small hole. Any light entering the box will have a very small likely hood of escaping & will eventually be absorbed by the gas or walls. For constant temperature walls, this is in thermodynamic equilibrium.



If this box is heated the walls will emit photons, filling the inside with radiation. A small fraction of the radiation will leak out of the hole, but so little that the gas within it remains in equilibrium. The emitted radiation is that of a black-body. Stars share properties of the black-body emitter, in the sense that a negligibly small fraction of the radiation escapes from each.

Vega (A0V)


BB=9400K
Vega (A0V)
Kurucz ATLAS9
Teff=9400K logg=3.95
Flux / 10^{-9}
Wavelength / 10^3
0
1
2
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10

Stefan – Boltzmann Law

Blackbody radiation is continuous and isotropic whose intensity varies only with wavelength and temperature. Following empirical (Josef Stefan in 1879) and theoretical (Ludwig Boltzmann in 1884) studies of black bodies, there is a well known relation between Flux and Temperature known as Stefan-Boltzmann law:

$$F=\sigma T^4$$

with $\sigma=5.6705\times10^{-5}$ erg/cm^2/s/K^4 (Note that Bohm-Vitense refers to `astronomical flux' which is defined as $F/\pi$ as `flux').

Effective temperatures of stars

Neglecting interstellar absorption, the total energy arriving above the Earth's atmosphere is its observed flux, f, corrected for the distance to the star, i.e. $L=4\pi d^2 f$

The same energy must be emitted by the star, i.e. $L=4\pi R^2 F$ where F is the surface flux, so $F=f(d/R)^2$. For the Sun, the angular radius is 959.6 arcsec and f=1.367x10^6 erg/cm^2/s so radiant flux at surface is F=6.317x10^{10} erg/cm^2/s.

The Stefan-Boltzmann law, $F=\sigma T^4_{eff}$, or alternatively $L/(4\pi R^2)=\sigma T^4_{eff}$ defines the `effective temperature' of a star, i.e. *the temperature which a black body would need to radiate the same amount of energy as the star, which is 5777K for the Sun.*

Planck formula

The black body intensity is defined (following discovery by Max Planck in 1900) as either

$$B_\lambda(T) = \frac{2hc^2}{\lambda^5}\frac{1}{e^{hc/\lambda kT}-1} \quad \text{or} \quad B_\nu(T) = \frac{2h\nu^3}{c^2}\frac{1}{e^{h\nu/kT}-1}$$

where c=2.99x10^{10} cm, h=6.57x20^{-27} erg s, k=1.38x10^{-16} erg/s. Using cgs units (λ in Angstroms) we have

$$B_\lambda(T) = \frac{1.19x10^{27}/\lambda^5}{e^{1.44x10^8/\lambda T}-1}$$

Wien's displacement law

For increasing temperatures, the black body intensity increases for all wavelengths. The maximum in the energy distribution shifts to shorter λ (longer ν) for higher temperatures.

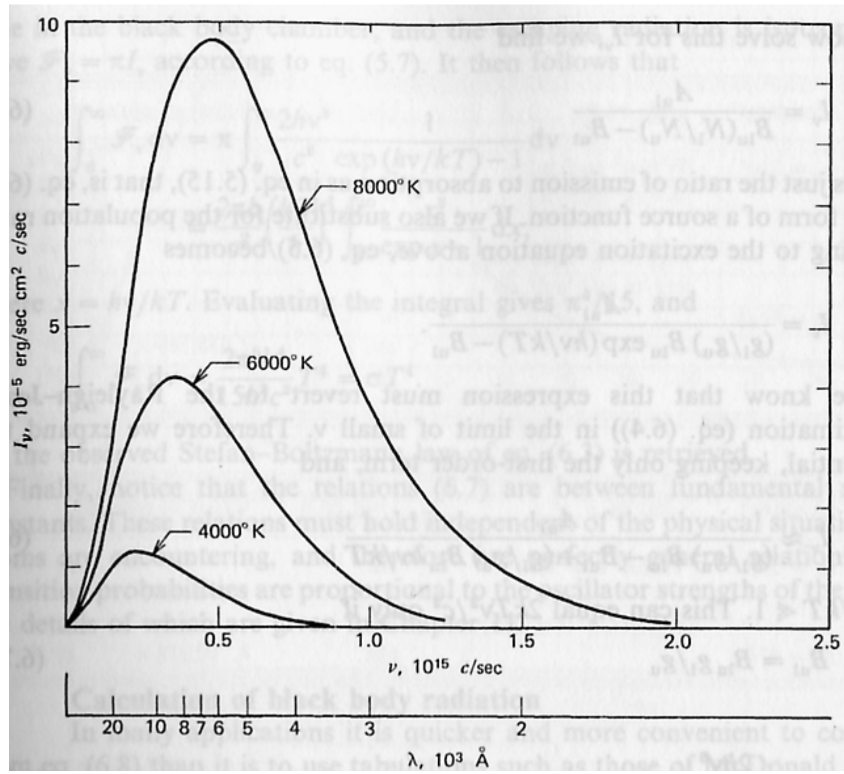
$\lambda_{max} T = 2.8978 \times 10^7$ Ang K

is *Wien's law* for the maximum I_{λ},

Alternatively

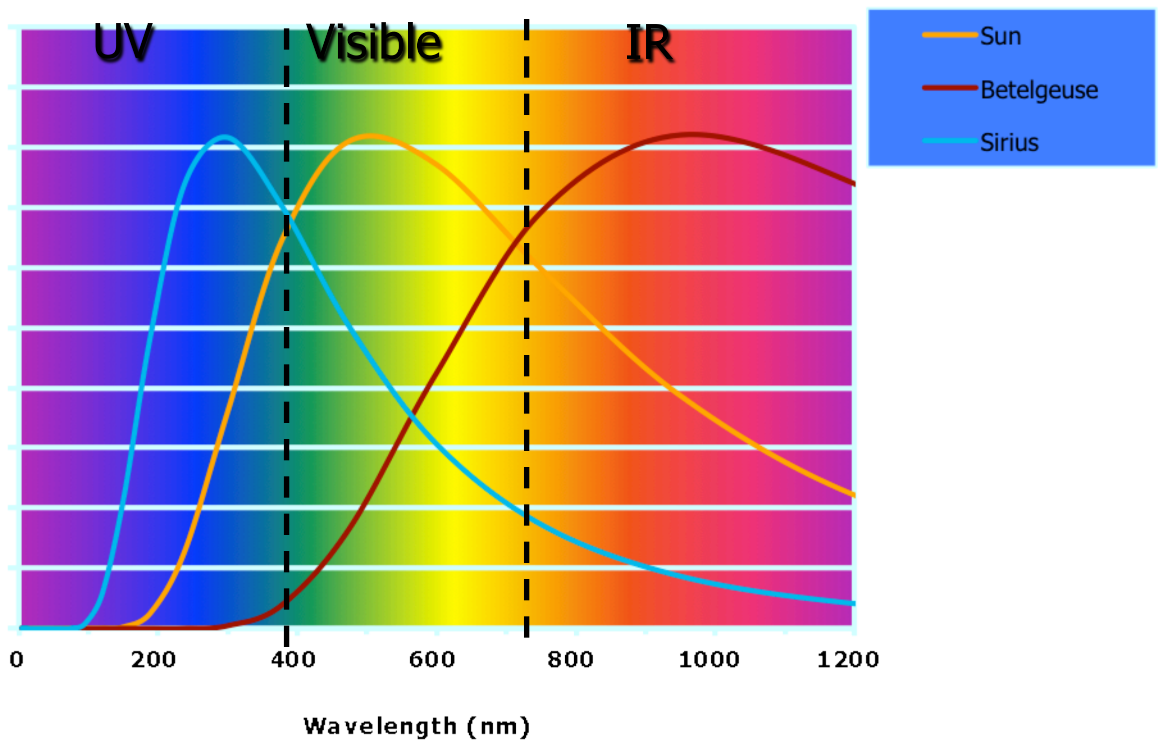
$\lambda'_{max} T = 5.0995 \times 10^7$ Ang K
Is Wien's law for the maximum I_{ν}

At long wavelengths, well beyond the peak in energy output, the Rayleigh-Jeans approximation is:
$I_{\nu} = 2kT\nu^2/c^2 = 2kT/\lambda^2$



Colour Corrections

- Examples of spectra



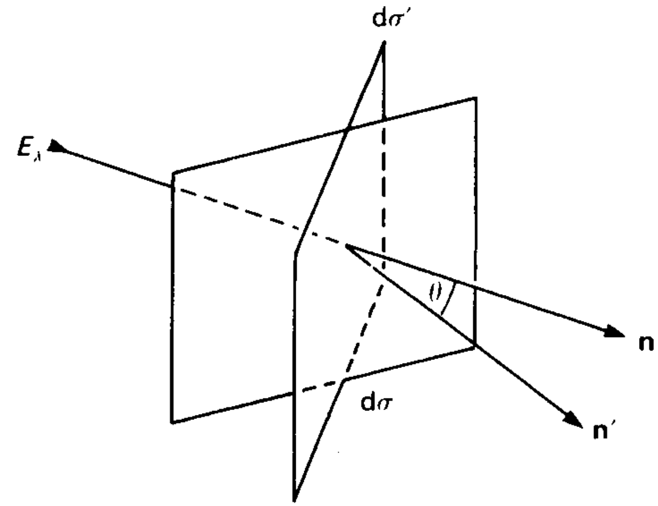
Example

The LMC B0 dwarf star discussed in the previous lecture was found to have a luminosity of $\log(L/L_\odot)=5.5$. What is its radius? At what wavelength (in Angstrom) does its radiation peak (assuming Wien's law)?

1. From Astrophysical Quantities, T_{eff}(B0V)=30,000K. Using Stefan-Boltzmann law defined above R=20.8 $R_\odot$
2. From above, λ_{max} T = 2.98978 x10^7 Ang K, so λ_{max} = 997 Angstrom in the far-UV (inaccessible except using FUSE satellite) although the Spectral Energy Distributions for such stars deviate from blackbodies.

Specific Intensity

Consider light passing through a surface area $d\sigma$ (at an angle θ with respect to the normal) in a narrow cone of opening solid angle $d\omega$. Intensity relates to energy via



$$E_\lambda = I_\lambda \cos\theta \, d\lambda \, d\sigma \, d\omega$$

The (specific) intensity is then a measure of brightness with units of erg/(s cm^2 rad^2 A). In model atmosphere calculations, I_λ is obtained from the `transfer equation' – which we shall introduce later on. I_λ is proportional to energy/solid angle, and so is independent of distance.

Mean Intensity and Flux

One can alternatively define intensity in frequency units such that

$$I_\lambda d\lambda = I_\nu d\nu$$

The two spectral distributions have different shapes for the same spectrum. The Solar Spectrum has a maximum in the green in I_λ (5000A) but the maximum is in the far-red (8800A) for I_ν.

Why?

$c=\lambda\nu$, $d\nu/d\lambda=-c/\lambda^2$,so equal intervals of λ correspond to different intervals of ν across the spectrum).

The *mean* intensity J_λ is the directional average (over 4π steradians) of the specific intensity, whilst the flux, $F_{\lambda,}$ is its projection in the radial direction, integrated over all solid angles. There is also a K-integral which we will use later on.

$$J_\lambda = \frac{1}{4\pi}\oint I_\lambda d\omega \qquad F_\lambda = \oint I_\lambda \cos\theta d\omega \qquad K_\nu = \oint I_\nu \cos^2\theta d\omega$$

A simple example

Let us consider a point on the boundary of a radiating sphere. From above,

$$F_\nu = \int_0^{2\pi} d\phi \int_0^{2\pi} I_\nu \sin\theta \cos\theta d\theta$$

If no flux enters the surface and if there is no azimuthal dependence for F_ν

$$F_\nu = 2\pi \int_0^{\pi/2} I_\nu \sin\theta \cos\theta d\theta$$

Finally, if I_ν is independent of direction over one hemisphere then $F_\nu=\pi I_\nu$ and $J_\nu = ½ I_\nu$. For the Sun, $I=2 \times 10^{10}$ ergs s^{-1} cm^{-2} $ster^{-1}$ and $J=1 \times 10^{10}$ ergs s^{-1} cm^{-2} $ster^{-1}$ (Note these are Bolometric quantities.)

<u>Important:</u> I is independent of distance from the source, and can only be measured directly if we resolve the radiating surface. In contrast, F obeys the inverse square law and is all that may be measured for most stars.

Radiative Transfer I

Voir Cours de Marianne Faurobert

Optical depth

Source function

Local Thermodynamic Equilibrium

Mean Free Path

- In the Sun, the characteristic distance over which the temperature varies (the Temperature scale height) is ~500km. How does this compare with the average distance travelled by an atom before hitting another atom?
- The density of the Solar photosphere is $\rho=2.5\text{x}10^{-7}$ g/cm^3 so the number of H atoms/cm^3 is $n(H)=\rho/m_H=1.5\text{x}10^{17}$ cm^3 where m_H is the mass of the H atom. Two atoms will collide if their centres pass within a radius of 2 Bohr radii ($2a_o$) of each other. The collision cross-section of the atom is $\sigma=\pi(2a_o)^2=3.5\text{x}10^{-16}$ cm$^{2.}$
- The mean free path between collisions is $1/\sigma$ x $1/n(H)=0.02$ cm. The atoms are confined within a limited volume of space in the photosphere at effectively fixed T (relative to the T scale height).
- In contrast, since the photosphere is the layer visible from Earth, photons must be able to escape freely into space, after 10^{21} scatterings and re-emissions (over 5000 yr!) from the centre.
- We now turn to the interaction between photons and particles in stellar atmospheres.

Optical Depth

- Consider radiation shining through a layer of material. The intensity of light is found experimentally to decrease by an amount dI_λ where

$$dI_\lambda = -\kappa_\lambda \rho I_\lambda ds$$

Here ρ is the density, and d*s* is a length and the so-called (mass) absorption coefficient (alias opacity) is κ. The photon mean free path is inversely proportional to $\kappa_\lambda \rho$

- Two physical processes contribute to κ; (i) true absorption where the photon is destroyed and the energy thermalized; (ii) scattering where the photon is deviated in direction and removed from the solid angle under consideration.
- The radiation sees a combination of κ ρ and ds over some path length, L given by a dimensionless quantity, the `optical depth'

$$\tau_\lambda = \int_0^L \kappa_\lambda \rho ds$$

Optical depth II

We can write the change in specific intensity over a path length as

$$dI_\lambda = -I_\lambda d\tau_\lambda$$

using the definition of optical depth. This may be directly integrated to give the usual extinction law:

$$I_\lambda(\tau_\lambda) = I_\lambda(0)e^{-\tau_\lambda}$$

An optical depth of zero corresponds to no extinction (i.e. top of photosphere for a star), whilst an optical depth of τ=1 corresponds to a reduction in intensity by a factor of e=2.7. If the optical depth is large (τ >>1) negligible intensity reaches the observer, In stellar atmospheres, typical photons originate from τ=2/3 (proof later).

Source function

We can also treat emission processes in the same way as absorption via an emission coefficient, $j_{\lambda,}$ with units of erg/s/rad^2/Hz/g.

$$dI_\lambda = j_\lambda \rho ds$$

Physical processes contributing to $j_{\lambda,}$ are (i) real emission – the creation of photons; (ii) scattering of photons into the direction being considered. The ratio of emission to absorption is called the Source function,

$$S_\lambda = j_\lambda / \kappa_\lambda$$

Einstein coefficients

Consider a simple 2-level atom with lower level l (with N_l ground-state atoms) and upper level u (with N_u excited atoms), energy separation $h\nu$. If A_{ul} is the Einstein probability coefficient for *spontaneous* emission (which is isotropic) of radiation from u to l:

$$j_\nu \rho = N_u A_{ul} h\nu.$$

Additional, *stimulated* emission (non-isotropic) is induced if a radiation field is present. The probability for stimulated emission producing a photon of energy $h\nu$ is $B_{ul} I_\nu$, where B_{ul} is the Einstein probability coefficient for stimulated emission. Finally, the Einstein probability coefficient for absorption of photons of energy $h\nu$ is called B_{lu}

$$\kappa_\nu \rho I_\nu = N_l (B_{lu} I_\nu) h\nu - N_u (B_{ul} I_\nu) h\nu$$

The source function, $S_\nu = j_\nu/\kappa_\nu = N_u A_{ul}/(N_l B_{lu} - N_u B_{ul})$

Local Thermodynamic Equil.

In the study of stellar atmospheres, the assumption of Local Thermodynamic Equilibrium (LTE) is described by:

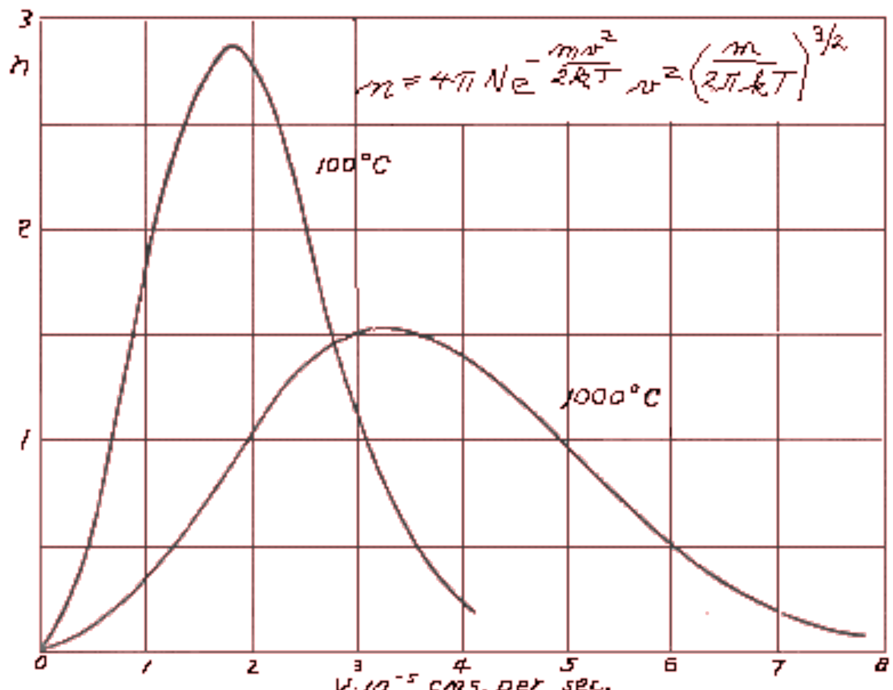
- Electron and ion velocity distributions are Maxwellian

$$f(v)=4\pi v^{2}(m/2\pi kT)^{1.5}e^{-mv^2/2kT}$$

- Excitation equilibrium given by Boltzmann equation
- Ionization equilibrium given by Saha equation (introduced later)
- The source function is given by the Planck function.

$S_\lambda = I_\lambda = B_\lambda(T)$ i,e. Kirchoff's law $j_\lambda = \kappa_\lambda B_\lambda(T)$

Maxwellian velocity distribution



Maxwellian distribution of velocities (from J. M. Cork "Heat")

Is LTE a valid assumption?

For LTE to be valid, the photon and particle mean free paths need to be much smaller than the length scale over which these temperature changes significantly.

Generally, when collisional processes dominate over radiative processes in the excitation and ionization of atoms, the state of the gas is close to LTE.

Consequently, *LTE is a good assumption in stellar interiors, but may break down in the atmosphere*. If LTE is no longer valid, all processes need to be calculated in detail via non-LTE. This is much more complicated, but needs to be considered in some cases (see much later).